Generalization of Contraction mapping in complex-valued metric space

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Abstract

The notion of complex-valued metric spaces was introduced by Azam et al. *[4]. He introduced the new concept and established a common fixed point result in the context of complex-valued metric spaces. In this paper, we generalized the concept of chatterjea [14] contraction mapping for multi-valued mappings on the complex-valued metric spaces.*

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Introduction 1.1.

It is well known fact that the mathematical results regarding fixed points of contraction-type mappings are very useful for determining the existence and uniqueness of solutions to various mathematical models. The theory of fixed points has been developed, regarding the results to finding the fixed points self and nonself over the last 50 years.

Many authors have proved fixed point results in the different kind of generalization in complex-valued metric spaces. Nadler [9] and Markin [8] was initiated the study of fixed points for multi-valued contraction mappings. Azam et al. [4] introduced the concept of complex-valued metric space and obtained sufficient conditions for the existence of common fixed points. Very recently, Ahmad et al. [2] obtained some new fixed point results for multi-valued mappings in the setting of complex-valued metric spaces. Some fixed point results by generalizing the contractive conditions in the context of complex-valued metric spaces was established by Sitthikul and Saejung [13] and Klin-eam and Suanoom [7]. The results presented in this paper substantially extend the results given by chatterjea et.al [14] for the multi-valued mappings.

Preliminaries 1.2.

Let C be the set of complex numbers and $z_1, z_2 \in \mathbb{C}$. Define a partial order $\precsim$ on $\mathbb{C}$ as follows:
$z_1 \precsim z_2$ if and only if $\operatorname{Re}(z_1) \leq \operatorname{Re}(z_2)$, $\operatorname{Im}(z_1) \leq \operatorname{Im}(z_2)$.

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It follows that $z_1 \precsim z_2$
if one of the following conditions is satisfied:

(i) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2),\ \mathrm{Im}(z_1) < \mathrm{Im}(z_2),$
(ii) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2),\ \mathrm{Im}(z_1) = \mathrm{Im}(z_2),$
(iii) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2),\ \mathrm{Im}(z_1) < \mathrm{Im}(z_2),$
(iv) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2),\ \mathrm{Im}(z_1) = \mathrm{Im}(z_2),$

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one of (i),(ii) and (iii) is satisfied and we will write $z_1 \prec z_2$ if only (iii) is satisfied. Note that

$$0 \precsim z_1 \precnsim z_2 \implies |z_1| < |z_2|,$$

$$z_1 \precsim z_2 ,\ z_2 \prec z_3 \implies z_1 \prec z_3 .$$

Definition 1.3. Let X be a nonempty set. Suppose that the mapping $d : X \times X \to \square$ satisfies:

1. $0 \precsim d(x,y)$ for all $x,y \in X$ and $d(x,y) = 0$ if and only if $x = y$;
2. $d(x,y) = d(y,x)$ for all $x,y \in X$
3. $d(x,y) \precsim d(y,x) + d(z,y)$ for all $x,y,z \in X$.

Then d is called a complex-valued metric on X, and (X,d) is called a complex-valued metric space. A point $x \in X$ is called an interior point of a set $A \subseteq X$ whenever there exists $0 \prec r \in \square$ such that

$B(x,r) = \{y \in X : d(x,y) \prec r\} \subseteq A.$

A point $x \in X$ is called a limit point of A whenever, for every $0 \prec r \in \square$ $B(x,r) \cap (A\backslash\{x\}) \neq \phi.$

A is called open whenever each element of *A* is an interior point of A. Moreover, a subset $B \subseteq X$ is called closed whenever each limit point of B belongs to B.

Let $\{x_n\}$ be a sequence in X and $x \in X$. If for every $c \in \square$ with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n,x) \prec c$, then $\{x_n\}$ is said to be convergent, $\{x_n\}$ converges to *x* and *x* is the limit point of $\{x_n\}$. We denote this by $\lim_{n \to \infty} x_n = x$, or $x_n \to x$ as $n \to \infty$.

