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Application of Complex Function Theory on Double Integral Problems

Chii-Huei Yu
Department of Information Technology, Nan Jeon University of Science and Technology, Tainan City, Taiwan
E-mail: chiihuei@mail.nju.edu.tw

Abstract

In this article, we study two types of double integrals. We can obtain the infinite series expressions of these double integrals using complex function theory. On the other hand, two examples are proposed to demonstrate the calculations, and we verify their answers using Maple.

Key Words: double integrals; infinite series expressions; complex function theory; Maple

1. Introduction

In calculus and engineering mathematics curricula, determining the surface area, the volume under a surface, and the center of mass of a lamina requires using double integrals. Therefore, both the evaluations and numerical calculations of double integrals possess significance. The study of related integral problems can refer to [1-14]. This paper considers the following two types of double integrals

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\exp(r^n\cos n\theta)\cos(m\theta+r^n\sin n\theta)drd\theta, \tag{1}$$

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\exp(r^n\cos n\theta)\sin(m\theta+r^n\sin n\theta)drd\theta, \tag{2}$$

where $r_1, r_2, \theta_1, \theta_2$ are real numbers, and m, n are positive integers. We can obtain the infinite series expressions of the double integrals (1) and (2) using complex function theory; these are the main results of this study (i.e., Theorems 1 and 2). On the other hand, we propose two double integrals to do calculation practically. The research methods adopted in this study is to find the solutions through manual calculations and verify these solutions using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

2. Preliminaries and Results

Firstly, we introduce some formulas and theorems used in this paper.

2.1 Formulas and theorems:

2.1.1 Euler's formula:

$e^{i\theta}=\cos\theta+i\sin\theta$, where θ is any real number.

2.1.2 DeMoivre's formula:

$(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$, where n is an integer, and θ is a real number.

2.1.3 Integration term by term ([15, p269]):

Assume that $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty}\int_I g_n$.

Before the major results can be derived, a lemma is needed, which is the complex function theory used in this paper.

Lemma *Suppose that z is a complex number, and m,n are positive integers, then*

$$z^m \exp(z^n) = \sum_{k=0}^{\infty}\frac{z^{nk+m}}{k!}. \tag{3}$$

Proof $z^m \exp(z^n)$

$$= z^m \sum_{k=0}^{\infty}\frac{(z^n)^k}{k!}$$

$$= \sum_{k=0}^{\infty}\frac{z^{nk+m}}{k!}. \qquad \text{q.e.d.}$$

In the following, the infinite series expressions of the double integral (1) can be obtained.

Theorem 1 *Assume that $r_1, r_2, \theta_1, \theta_2$ are real numbers, and m,n are positive integers, then the double integral*

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\exp(r^n\cos n\theta)\cos(m\theta + r^n \sin n\theta)drd\theta$$
$$= \sum_{k=0}^{\infty}\frac{r_2^{nk+1}-r_1^{nk+1}}{k!(nk+1)(nk+m)}\{\sin[(nk+m)\theta_2]-\sin[(nk+m)\theta_1]\}. \tag{4}$$

Proof Let $z = re^{i\theta}$ in Eq. (3), then by Euler's formula and DeMoivre's formula, we have

$$(r^m e^{im\theta})\exp(r^n e^{in\theta}) = \sum_{k=0}^{\infty}\frac{r^{nk+m}e^{i(nk+m)\theta}}{k!}. \tag{5}$$

Using the equality of the real parts of both sides of Eq. (5) yields

$$\exp(r^n\cos n\theta)\cos(m\theta + r^n\sin n\theta)$$
$$= \sum_{k=0}^{\infty}\frac{1}{k!}r^{nk}\cos[(nk+m)\theta]. \tag{6}$$

By integration term by term, we obtain the double integral

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\exp(r^n\cos n\theta)\cos(m\theta + r^n\sin n\theta)drd\theta$$
$$= \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\sum_{k=0}^{\infty}\frac{1}{k!}r^{nk}\cos[(nk+m)\theta]drd\theta$$
$$= \sum_{k=0}^{\infty}\frac{r_2^{nk+1}-r_1^{nk+1}}{k!(nk+1)(nk+m)}\{\sin[(nk+m)\theta_2]-\sin[(nk+m)\theta_1]\}.$$

q.e.d.

