

Performance and Analysis of MIMO Multichannel Beamforming under the Double Scattering Channel

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Abstract.

We present the new method to investigate and analyze the symbol error rate performance of the multiple-input multiple-output (MIMO) multi-channel beam forming (MB) under the double-scattering channel. In the proposed method we derive an asymptotic expansion on the marginal eigenvalue distribution of the MIMO channel matrix, and use the result to obtain an approximate expression on the average SER at high signal-to-noise ratio (SNR). Two major parameters of the SER, i.e., the diversity gain and the coding gain are analyzed. Our simulation results explain that it suffices for the double-scattering channel to have only limited scatterers, if the same diversity gain as the Rayleigh channel is desired; however, when the number of scatterers is below a certain level, the coding

gain in the double-scattering channel will vary with the SNR logarithmically.

1.INTRODUCTION:

The prospect of extraordinary improvements in the capacity of wireless networks has drawn considerable attention to multiple-input-multiple-output (MIMO) communication techniques. MIMO methods make use of multi-element antenna coding at both the transmit and the receive side of a radio link to drastically improve the capacity over more traditional single-input-multiple-output (SIMO) systems. Multiple-input multiple-output (MIMO) multichannel beam-forming (MB) [3] a linear transmission scheme which applies perfect channel state information (CSI) at the transmitter and receiver to guide multiple data streams along the strongest eigen-direction of the MIMO channel. Previous studies shown that, even for non- Gaussian codes, MIMO MB still corresponds to the

optimal choice of linear transmit-receive processing under various practical criteria, such as symbol error rate (SER) and mean square error[9].

MIMO MB has been well investigated in various Rayleigh and Rician fading channel scenarios [3]-[6], [8] evaluating the performance in terms of average SER, outage probability, and diversity-and-multiplexing tradeoff because of its theoretical importance. These prior studies, however, all made the key assumption that the scattering environment was sufficient enough to render full-rank MIMO channel matrices. Recently, it has been observed via experimental studies that, for various practical environments (such as outdoor large-distance propagation, indoor keyhole propagation, and rooftop-diffracting propagation etc.) the channel may in fact exhibit reduced-rank behavior due to a lack of scattering around the transmitter and the receiver. A more general channel model that embraces this aspect of the MIMO channel had been proposed and referred to as the double-scattering model which is characterized as the matrix product of two statistically independent complex Gaussian matrices [9].

There are very few analytical results on pertaining to the double scattering model despite of its generality and practical significance. These few results mainly focus on single stream beam-forming, space-time block codes, ergodic channel capacity and diversity-multiplexing tradeoff. The performance of MIMO MB has not been studied yet. In this context, we presented in some analytical results on the average SER of the MIMO MB system. These results, though applicable to the whole range of SNR, are extremely complex, and thus provide very few insights. To gain more insights into the system and the channel, in this paper we focus on the SER performance in the high-SNR regime. Our purpose is to get an approximate expression for the average SER, which becomes accurate at high SNR. The main difficulty in doing this is to derive the asymptotic expansion on the eigen-value distribution of the channel matrix. The conventional deriving technique, i.e., the *differential-based method* [3],[10],[11] is not applicable here as the eigen-value distribution is not even continuous. To solve the problem, we herein propose a new technique, called the *Expand-Remove-Omit method*, which can be applied to both differentiable and non-

differentiable functions. By applying the new technique, we get the desired asymptotic expansion, as well as the approximate SER expression. The average SER at high-SNR turns out to be completely characterized by two parameters, the diversity gain and the coding gain, where the diversity gain determines the slope of the SER curve (on a log-to-log scale), while the coding gain determines the SNR gap between the SER curve and the benchmark curve. We prove that the diversity gain of MIMO MB in the double-scattering channel is upper bounded by the diversity gain in the corresponding Rayleigh channel. If the number of scatterers in the double-scattering channel is above a certain level, the same diversity gain as the Rayleigh channel can be achieved. We also show that the double-scattering channel is distinctly different from the Rayleigh and Rician channels in terms of coding gain. Although the coding gain of MIMO MB in Rayleigh and Rician channels is well known to be a constant independent of the SNR [3],[5] the coding gain in the double-scattering channel will vary with the SNR logarithmically, if the number of scatterers is below a certain level. The organization of this paper is as follows: Section-2 presents the MIMO MB system