If for every $c \in \square$ with $0 \prec c$ there is $n_0 \in N$ such that for all $n > n_0$, $d(x_n,x_{n+m}) \prec c$, where $m \in N$, then $\{x_n\}$ is called a Cauchy sequence in (X,d). If every Cauchy sequence is convergent in (X,d), then (X,d) is called a complete complex-valued metric space. We require the following lemmas.

Lemma 1.4. [4] Let (X,d) be a complex-valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $|d(x_n,x)| \to 0$ as $n \to \infty$.

Lemma 1.5. [4] Let (X,d) be a complex-valued metric space and let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$, where $m \in N$.

Definition 1.6 [4]

Let (X,d) be a complex-valued metric space.

We denote the family of nonempty, closed and bounded subsets of a complex valued metric space by CB(X).

From now on, we denote $s(z_1) = \{z_2 \in \square \ : z_1 \preceq z_2\}$ for $z_1 \in \square$, and

$s(a,B) = \underset{b \in B}{\cup} s(d(a,b)) = \underset{b \in B}{\cup} \{z \in \square : d (a,b) \preceq z\}$ for $a \in X$ and $B \in CB(X)$.

For $A, B \in CB(X)$, we denote

$$s(A,B) = (\underset{a \in A}{\cap} s(a,B)) \cap (\underset{b \in B}{\cap} s(b,A)).$$

Definition 1.7. [2] Let (X,d) be a complex-valued metric space. Let $T : X \to CB(X)$ be a multi-valued map. For $x \in X$ and $A \in CB(X)$, define $W_x(A) = \{d (x,a) : a \in A\}$. Thus, for $x,y \in X$, $W_x(T_y) = \{d(x,u) : u \in Ty\}$.

Definition 1.8. [2] Let (X,d) be a complex-valued metric space. A subset A of X is called bounded from below if there exists some $z \in X$ such that $z \preceq a$ for all $a \in A$.

Definition 1.9. [2] Let (X,d) be a complex - valued metric space. A multi - valued mapping $F : X \to 2^C$ is called bounded from below if for each $x \in X$ there exists $z_x \in \square$ such that $z_x \preceq u$ for all $u \in Fx$.

Definition 1.10. [2] Let (X,d) be a complex-valued metric space. The multi - valued mapping $T:X \to CB(X)$ is said to have the lower bound property (l.b property) on (X,d) if for any $x \in X$, the multi-valued mapping $F_x : X \to 2^C$ defined by $F_x(y) = W_x(Ty)$ is bounded from below. That is, for $x,y \in X$, there exists an element $l_x(Ty) \in \square$ such that

$$l_x(Ty) \preceq u$$

for all $u \in W_x(Ty)$, where $l_x(Ty)$ is called a lower bound of T associated with (x,y).

Definition 1.11. [2] Let (X,d) be a complex-valued metric space. The multi-valued mapping $T:X \to CB(X)$ is said to have the greatest lower bound property (g.l.b property) on (X,d) if a greatest lower bound of $W_x(Ty)$ exists in $\square$ for all $x,y \in X$. We denote d(x,Ty) by the g.l.b of $W_x(Ty)$. That is,

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$d(x,Ty) = \inf\{d(x,u) : u \in Ty\}$.

Main Result

Theorem 1.12. Let (X, d) be a complete complex-valued metric space and let $S, T: X \to CB(X)$ be multi-valued mappings with g.l.b property such that

$$(1.1) \quad \alpha\left\{\frac{d(x,Sx)d(x,Ty)+d(y,Sx)d(y,Sx)}{d(x,Ty)+d(y,Sx)}\right\} +\beta\left\{\frac{d(x,y)d(y,Ty)+d(x,Sx)d(y,Sx)}{d(y,Ty)}\right\}$$

$$+\gamma\left\{\frac{d(x,Sx)d(y,Ty)+d(Sx,y)d(Ty,Sx)}{d(Sx,Ty)+d(y,Sx)}\right\} +\delta\left\{\frac{d(x,y)d(x,Ty)+d(Sx,y)}{d(x,Ty)+d(Sx,y)}\right\}$$

$$+\lambda\left\{\frac{d(x,y)d(Sx,Ty)+d(y,Sx)}{d(y,Ty)+d(y,Sx)}\right\} \in s(Sx,Ty)$$