Next, we determine the double integral (2).

Theorem 2 *If the assumptions are the same as Theorem 1, then*

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\exp(r^n\cos n\theta)\sin(m\theta + r^n\sin n\theta)drd\theta$$
$$= -\sum_{k=0}^{\infty}\frac{r_2^{nk+1}-r_1^{nk+1}}{k!(nk+1)(nk+m)}\{\cos[(nk+m)\theta_2]-\cos[(nk+m)\theta_1]\}. \tag{7}$$

Proof By the equality of the imaginary parts of both sides of Eq. (5), we have

$$\exp(r^n\cos n\theta)\sin(m\theta + r^n\sin n\theta)$$
$$= \sum_{k=0}^{\infty}\frac{1}{k!}r^{nk}\sin[(nk+m)\theta]. \tag{8}$$

Also, using integration term by term yields

$$\int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\exp(r^n\cos n\theta)\sin(m\theta + r^n\sin n\theta)drd\theta$$
$$= \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2}\sum_{k=0}^{\infty}\frac{1}{k!}r^{nk}\sin[(nk+m)\theta]drd\theta$$

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$$= -\sum_{k=0}^{\infty} \frac{r_2^{nk+1} - r_1^{nk+1}}{k!(nk+1)(nk+m)} \{\cos[(nk+m)\theta_2] - \cos[(nk+m)\theta_1]\}.$$

q.e.d.

3. Example

For the two types of double integrals in this article, we will propose two examples and use Theorems 1 and 2 to determine their infinite series forms. Moreover, we employ Maple to calculate the approximations of these double integrals and their infinite series forms to verify our answers.

Example 1 Using Eq. (4) yields the double integral

$$\int_{\pi/6}^{\pi/2}\int_2^5 \exp(r^2\cos 2\theta)\cos(3\theta + r^2\sin 2\theta)drd\theta$$
$$= \sum_{k=0}^{\infty} \frac{5^{2k+1} - 2^{2k+1}}{k!(2k+1)(2k+3)}\left[\sin\frac{(2k+3)\pi}{2} - \sin\frac{(2k+3)\pi}{6}\right]. \quad (9)$$

Using Maple to verify the correctness of Eq. (9) as follows:

>evalf(Doubleint(exp(r^2*cos(2*theta))*cos(3*theta+r^2*sin(2*theta)),r=2..5,theta=Pi/6..Pi/2),18);

-428.396085871952629

>evalf(sum((5^(2*k+1)-2^(2*k+1))/(k!*(2*k+1)*(2*k+3))*(sin((2*k+3)*Pi/2)-sin((2*k+3)*Pi/6)),k=0..infinity),18);

-428.396085871952647

Example 2 By Eq. (7) , we have

$$\int_{\pi/4}^{\pi/2}\int_3^8 \exp(r^3\cos 3\theta)\sin(6\theta + r^3\sin 3\theta)drd\theta$$
$$= -\sum_{k=0}^{\infty} \frac{8^{3k+1} - 3^{3k+1}}{k!(3k+1)(3k+6)}\left[\cos\frac{(3k+6)\pi}{2} - \cos\frac{(3k+6)\pi}{4}\right]. \quad (10)$$

Also, we employ Maple to verify the correctness of Eq. (10).

>evalf(Doubleint(exp(r^3*cos(3*theta))*sin(6*theta+r^3*sin(3*theta)),r=3..8,theta=Pi/4..Pi/2),18);

-0.000119223862333110506

>evalf(-sum((8^(3*k+1)-3^(3*k+1))/(k!*(3*k+1)*(3*k+6))*(cos((3*k+6)*Pi/2)-cos((3*k+6)*Pi/4)),k=0..infinity),18);

-0.0001192238623331105

4. Conclusion

As mentioned, complex function theory plays a significant role in this study. In fact, its application is extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related application.

On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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