

On Fundamental Paradox of Mathematics

¹Adewunmi Olusola A. ; ²Sulaimon Mutiu O.; ³Olagbegi Moses & ⁴Ogunsanya B. G.

^{1,2,3,4}Department of Statistics & Mathematics Moshood Abiola Polytechnic, Abeokuta, Ogun State, Nigeria.

¹dokunsola@yahoo.com; ²mtsulaimon@gmail.com;
³olagbegi12@gmail.com ⁴ambassbgo2014@yahoo.com

ABSTRACT

For centuries ago, we have been made to learn and understand a kind of too abstract mathematics which diverges from practice and nature. In nature as well as in practice, no number smaller than 0 (<0) can be found, but in "Mathematics" we are taught to accept and understand so many things smaller than 0 (<0). For example, there are such values as ($0 > -1 > -2 > -3 > \dots > -\infty$). Due to Mathematics' heavy abstraction, it forces us to use absolute value ($| |$), which is like a witchcraft capable of transforming negative values ($-$) smaller than zero ($- < 0$) into positive values greater than zero ($+ > 0$). This paper identifies some Fundamental Paradox of Mathematics about negative value which is smaller than 0 ($- < 0$).

Keywords: Mathematics; Paradox

INTRODUCTION

Mathematics, the queen of all sciences, remains and will remain as a subject with great charm having an intrinsic value and beauty of its own. It plays an indispensable role in sciences, engineering and other subjects as well. So, mathematical knowledge is essential for the growth of science and technology, and for any individual to shine well in the field of one's choice (Government of Tamil Nadu, 2015).

Mathematics has undergone centuries, when mathematicians invented negative values ($-$) and positive values ($+$), including value (zero), also known as value (0), always greater than the negative ($-$), through the "axiom" without giving any equation, or mathematical formulas, prove to us that the positive values ($+$) is greater than the negative values ($-$) in a particular way.

The number line currently one of the important tools for teaching basic arithmetical concepts such as natural and real numbers in primary and secondary education. Freudenthal Hans (1983) calls this mental object a "device beyond praise" and considers it a preferred vehicle to teach negative numbers. In many countries the ordering of negative numbers by means of the number line is taught by the fifth grade (Howson, Harries and Sutherland, 1999). Despite its wide acceptance, the use of the number line in mathematics education is rather new. It seems to have originated in the 1950's. Max Beberman, credited for many innovations in math teaching, used the earlier term 'number scale': "In teaching subtraction of signed numbers, I first draw a number scale" (Beberman and Meserve, 1956). However, not everyone is convinced of the benefits of using the number line for teaching negative numbers in primary

education. In fact, the very teaching of operations on negative numbers is no longer allowed in basic education in Belgium. But even the question of the historical acceptance of negative numbers is problematic (Albrecht Heffer, NA).

Algebraic practice of solving linear problems has lead repeatedly to situations in which one arrives at "a negative value". Before the sixteenth century, such solutions were consistently called 'absurd' or 'impossible'. The abacus master, convinced of the correctness of his algebraic derivations, could interpret the negative value in some contexts as a debt. This does not imply that he accepted the solution as a negative value (Albrecht Heffer, NA).

On the contrary, by interpreting the solution as a debt, he removed the negative. Only from the beginning of the sixteenth century onwards, we see the first step towards negative values, in the form of algebraic terms affected by a negative sign. The fact that negative solutions were considered absurd for several centuries of algebraic practice is of significance to the teaching of mathematics. When teachers are aware that isolated negative quantities formed a conceptual barrier for the Renaissance habit of mind, it prepares them for potential difficulties in the student's understanding of the concept (Albrecht Heffer, NA).

Mathematicians, accidentally turned from a mathematical "logic and reality" to a mathematical "abstract and unrealistic". There is no mathematical formula to prove by how many units that $+5 > -5$. It is indeed difficult to understand why a mathematical foundation could be "misleading" as such. Facing a mathematical foundation to be "unrealistic" and "illogical" as such, this paper use the math to point out "paradox" of the "axioms and

conventions", {assuming that the positive values (+) is greater than the negative values (-)}.

JUSTIFICATION

(a) In the addition operation

For simplicity, we take value 30 (added), as an example, to perform numeric calculation for easy understanding as follows:

Problem 1

$$30 + (-5) = 25 \quad ; \quad (1)$$

$$30 + (-10) = 20 \quad ; \quad (2)$$

From (1) and (2), we have ($25 > 20$), so it is inferred that ($-10 > -5$)

Problem 2

$$30 + (-5) = 25 \quad ; \quad (3)$$

$$30 + (-0) = 30 \quad ; \quad (4)$$

From (3) and (4), we have ($30 > 25$), and so it is inferred that ($-5 > 0$).

Way of Argument

With the same added value 30, after we add another value to it, what will happen? If the result gives a greater value, the added number has a smaller value and vice versa if the result gives a smaller value, the added value has a greater value?

In problem (1), the result $25 > 20$, so we infer that (the added value -10 is greater than the added value -5 , which means $-10 > -5$).

In problem (2), the result $30 > 25$, then we infer that the added value -5 is greater than the added value 0 , which means $-5 > 0$.

With the argument method, all negative numbers will be greater than zero, thus in the range of negative number there is this order: $-\infty > \dots > -4 > -3 > -2 > -1 > 0$

Note: the results of the two above problems are always smaller or equal to the added number 30.