For all $x, y \in X$ and $0 \le \alpha+\beta+\gamma+\delta+\lambda < 1$. Then S and T have a common fixed point.

Proof : Let $x_0 \in X$ and $x_1 \in Sx_0$ *and* Tx_0 from (1.1), we get

$$\alpha\left\{\frac{d(x_0,Sx_0)d(x_0,Tx_1)+d(x_1,Sx_0)d(x_1,Sx_0)}{d(x_0,Tx_1)+d(x_1,Sx_0)}\right\} +\beta\left\{\frac{d(x_0,x_1)d(x_1,Tx_1)+d(x_0,Sx_0)d(x_1,Sx_0)}{d(x_1,Tx_1)}\right\}$$

$$+\gamma\left\{\frac{d(x_0,Sx_0)d(x_1,Tx_1)+d(Sx_0,x_1)d(Tx_1,Sx_0)}{d(Sx_0,Tx_1)+d(x_1,Sx_0)}\right\} +\delta\left\{\frac{d(x_0,x_1)d(x_0,Tx_1)+d(Sx_0,x_1)}{d(x_0,Tx_1)+d(Sx_0,x_1)}\right\}$$

$$+\lambda\left\{\frac{d(x_0,x_1)d(Sx_0,Tx_1)+d(x_1,Sx_0)}{d(x_1,Tx_1)+d(x_1,Sx_0)}\right\} \in s(Sx_0,Tx_1) \in \bigcap_{x\in Sx_0} s(x,Tx_1)$$

That is

$$\alpha\left\{\frac{d(x_0,Sx_0)d(x_0,Tx_1)+d(x_1,Sx_0)d(x_1,Sx_0)}{d(x_0,Tx_1)+d(x_1,Sx_0)}\right\} +\beta\left\{\frac{d(x_0,x_1)d(x_1,Tx_1)+d(x_0,Sx_0)d(x_1,Sx_0)}{d(x_1,Tx_1)}\right\}$$

$$+\gamma\left\{\frac{d(x_0,Sx_0)d(x_1,Tx_1)+d(Sx_0,x_1)d(Tx_1,Sx_0)}{d(Sx_0,Tx_1)+d(x_1,Sx_0)}\right\} +\delta\left\{\frac{d(x_0,x_1)d(x_0,Tx_1)+d(Sx_0,x_1)}{d(x_0,Tx_1)+d(Sx_0,x_1)}\right\}$$

$$+\lambda\left\{\frac{d(x_0,x_1)d(Sx_0,Tx_1)+d(x_1,Sx_0)}{d(x_1,Tx_1)+d(x_1,Sx_0)}\right\}$$

$$\in s\left(x, Tx_1\right) \text{ for all } x \in Sx_0$$

Since $x_1 \in Sx_0$, so we have

$$\alpha\left\{\frac{d\left(x_0, Sx_0\right)d\left(x_0, Tx_1\right)+d\left(x_1, Sx_0\right)d\left(x_1, Sx_0\right)}{d\left(x_0, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}+\beta\left\{\frac{d\left(x_0, x_1\right)d\left(x_1, Tx_1\right)+d\left(x_0, Sx_0\right)d\left(x_1, Sx_0\right)}{d\left(x_1, Tx_1\right)}\right\}$$