model. Section-3 derives the asymptotic expansion on the eigen-value distribution. In Section-4 we apply the asymptotic expansion to analyze the SER performance of MIMO MB. The summary and conclusion is given in last section.

2. SYSTEM MODEL AND PROBLEM FORMULATION :

A. MIMO MB System Model

We consider a MIMO wireless communication system having n_T transmit and n_R receive antennas. The received vector is given is given by

$$Y = HX + n \quad (1)$$

where $H \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $X \in \mathbb{C}^{n_T \times 1}$ is the transmitted signal vector, and $n \in \mathbb{C}^{n_R \times 1}$ is the complex additive white Gaussian noise (AWGN) vector with zero mean and identity covariance matrix. In MIMO MB, under the assumption of perfect CSI at the transmitter, the transmit vector X is formed by mapping L ($\leq \text{rank}(H)$) modulated symbols \hat{d} ($\triangleq (d_1, \dots, d_L)^T$) onto n_T transmit antennas via linear pre-coding

$$X = Pd \quad (2)$$

with $P \in \mathbb{C}^{n_T \times L}$ denoting the spatial pre-coding matrix. Here, the columns of are the right singular vectors of H , which correspond to the L largest singular values.

Under the assumption of perfect CSI at the receiver, the combiner of MIMO MB forms the decision statistics $\hat{d} (\triangleq (\hat{d}_1, \dots, \hat{d}_L) T)$ by weighting the received vector y with a spatial equalizing matrix $Q \in \mathbb{C}^{nR \times L}$

$$\hat{d} = QH Y \quad (3)$$

where the columns of Q are the left singular vectors of H , which corresponds to the L largest singular value. After such pre-coding and equalization, the MIMO channel is decomposed into a set of equivalent single-input single-output (SISO) channels, whose input-output relation is where the columns of Q are the left singular vectors of H , which correspond to the L largest singular values. After such pre-coding and equalization, the MIMO channel is decomposed into a set of equivalent single-input single-output (SISO) channels, whose input-output relation is given by

$$\hat{d}_k = \sqrt{\lambda_k} d_k + n_k, \quad (k=1, \dots, L) \quad (4)$$

where λ_k is the k -th largest eigen-value of HH^H , and n_k is the complex AWGN with zero mean and unit variance. In this paper, we denote each SISO channel a sub-stream of the MIMO MB system. Letting ρ_k denote the power allocated to the k -th sub-stream, the instantaneous output SNR of this sub-stream is given by

$$\gamma_k = \rho_k \lambda_k, \quad (k=1, \dots, L) \quad (5)$$

Clearly, the output SNRs and the average SERs of the sub-streams depend directly on the distributions of the eigen-value λ_k s. It is worth noting that the power allocating strategy considered here is the so-called fixed power allocation [3], [5], i.e., $\rho_k = \phi_k \rho$ subject to $\sum_{k=1}^L \phi_k = 1$, where ρ is the total transmit power, and ϕ_k is a constant satisfying $0 < \phi_k \leq 1$. The reason for adopting this simple strategy is: in the high SNR regime, the optimal water-filling strategy tends to the uniform power allocation, i.e., a special case of the fixed allocation strategy ($\phi_k = 1/L$) [4, App.IV]. As the main focus of this paper is on the system performance at high SNR, fixed power allocation serves that purpose very well.

