International Journal of Research
Available at https://edupediapublications.org/journals
p-ISSN: 2348-6848
e-ISSN: 2348-795X
Volume 03 Issue 01
January 2015

A Study of an Integral Related to the Logarithmic Function with Maple

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Abstract

This paper studies the integral problem related to the logarithmic function. We can determine the analytic form of this type of integral mainly using the integration term by term theorem. Moreover, two examples are proposed to do a calculation practically. The research methods adopted in this paper is to find solutions through manual calculations and verify the answers using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations.

Key Words: integral; logarithmic function; analytic form; integration term by term theorem; Maple

1. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research.

In calculus and engineering mathematics, there are many methods to solve the integral problems, for example, change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. This paper considers the following some type of indefinite integral related to the logarithmic function, which are not easy to obtain their answers using the methods mentioned above.

$$\int \frac{x^a}{(\ln x)^r} dx, \tag{1}$$

where a, r, x are real numbers, $r > 0$, $x > 0$, and $x \neq 1$. The analytic form of this type of indefinite integral can be obtained by using integration term by term theorem; this is the major result of this study (i.e., Theorem A). Adams et al. [1], Nyblom [2], and Oster [3] provided some techniques to solve the integral problems. Moreover, Yu [4-27], Yu and Chen [28], and Yu and Sheu [29-31] used complex power series method, integration term by term theorem, Parseval's theorem, area mean value theorem, and generalized Cauchy integral formula to evaluate some types of integral problems. In this paper, we propose two examples to demonstrate the manual calculations, and verify the results using Maple.

2. Preliminaries and Main Result

First, an important theorem used in this paper is introduced below which can be found in ([32, p269]).

2.1 Integration term by term theorem:

Suppose that $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty}\int_I g_n$.

In the following, we obtain the analytic form of the indefinite integral (1).

Theorem A *Suppose that* a, r, x *are real numbers,* $r > 0$, $x > 0$, $x \neq 1$, *and* C *is a constant.*

Case 1. If r *is not a positive integer, then*

$$\int \frac{x^a}{(\ln x)^r}dx = \sum_{k=0}^{\infty}\frac{(a+1)^k}{k!(k-r+1)}(\ln x)^{k-r+1} + C. \tag{2}$$

Case 2. If r *is a positive integer and* $x > 1$, *then*

$$\int \frac{x^a}{(\ln x)^r}dx = \sum_{\substack{k=0 \\ k\neq r-1}}^{\infty}\frac{(a+1)^k}{k!(k-r+1)}(\ln x)^{k-r+1} + \frac{(a+1)^{r-1}}{(r-1)!}\ln(\ln x) + C. \tag{3}$$

Proof *Case 1.* If r is not a positive integer, then

$$\int \frac{x^a}{(\ln x)^r}dx,$$

$$= \int \frac{e^{at}}{t^r}e^t dt \quad (\text{let } x = e^t)$$

$$= \int t^{-r}e^{(a+1)t}dt$$

$$= \int \sum_{k=0}^{\infty}\frac{(a+1)^k}{k!}t^{k-r}dt$$

$$= \sum_{k=0}^{\infty}\frac{(a+1)^k}{k!}\int t^{k-r}dt \tag{4}$$

(by integration term by term theorem)

$$= \sum_{k=0}^{\infty}\frac{(a+1)^k}{k!(k-r+1)}t^{k-r+1} + C$$

$$= \sum_{k=0}^{\infty}\frac{(a+1)^k}{k!(k-r+1)}(\ln x)^{k-r+1} + C.$$

Case 2. If r is a positive integer, then using Eq. (4) yields

$$\int \frac{x^a}{(\ln x)^r}dx$$

$$= \sum_{\substack{k=0 \\ k\neq r-1}}^{\infty}\frac{(a+1)^k}{k!(k-r+1)}t^{k-r+1} + \frac{(a+1)^{r-1}}{(r-1)!}\ln t + C$$

$$= \sum_{\substack{k=0 \\ k\neq r-1}}^{\infty}\frac{(a+1)^k}{k!(k-r+1)}(\ln x)^{k-r+1} + \frac{(a+1)^{r-1}}{(r-1)!}\ln(\ln x) + C.$$

q.e.d.

3. Example

In the following, for the indefinite integral in this paper, we provide two examples and use Theorem A to determine their analytic forms. Additionally, Maple is used to calculate the approximations of some definite integrals and their solutions to verify our answers.

Example 3.1
By Eq. (2), we obtain

$$\int \frac{x^{2/3}}{(\ln x)^{6/7}}dx = \sum_{k=0}^{\infty} \frac{(5/3)^k}{k!(k+1/7)}(\ln x)^{k+1/7} + C, \tag{5}$$

for all $x > 0$, and $x \neq 1$.
Therefore, the definite integral

$$\int_{1/4}^{1/2} \frac{x^{2/3}}{(\ln x)^{6/7}}dx$$

$$= \sum_{k=0}^{\infty} \frac{(5/3)^k}{k!(k+1/7)}\left[\left(\ln\frac{1}{2}\right)^{k+1/7} - \left(\ln\frac{1}{4}\right)^{k+1/7}\right]. \tag{6}$$

Next, we use Maple to verify the correctness of Eq. (6).

>evalf(int(x^(2/3)/(ln(x))^(6/7),x=1/4..1/2), 22);

-0.1229745279043421480677

>evalf(sum((5/3)^k/(k!*(k+1/7))*((ln(1/2))^(k+1/7)-(ln(1/4))^(k+1/7)),k=0..infinity),22);

-0.1229745279043421480066

Example 3.2
Using Eq. (3) yields

$$\int \frac{x^{8/5}}{(\ln x)^4}dx = \sum_{\substack{k=0\\k\neq 3}}^{\infty} \frac{(13/5)^k}{k!(k-3)}(\ln x)^{k-3} + \frac{(13/5)^3}{6}\ln(\ln x) + C, \tag{7}$$

for all $x > 1$.
Thus, we have

$$\int_2^8 \frac{x^{8/5}}{(\ln x)^4}dx = \sum_{\substack{k=0\\k\neq 3}}^{\infty} \frac{(13/5)^k}{k!(k-3)}[(\ln 8)^{k-3} - (\ln 2)^{k-3}] + \frac{(13/5)^3}{6}[\ln(\ln 8) - \ln(\ln 2)]. \tag{8}$$

We also use Maple to verify the correctness of Eq. (8).

>evalf(int(x^(8/5)/(ln(x))^4,x=2..8),22);

17.11575989819930118353

>evalf(sum((13/5)^k/(k!*(k-3))*((ln(8))^(k-3)-(ln(2))^(k-3)),k=0..2)+sum((13/5)^k/(k!*(k-3))*((ln(8))^(k-3)-(ln(2))^(k-3)),k=4..infinity)+(13/5)^3/6*(ln(ln(8))-ln(ln(2))),22);

17.11575989819930118361

4. Conclusion

In this study, we use integration term by term theorem to study an integral problem related to the logarithmic function. In fact, the applications of this theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and use Maple to verify our answers.

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p-ISSN: 2348-6848
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