$$+\gamma\left\{\frac{d\left(x_0, Sx_0\right)d\left(x_1, Tx_1\right)+d\left(Sx_0, x_1\right)d\left(Tx_1, Sx_0\right)}{d\left(Sx_0, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}+\delta\left\{\frac{d\left(x_0, x_1\right)d\left(x_0, Tx_1\right)+d\left(Sx_0, x_1\right)}{d\left(x_0, Tx_1\right)+d\left(Sx_0, x_1\right)}\right\}$$

$$+\lambda\left\{\frac{d\left(x_0, x_1\right)d\left(Sx_0, Tx_1\right)+d\left(x_1, Sx_0\right)}{d\left(x_1, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\} \in s\left(x_1, Tx_1\right),$$

$$=\bigcup_{x \in Tx_1} s\left(d\left(x_1, x\right)\right)$$

So there exists some $x_2 \in Tx_1$ such that

$$\alpha\left\{\frac{d\left(x_0, Sx_0\right)d\left(x_0, Tx_1\right)+d\left(x_1, Sx_0\right)d\left(x_1, Sx_0\right)}{d\left(x_0, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}+\beta\left\{\frac{d\left(x_0, x_1\right)d\left(x_1, Tx_1\right)+d\left(x_0, Sx_0\right)d\left(x_1, Sx_0\right)}{d\left(x_1, Tx_1\right)}\right\}$$

$$+\gamma\left\{\frac{d\left(x_0, Sx_0\right)d\left(x_1, Tx_1\right)+d\left(Sx_0, x_1\right)d\left(Tx_1, Sx_0\right)}{d\left(Sx_0, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}+\delta\left\{\frac{d\left(x_0, x_1\right)d\left(x_0, Tx_1\right)+d\left(Sx_0, x_1\right)}{d\left(x_0, Tx_1\right)+d\left(Sx_0, x_1\right)}\right\}$$

$$+\lambda\left\{\frac{d\left(x_0, x_1\right)d\left(Sx_0, Tx_1\right)+d\left(x_1, Sx_0\right)}{d\left(x_1, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\} \in s\left(d\left(x_1, x_2\right)\right)$$

That is,

$$d\left(x_1, x_2\right) \leq \alpha\left\{\frac{d\left(x_0, Sx_0\right)d\left(x_0, Tx_1\right)+d\left(x_1, Sx_0\right)d\left(x_1, Sx_0\right)}{d\left(x_0, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}+\beta\left\{\frac{d\left(x_0, x_1\right)d\left(x_1, Tx_1\right)+d\left(x_0, Sx_0\right)d\left(x_1, Sx_0\right)}{d\left(x_1, Tx_1\right)}\right\}$$

$$+\gamma\left\{\frac{d\left(x_0, Sx_0\right)d\left(x_1, Tx_1\right)+d\left(Sx_0, x_1\right)d\left(Tx_1, Sx_0\right)}{d\left(Sx_0, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}+\delta\left\{\frac{d\left(x_0, x_1\right)d\left(x_0, Tx_1\right)+d\left(Sx_0, x_1\right)}{d\left(x_0, Tx_1\right)+d\left(Sx_0, x_1\right)}\right\}$$

$$+\lambda\left\{\frac{d\left(x_0, x_1\right)d\left(Sx_0, Tx_1\right)+d\left(x_1, Sx_0\right)}{d\left(x_1, Tx_1\right)+d\left(x_1, Sx_0\right)}\right\}$$

By using the greatest lower property (g.l.b. property) of S and T , we get.

$$d(x_1,x_2) \leq \alpha\left\{\frac{d(x_0,x_1)d(x_0,x_2)+d(x_1,x_1)d(x_1,x_1)}{d(x_0,x_2)+d(x_1,x_1)}\right\}+\beta\left\{\frac{d(x_0,x_1)d(x_1,x_2)+d(x_0,x_1)d(x_1,x_1)}{d(x_1,x_2)}\right\}$$