In the (uncorrelated) double-scattering model, the channel matrix H is given by

$$H = (1/\sqrt{nS}) H_1 H_2 \quad (6)$$

where $H_1 \in \mathbb{C}^{nR \times nS}$ and $H_2 \in \mathbb{C}^{nS \times nT}$ are mutually independent complex Gaussian matrices, whose elements are independent and identically distributed with zero mean and unit variance. By controlling the number of scatterers (nS), the double scattering model embraces a broad family of fading channels. For instances, when $nS = 1$, it models the keyhole channel; when $nS \rightarrow \infty$,

it models the standard Rayleigh fading (due to the law of large numbers). For brevity, we hereafter use the three-tuple, (n_R, n_S, n_T) to denote the double-scattering channel above.

B. Problem Formulation

This paper investigates the average SER of the sub-streams of the MIMO MB system above. In particular, we study two important parameters pertaining to the average SER at high SNR, i.e., the diversity gain and the coding gain. To give definitions for the two gains, we reproduce below the analysis framework proposed by Wang and Giannakis [12]. The instantaneous SER of general modulation formats (BPSK, BFSK, M -PAM, etc.) in the AWGN channel can be expressed as a function of the instantaneous received SNR [13]

$$SER(\gamma) = Q(\sqrt{2b\gamma}), \quad (7)$$

where $Q(\cdot)$ is the Gaussian Q -function, a and b are modulation specific constants, e.g., $a = 1$ and $b = 1$ for BPSK. When channel fading is taken into account, the concept of average SER becomes more useful as it reflects the influence of the fading. The average SER is obtained by averaging the instantaneous SER, $SER(\gamma)$, over all random realizations of γ . Assuming that the instantaneous SNR γ is given by the product

of a channel-dependent parameter ζ and a deterministic positive quantity γ [10], i.e.

$$\gamma = \zeta \gamma \quad (8)$$

The average SER, denoted by $SER(\zeta, \gamma)$, is then given by,

$$SER(\zeta, \gamma) = \int_0^\infty SER(\zeta \gamma) f_\zeta(x/\zeta) dx, \quad (9)$$

where $F_\zeta(\cdot)$ is the cumulative distribution function (CDF) of the random variable ζ . Generally speaking, obtaining closed form expression for the average SER is difficult as the integral in (9) may yield no analytical result [3]. Although in a few cases closed-form results exist, the exact expressions there provide very limited insights as they are prohibitively complex, e.g., see [13]. To avoid such intractability and to gain more insights into the system, the approximate average SER, which becomes accurate at high SNR, is studied instead. This is where Wang and Giannakis's analysis framework [12] came in. In their work, they assumed that the CDF of ζ around zero could be approximated by a single-term polynomial, i.e.,

$$F_\zeta(x) = \alpha x^{Gd} + o(x^{Gd}), \quad (10)$$

where α and Gd are two positive constants, $o(x^{Gd})$ is the higher-order infinitesimal of x^{Gd} as x approaches zero. By substituting

$F\zeta(x)$ back into (9), they finally arrived at a conclusion that the average SER at high SNR was characterized by two parameters, the diversity gain and the coding gain, i.e.,

$$\text{SER}(\gamma) \approx (G_a \gamma)^{-G_d} \quad (11)$$

with G_d being the diversity gain, and $G_a(\alpha)$ a function of α being the coding gain.

In this paper, we apply Wang and Giannakis's framework to analyze the average SERs of the MIMO MB sub-streams at high SNR. Since the key step in the framework is the asymptotic expansion of $F\zeta(x)$, our focus in next section is on the asymptotic expansion of the marginal CDF of the Eigen value $\lambda_k (k=1 \dots M)$.

3. ASYMPTOTIC EXPANSION OF THE EIGENVALUE DISTRIBUTION:

First of all we present the exact expression on the Eigen value distribution of HHH. Based on the exact distribution, we then derive its asymptotic expansion. For notational convenience, we define through the rest of this paper: $S \triangleq \min(n_R, n_S)$, $T \triangleq \max(n_R, n_S)$, $M \triangleq \min(S, n_T)$, $N \triangleq \max(S, n_T)$, $P \triangleq \min(N, T)$, $Q \triangleq \max(N, T)$, $R \triangleq \min(T, n_T)$.