(b) In the greater (>) or smaller (<) comparison:

Mathematics states that we can compare any negative (-), positive (+) values with each other and the positive values (+) are always considered to be the base to compare with negative values(-) and the result is that the

positive value (+) is always greater than the negative number (-):

$$+3 > -3; +1 > -1.000\dots\dots\text{etc.} \dots \text{or } -1 > -3; 10 > 1\dots\dots\text{etc.}\dots$$

Because all positive values (+) are always greater than any negative values (-), then Mathematics is prone to the "paradox" in such problems as follows:

Problem 1

Comparison between positive values: always "True", because with two people having money, we can easily compare their money to see who is richer:

$6 > 4; 8 > 5$ so $6 + 8 > 4 + 5$; which means $14 > 9$; Is it correct? From that comparison, we can infer that $6 \times 8 > 4 \times 5$, which means $48 > 20$, that comparison is totally correct.

Problem 2

For the comparison between a negative (-) and positive (+) value, we always assumes the positive number is greater than the negative number. This is absolutely a paradox. Why? Let's take a look at this:

$3 > -6; 4 > -3$; so $3 + 4 > (-6) + (-3)$: is this is right or wrong?

But we can't infer that:

$3 \times 4 > (-6) \times (-3)$; because the result is ($12 > 18$). This is completely a paradox.

Problem 3

For the comparison of a negative (-) value with another negative (-) value, it is assumed that the small negative value is bigger than the big negative value.

For example:

$-3 > -5$; this is completely a paradox in both Mathematics and practice. Why can we assert that? Here are our arguments:

* Wrong in Mathematics:

$$-3 > -5 \quad ;$$

$$-2 > -3 \quad ;$$

Then we infer $(-3) + (-2) > (-5) + (-3)$. Is this right? But from this comparison, we can't infer:

$(-3) \times (-2) > (-5) \times (-3)$, because the result will be $+6 > +15$, which is completely a paradox.

* Wrong in practical:

In case of debtor: If A owes ~~N~~4, it means that A has ~~N~~(-4), if B owes ~~N~~8, then B's property is negative ~~N~~8 {~~N~~(-8)}. So, in the aspect of "debt" or "deficit", B owes much more than A to payback {B's property is more negative (indebted) than A's}, thus $-8 > -4$ holds true in both mathematics and practice.

In case of owner: If A and B are given ₦30, then A's money will be ₦30 - ₦4 = ₦26, B's money will be ₦30 - ₦8 = ₦22. So, in the view of "owning", A possesses more money than B, which results in $26 > 22$, this holds true in both mathematics and practice.

The above-mentioned problems are some fundamental ones which show the "paradox" of Mathematics, which we have been forced to accept so far.

CONCLUSION

Due to the very abstraction and unreality of Mathematics, it contains inexplicable inherent conflicts as well as axioms which force the user to accept them without demonstration or explanation. Mathematics is prone to impasse when facing such questions as:

1. In nature as well in practice, nothing or no value is smaller than 0 (<0), then why do Mathematics force us to accept and understand so many thing smaller than 0 (<0)?
2. Let's look at the range of negative (-) and positive (+) numbers that Mathematics stipulates:
 $-\infty \dots -4, -3, -2, -1, 0, +1, +2, +3, +4, \dots +\infty$
 It stipulates that all numbers to the left of zero (0) bearing the minus sign are all smaller than zero (<0), and all numbers to the right of zero (0) bearing the plus sign are all greater than zero (>0). Then let us consider this question: If we multiply two negative numbers together, for example ($-3 < 0$) and ($-5 < 0$), why is the result a positive number which is greater than zero ($15 > 0$)?
3. Similarly, consider a negative value 1 ($-1 < 0$) and positive value 1000 ($+1000 > 0$), the latter is very much greater than zero, why is the result of multiplying them (1000, the number represents a healthy rich owner and -1, the number represents a weak poor debtor) so negative (-1000)?
4. Mathematics forces us to assume all positive numbers are greater than negative numbers, but most mathematician cannot demonstrate how ($+5 > -5$)? Then why for so many centuries must human still learn "old mathematics basis" that is very paradoxical.

Why does mathematics reach an impasse when facing such simple mathematic questions? In our opinion, the reason is that Mathematics takes wrong arguments from the beginning, namely the two fundamental problems that we introduced above. Another reason is that the stipulation in the comparison of negative numbers, with positive

numbers, which states that positive numbers are always greater than negative numbers.
 ($+3 > -3$); ($+1 > -100$); ($-1 > -10$).....etc.....

RECOMMENDATIONS

Facing such an old mathematics basis which is heavily abstract and diverging from practice and unable to solve the above-mentioned fundamental paradoxes, we would like to suggest a new solution in order to correct wrong ideals of "old mathematics basis". In our view, a healthy and correct mathematics basis must have factors and features that comply with nature and practice as follows:

1. In nature as well as in practice, no value is smaller than zero (<0), so mathematics should not use any numbers smaller than zero (<0), even when this value bears negative calculation (-) {it is called negative value (-)}.
2. In nature as well as in practice, all problems originate from the zero (0), therefore in mathematics, we should also take zero (0) as the starting number and the centre of all systems of coordinate or in all comparisons. We should not use positive infinite (+) as well as negative infinite (-) as a starting points or reference number in comparisons. This is the way Mathematics intensively uses.

REFERENCES

- [1] Albrecht H. (NA). Negative Numbers As An Epistemic Difficult Concept: Some Lessons From History. *Center for Logic and Philosophy of Science Ghent University, Belgium*. Pg 1 -13.
- [2] Beberman, Max and Meserve, Bruce E., (1956). "An exploratory approach to solving equations", *The Mathematics Teacher*, January.
- [3] Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*, Dordrecht: *Reidel*.
- [4] Howson, A. G., Harris T. and R Sutherland (1999). *Primary school mathematics textbooks*, London: *Qualifications and Curriculum Authority*.