$$+\gamma\left\{\frac{d(x_0,x_1)d(x_1,x_2)+d(x_1,x_1)d(x_2,x_1)}{d(x_1,x_2)+d(x_1,x_1)}\right\}+\delta\left\{\frac{d(x_0,x_1)d(x_0,x_2)+d(x_1,x_1)}{d(x_0,x_2)+d(x_1,x_1)}\right\}$$

$$+\lambda\left\{\frac{d(x_0,x_1)d(x_1,x_2)+d(x_1,x_1)}{d(x_1,x_2)+d(x_1,x_1)}\right\}$$

which implies that

$$d(x_1,x_2) \leq \alpha d(x_0,x_1)+\beta d(x_0,x_1)+\gamma d(x_0,x_1)+\delta d(x_0,x_1)+\lambda d(x_0,x_1)$$

$$d(x_1,x_2) \leq (\alpha+\beta+\gamma+\delta+\lambda)d(x_0,x_1)$$

Similarly

$$d(x_2,x_3) \leq (\alpha+\beta+\gamma+\delta+\lambda)^2 d(x_0,x_1)$$

$$d(x_3,x_4) \leq (\alpha+\beta+\gamma+\delta+\lambda)^3 d(x_0,x_1)$$

| |
| |

Inductively, we can construct a sequence $\{x_n\}$ in X such that for n = 1,2 ………., $|d(x_n,x_{n+1})| \leq$ $(\alpha+\beta+\gamma+\delta+\lambda)^n |d(x_0,x_1)|$ with $(\alpha+\beta+\gamma+\delta+\lambda)<1, x_{2n+1} \in Sx_{2n}$ and $x_{2n+2} \in Tx_{2n+1}$.

Now for $m>n$, we get

$$|d(x_n,x_m)| \leq |d(x_n,x_{n+1})|+|d(x_{n+1},x_{n+2})|+----+|d(x_{m-1},x_m)| \leq \left[a^n+a^{n+1}+...+a^{m-1}\right]|d(x_0,x_1)|$$

where $a=\alpha+\beta+\gamma+\delta+\lambda$

$$|d(x_n,x_m)| \leq \left[\frac{a^n}{1-a}\right]|d(x_0,x_1)|, \; since \; 0\leq a <1.$$

And so $\left| d\left(x_n, x_m \right) \right| \to 0$ as m, n $\to \infty$.

This implies that $\{x_n\}$ is a Cauchy Sequence in X. Since X is complete , so there exists $v \in X$ such that $x_n \to v$ as $n \to \infty$. We now show that '$v \in Tv$ and $v \in Sv$. From (1.1),we have

$$\alpha \left\{ \frac{d\left(x_{2n}, Sx_{2n}\right)d\left(x_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)d\left(v, Sx_{2n}\right)}{d\left(x_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)} \right\}$$

$$+ \beta \left\{ \frac{d\left(x_{2n}, v\right)d\left(v, Tv\right) + d\left(x_{2n}, Sx_{2n}\right)d\left(v, Sx_{2n}\right)}{d\left(v, Tv\right)} \right\}$$

$$+ \gamma \left\{ \frac{d\left(x_{2n}, Sx_{2n}\right)d\left(v, Tv\right) + d\left(Sx_{2n}, v\right)d\left(Tv, Sx_{2n}\right)}{d\left(Sx_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)} \right\} + \delta \left\{ \frac{d\left(x_{2n}, v\right)d\left(x_{2n}, Tv\right) + d\left(Sx_{2n}, v\right)}{d\left(x_{2n}, Tv\right) + d\left(Sx_{2n}, v\right)} \right\}$$

$$+ \lambda \left\{ \frac{d\left(x_{2n}, v\right)d\left(Sx_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)}{d\left(v, Tv\right) + d\left(v, Sx_{2n}\right)} \right\} \in s\left(Sx_{2n}, Tv\right)$$

$$\in \bigcap_{x \in Sx_{2n}} s\left(x, Tv\right)$$

and so

$$\alpha \left\{ \frac{d\left(x_{2n}, Sx_{2n}\right)d\left(x_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)d\left(v, Sx_{2n}\right)}{d\left(x_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)} \right\}$$