From [2, Eq.(12)]
$$F\lambda_k(z) = \mathcal{K}ns \sum_{l=0}^{M-k} (-1)^l \binom{M-k-l}{l} \times \det Y_{s, l, \beta} \beta_1 \dots \beta_{k-l-1} \beta_{k-1} \dots \beta_M, (z \geq 0), \quad (12)$$

$$h(z, a, b, c) = c! \int_0^1 (a+c+1)! b^{a+c+2} - 2b^{(a+c+2-n)/2} n! c^n = 0 \times z^{(a+c+2+n)/2} K_{a+c+2-n}(2\sqrt{z/b}) \quad (13)$$

with $K_\nu(\cdot)$ being the modified Bessel function of the second kind [14, Eq.(8.432.6)].

To see the complexity of the exact SER result, we substitute (12) back into (9), and get an expression consisting of special functions, determinants, and integrals. Knowing this, we turn our attention to the approximate SER. Our first step is to derive an asymptotic expansion on the eigen-value distribution, but unfortunately, we find that the conventional deriving technique not applicable here. This conventional technique, termed as *differential-based method* [3], [10], [11], requires the function to be expanded to be differentiable around zero. However, the CDF here is not even continuous around zero (as the modified Bessel function $K(\cdot)$ is discontinuous at the origin for $\nu \in \mathbb{Z}$). A deriving technique that applies for double scattering channels of arbitrary configurations (n_R, n_S, n_T) is needed.

4. PROPOSED METHOD:

In this context, we propose here the Expand Remove-Omit method, which does not require the differentiability of the CDF, and, more importantly, is applicable to arbitrary double-scattering channels.

1) For a given vector β , let $(\alpha_1, \dots, \alpha_{M+1-k})$ is a permutation of $(\beta_k, \dots, \beta_M)$, $\Delta_0 = \det Y_s z, 0, \beta$ and $i = 1$;

2) Expand the α_i -th column of the matrix Δ using the multi-linear property of the determinant (see below), and get multiple matrices with exponential terms of different orders (let $h_{i,j}(\cdot)$ denote a generic function, and “\” denote “except”)

$$\det \begin{cases} \sum_l h_{i,j}(\cdot), & i = 1 \dots n; j = m; \\ \alpha_i, j, & i = 1 \dots n; j = \{1, \dots, n\} \setminus m; \\ h_{i,j}(\cdot), & i = 1 \dots n; j = m; \end{cases} \\ = \sum_l \{ \alpha_i, j, & i = 1 \dots n; j = \{1, \dots, n\} \setminus m; \}$$

3) Remove matrices with co-linear columns as their determinants are zero-valued;

4) Omit other matrices, leaving only the one with the lowest-order exponential term; denote the remaining matrix as Δ_i ;

5) Let $i = i + 1$;

6) If $i \leq M + 1 - k$, go back to 2); otherwise, continue;

7) If all permutations of $(\beta_k, \dots, \beta_M)$ have been used, continue; otherwise, update $(\alpha_1, \dots, \alpha_{M+1-k})$ with a new

permutation of $(\beta_k, \dots, \beta_M)$ and go back to 2);

8) Sum up determinants of the remaining matrices for all possible $(\alpha_1, \dots, \alpha_{M+1-k})$, and get the following equality:

$$\det[\Delta_0] = \alpha_1, \dots, \alpha_{M+1-k} \det[\Delta_{M+1-k}] + \mathcal{O},$$

where \mathcal{O} denotes the higher-order infinitesimal;

9) Factor out all exponential terms of $\det[\Delta_0]$ into z^{dk} minimize dk over all possible β , and finally get the desired term dk . The remaining part of $\det[\Delta_0]$ after the factorization then equals $c_k(z)$.

Detailed description of the method is in [1, App. A], here we present directly its expanding result.

Theorem 1. The marginal CDF $F_{\lambda k}(z)$ can be expanded as

$$(k = 1, \dots, M)$$

$$F_{\lambda k}(z) = c_k(z) z^{dk} + o(z^{dk}), \quad (14)$$

$$\text{where } c_k(z) = \sum_{i \in S_k} C_{k,i} (\ln z)^i$$

$$\text{and } d_k = \frac{(nR+1-k)(nT+1-k)(N+1-k)}{\max(nR, nS, nT)+1-k} \\ \left[\frac{[(nR+nS+nT-2\max(nR, nS, nT))+1-k]^2}{4} \right].$$

$o(z^{dk})$ is the higher-order infinitesimal of z^{dk} as z approaches zero, $C_{k,i}$ is a constant coefficient, and S_k is a set of nonnegative integer numbers. Both S_k and $C_{k,i}$ are uniquely determined by [2, Eq. (21)].



Proof: See [2, App. A]

From the [2, corollary 1]

$$F_{\lambda k}(z) = C_k, 0 zdk + o(zdk), (15)$$

$$\text{where } d_k = \frac{(nR+1-k)(nT+1-k)(N+1-k)}{\max(nR, nT, nT)+1-k}$$

$C_{k,0} = \frac{(-1)^{(S-M)(S+M-1)/2}}{\prod_{i=1}^M (R-i)! \prod_{i=1}^S (S-i)! (Q-i)!} \det(\Xi_S)$, with Ξ_S being an S matrix, whose (i,j) -th element is

$$\{\Xi_S\}_{i,j} = \begin{cases} \left(\sum_{i=0}^{M-k} \frac{(-1)^{S-M+i}}{(S-M+i)(P-M+j+i)} (Q+P+i-2-l)! nT^{P-i+1+l} \right) & i = 1 \dots S; j = 1 \dots M+1-k, \\ (P-S+j-1)! (Q-M+i+j-2)! nT^{M-i-j+1} & i = 1 \dots S; j = M+2-k \dots M, \\ (-1)^{(S-j)} (Q-M-P+i+j-2)! nT^{-(M+P-N+i-j+1)} & \text{for } i = 1, \dots, S; j = M+1, \dots, S, \end{cases}$$

Proof: See [2, App B].

5. SYSTEM PERFORMANCE IN THE HIGH-SNR REGIME :

Now, we apply the asymptotic expansion to analyze the performance of MIMO MB in the high-SNR regime. We express the average SER of the k -th strongest sub-stream as ($k = 1 \dots L$)

$$SER_k(\rho_k) = \frac{ak \sqrt{b}}{2\sqrt{\pi}} \int_0^\infty x^{-1/2} e^{-b k x} F_{\lambda k}(x/\phi_k \rho) dx, (16)$$

Theorem 2. *At high SNR, the average SER of the k -th strongest sub-stream of the MIMO MB system can be approximated as ($k = 1, \dots, L$)*

$$SER(\rho_k) \approx [Ga(k) \rho k]^{-Gd(k)}, (17)$$

$$\text{where } G_d(k) = \frac{(nR+1-k)(nT+1-k)(nT+1-k)}{\max(nR, nT, nT)+1-k} \left[\frac{((nR+nT+nT-2\max(nR, nT, nT)+1-k))^2}{4} \right]$$

And if (and only if) $nR + nT + nS + 1 - k = 2 \max(nR, nS, nT)$, $Ga(k)$ is a constant independent of the SNR ρ , given by

$$Ga(k) = b_k \left(\frac{ak C_{k,0} \Gamma[G(k)+1/2]}{2\sqrt{\pi}} \right)^{-1/Gd(k)}$$

Proof: The desired result is easily obtained by substituting (20) into (16), invoking the binomial theorem, and omitting the higher-order infinitesimal.