$$+ \beta \left\{ \frac{d\left(x_{2n}, v\right)d\left(v, Tv\right) + d\left(x_{2n}, Sx_{2n}\right)d\left(v, Sx_{2n}\right)}{d\left(v, Tv\right)} \right\}$$

$$+ \gamma \left\{ \frac{d\left(x_{2n}, Sx_{2n}\right)d\left(v, Tv\right) + d\left(Sx_{2n}, v\right)d\left(Tv, Sx_{2n}\right)}{d\left(Sx_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)} \right\} + \delta \left\{ \frac{d\left(x_{2n}, v\right)d\left(x_{2n}, Tv\right) + d\left(Sx_{2n}, v\right)}{d\left(x_{2n}, Tv\right) + d\left(Sx_{2n}, v\right)} \right\}$$

$$+ \lambda \left\{ \frac{d\left(x_{2n}, v\right)d\left(Sx_{2n}, Tv\right) + d\left(v, Sx_{2n}\right)}{d\left(v, Tv\right) + d\left(v, Sx_{2n}\right)} \right\} \in s\left(x, Tv\right) \text{ for all } x \in Sx_{2n}$$

Since $x_{2n+1} \in Sx_{2n}$, so we have

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$$\alpha \left\{ \frac{d(x_{2n},Sx_{2n})d(x_{2n},Tv)+d(v,Sx_{2n})d(v,Sx_{2n})}{d(x_{2n},Tv)+d(v,Sx_{2n})} \right\}$$

$$+\beta \left\{ \frac{d(x_{2n},v)d(v,Tv)+d(x_{2n},Sx_{2n})d(v,Sx_{2n})}{d(v,Tv)} \right\}$$

$$+\gamma \left\{ \frac{d(x_{2n},Sx_{2n})d(v,Tv)+d(Sx_{2n},v)d(Tv,Sx_{2n})}{d(Sx_{2n},Tv)+d(v,Sx_{2n})} \right\} +\delta \left\{ \frac{d(x_{2n},v)d(x_{2n},Tv)+d(Sx_{2n},v)}{d(x_{2n},Tv)+d(Sx_{2n},v)} \right\}$$

$$+\lambda \left\{ \frac{d(x_{2n},v)d(Sx_{2n},Tv)+d(v,Sx_{2n})}{d(v,Tv)+d(v,Sx_{2n})} \right\} \quad \in s(x_{2n+1},Tv)$$

$$\in s(x_{2n+1},Tv) = \bigcup_{u'\in Tv} s\big(d(x_{2n+1},u')\big)$$

so there exists some $v_n \in Tv$ such that

$$\alpha \left\{ \frac{d(x_{2n},Sx_{2n})d(x_{2n},Tv)+d(v,Sx_{2n})d(v,Sx_{2n})}{d(x_{2n},Tv)+d(v,Sx_{2n})} \right\} +\beta \left\{ \frac{d(x_{2n},v)d(v,Tv)+d(x_{2n},Sx_{2n})d(v,Sx_{2n})}{d(v,Tv)} \right\}$$

$$+\gamma \left\{ \frac{d(x_{2n},Sx_{2n})d(v,Tv)+d(Sx_{2n},v)d(Tv,Sx_{2n})}{d(Sx_{2n},Tv)+d(v,Sx_{2n})} \right\} +\delta \left\{ \frac{d(x_{2n},v)d(x_{2n},Tv)+d(Sx_{2n},v)}{d(x_{2n},Tv)+d(Sx_{2n},v)} \right\}$$

$$+\lambda \left\{ \frac{d(x_{2n},v)d(Sx_{2n},Tv)+d(v,Sx_{2n})}{d(v,Tv)+d(v,Sx_{2n})} \right\} \quad \in s\big(d(x_{2n+1},v_n)\big),$$

That is,

$$d(x_{2n+1},v_n) \preceq \; \alpha \left\{ \frac{d(x_{2n},Sx_{2n})d(x_{2n},Tv)+d(v,Sx_{2n})d(v,Sx_{2n})}{d(x_{2n},Tv)+d(v,Sx_{2n})} \right\}$$

$$+\beta \left\{ \frac{d(x_{2n},v)d(v,Tv)+d(x_{2n},Sx_{2n})d(v,Sx_{2n})}{d(v,Tv)} \right\}$$

$$+\gamma \left\{ \frac{d(x_{2n},Sx_{2n})d(v,Tv)+d(Sx_{2n},v)d(Tv,Sx_{2n})}{d(Sx_{2n},Tv)+d(v,Sx_{2n})} \right\}$$

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$$+\delta\left\{\frac{d\left(x_{2n},v\right)d\left(x_{2n},Tv\right)+d\left(Sx_{2n},v\right)}{d\left(x_{2n},Tv\right)+d\left(Sx_{2n},v\right)}\right\}+\lambda\left\{\frac{d\left(x_{2n},v\right)d\left(Sx_{2n},Tv\right)+d\left(v,Sx_{2n}\right)}{d\left(v,Tv\right)+d\left(v,Sx_{2n}\right)}\right\}$$

By using the greatest lower bound property (g.l.b property) of S and T, we get

$$d\left(x_{2n+1},v_n\right)\preceq \alpha\left\{\frac{d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n},v_n\right)+d\left(v,x_{2n+1}\right)d\left(v,x_{2n+1}\right)}{d\left(x_{2n},v_n\right)+d\left(v,x_{2n+1}\right)}\right\}$$

$$+\beta\left\{\frac{d\left(x_{2n},v\right)d\left(v,v_n\right)+d\left(x_{2n},x_{2n+1}\right)d\left(v,x_{2n+1}\right)}{d\left(v,v_n\right)}\right\}$$

$$+\gamma\left\{\frac{d\left(x_{2n},x_{2n+1}\right)d\left(v,v_n\right)+d\left(x_{2n+1},v\right)d\left(v_n,x_{2n+1}\right)}{d\left(x_{2n+1},v_n\right)+d\left(v,x_{2n+1}\right)}\right\}$$

$$+\delta\left\{\frac{d\left(x_{2n},v\right)d\left(x_{2n},v_n\right)+d\left(x_{2n+1},v\right)}{d\left(x_{2n},v_n\right)+d\left(x_{2n+1},v\right)}\right\}+\lambda\left\{\frac{d\left(x_{2n},v\right)d\left(x_{2n+1},v_n\right)+d\left(v,x_{2n+1}\right)}{d\left(v,v_n\right)+d\left(v,x_{2n+1}\right)}\right\}$$

By using again the triangular inequality, we get

$$d\left(v,v_n\right)\preceq d\left(v,x_{2n+1}\right)+d\left(x_{2n+1},v_n\right).$$

Then we have

$$d\left(v,v_n\right)\preceq d\left(v,x_{2n+1}\right)+\alpha\left\{\frac{d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n},v_n\right)+d\left(v,x_{2n+1}\right)d\left(v,x_{2n+1}\right)}{d\left(x_{2n},v_n\right)+d\left(v,x_{2n+1}\right)}\right\}$$

$$+\beta\left\{\frac{d\left(x_{2n},v\right)d\left(v,v_n\right)+d\left(x_{2n},x_{2n+1}\right)d\left(v,x_{2n+1}\right)}{d\left(v,v_n\right)}\right\}$$

$$+\gamma\left\{\frac{d\left(x_{2n},x_{2n+1}\right)d\left(v,v_n\right)+d\left(x_{2n+1},v\right)d\left(v_n,x_{2n+1}\right)}{d\left(x_{2n+1},v_n\right)+d\left(v,x_{2n+1}\right)}\right\}$$

$$+\delta\left\{\frac{d\left(x_{2n},v\right)d\left(x_{2n},v_n\right)+d\left(x_{2n+1},v\right)}{d\left(x_{2n},v_n\right)+d\left(x_{2n+1},v\right)}\right\}+\lambda\left\{\frac{d\left(x_{2n},v\right)d\left(x_{2n+1},v_n\right)+d\left(v,x_{2n+1}\right)}{d\left(v,v_n\right)+d\left(v,x_{2n+1}\right)}\right\}$$

and we obtain

$$\left| d\left(v,v_{n}\right)\right| \leq \left| d\left(v,x_{2n+1}\right)\right| + \alpha \left| \frac{d\left(x_{2n},x_{2n+1}\right)d\left(x_{2n},v_{n}\right)+d\left(v,x_{2n+1}\right)d\left(v,x_{2n+1}\right)}{d\left(x_{2n},v_{n}\right)+d\left(v,x_{2n+1}\right)}\right|$$

$$+\beta \left| \frac{d\left(x_{2n},v\right)d\left(v,v_{n}\right)+d\left(x_{2n},x_{2n+1}\right)d\left(v,x_{2n+1}\right)}{d\left(v,v_{n}\right)}\right|$$

$$+\gamma \left| \frac{d\left(x_{2n},x_{2n+1}\right)d\left(v,v_{n}\right)+d\left(x_{2n+1},v\right)d\left(v_{n},x_{2n+1}\right)}{d\left(x_{2n+1},v_{n}\right)+d\left(v,x_{2n+1}\right)}\right|$$

$$+\delta \left| \frac{d\left(x_{2n},v\right)d\left(x_{2n},v_{n}\right)+d\left(x_{2n+1},v\right)}{d\left(x_{2n},v_{n}\right)+d\left(x_{2n+1},v\right)}\right| + \lambda \left| \frac{d\left(x_{2n},v\right)d\left(x_{2n+1},v_{n}\right)+d\left(v,x_{2n+1}\right)}{d\left(v,v_{n}\right)+d\left(v,x_{2n+1}\right)}\right|$$

By letting $n \to \infty$ in the above inequality , we get $\left| d\left(v,v_{n}\right)\right| \to 0$ as n $\to \infty$. By Lemma (1.4), we have $v_{n} \to v$ as n $\to \infty$. Since Tv is closed , so $v \in Tv$. Similarly, it follows that $v \in Sv$. Thus S and T have a common fixed point.

Corollary 1.13 Let $\left(X,d\right)$ be a complete complex –valued metric space and let T:X $\to$ CB(X) be a multi-valued mapping with g.l.b property such that

$$\alpha \left\{ \frac{d\left(x,Tx\right)d\left(x,Ty\right)+d\left(y,Tx\right)d\left(y,Tx\right)}{d\left(x,Ty\right)+d\left(y,Tx\right)}\right\} + \beta \left\{ \frac{d\left(x,y\right)d\left(y,Ty\right)+d\left(x,Tx\right)d(y,Tx)}{d\left(y,Ty\right)}\right\}$$

$$+\gamma \left\{ \frac{d\left(x,Tx\right)d\left(y,Ty\right)+d\left(Tx,y\right)+d\left(Ty,Tx\right)}{d\left(Tx,Ty\right)+d\left(y,Tx\right)}\right\} + \delta \left\{ \frac{d\left(x,y\right)d\left(x,Ty\right)+d\left(Tx,y\right)}{d\left(x,Ty\right)+d\left(Tx,y\right)}\right\}$$

$$+\lambda \left\{ \frac{d\left(x,y\right)d\left(Tx,Ty\right)+d\left(y,Tx\right)}{d\left(y,Ty\right)+d\left(y,Tx\right)}\right\} \in s\left(Tx,Ty\right)$$

For all $x, y \in X$ and $0 \leq \alpha+\beta+\gamma+\delta+\lambda <1$, Then T has a fixed point

Proof. We can prove it by setting S = T in the aboveTheorem.

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