Comparing Theorem 2 to Wang's results in Section 2-B, we find that, for those cases where $nR + nT + nS + 1 - k > 2 \max(nR, nS, nT)$, the term $Ga(k)$ does not meet the definition of the coding gain. The coding gain is defined as a constant independent of the SNR, but here it may vary with the SNR. However, $Gd(k)$ here agrees completely with the conventional definition of the diversity gain [15], i.e., Diversity Gain $\triangleq -\lim_{SNR \rightarrow \infty} \log SER(SNR) / \log SNR$. Noticing that (k) is exponentially

equal to a constant, we now extend Wang's definitions to cover the general double scattering channels. Discussions the two gains are given as follows:

A. Diversity Gain

According to Theorem 2, the diversity gain of the k -th sub stream is ($k = 1, \dots, L$)

$$G_d(k) = \frac{(nR+1-k)(nT+1-k)(nT+1-k)}{\max(nR, nT, nT)+1-k} - \frac{[(nR+nT+nT-2\max(nR, nT, nT)+1-k)^2]}{4} \quad (18)$$

Where the subtrahend, i.e., the $\frac{[(nR+nT+nT-2\max(nR, nT, nT)+1-k)^2]}{4}$ term, vanishes if and only if $nR+nT+nT+1-k \leq 2\max(nR, nT, nT)$. First of all, we use a (3, 3, 3) double-scattering channel to verify the analytical expression on the diversity gain. For simplicity, we assume that all MIMO MB sub-streams are active, upon which uniform power allocation and coherent BPSK are employed. The average SERs of all the sub-streams are plotted in Fig.1, where each “Analytical SER” curve is

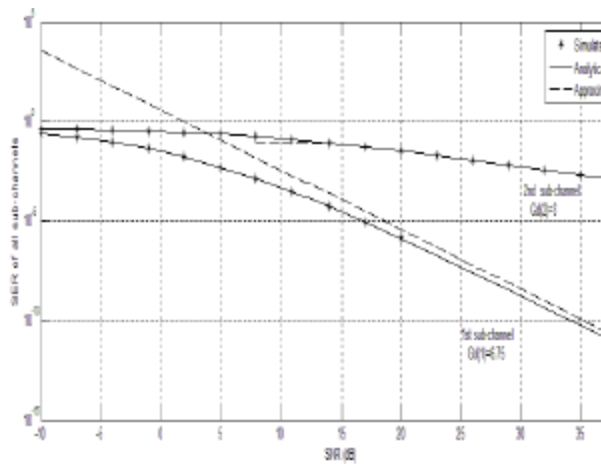


Fig.1. SER of all MIMO MB sub-streams in a (3, 3, 3) double-scattering channel

computed by substituting (12) into (16). Clearly, we can see that two diversity gains, 6.75 and 3, are attained by the two sub-

streams, respectively, which is in perfect agreement with our theoretical result (18).

From (18), we can say that the diversity gain of a (nR, nS, nT) double-scattering channel is less than or equal to $(nR+1-k)(nT+1-k)$ which equals the diversity gain of the corresponding Rayleigh channel (nR, ∞, nT) . However, the question that follows is not as intuitive and deserves more discussions. Whether or not a double-scattering channel can achieve the same diversity gain as the Rayleigh channel, when the number of scatterers is limited? To answer this, we need to revisit (18) which says that, when $nT \geq nR+nT-1$ is true the upper-bound diversity gain is attained, which further indicates that, for finite nR and nT , it suffices for the double-scattering channel to have only $nR+nT-1$ scatterers, if the full diversity gain $nR \times nT$ is required. The explanation for this result shows the basic idea of diversity. General speaking, the diversity gain is proportional to the number of independent fading coefficients in the MIMO channel matrix. For example in a (nR, ∞, nT) Rayleigh channel, there exists $nR \times nT$ independent fading coefficients. Hence, the maximum diversity gain of the channel equals $nR \times nT$. The situation in the double-scattering channel is almost similar,

except that the maximum number of independent fading coefficients maybe less than $nR \times nT$, which is due to faded replicas which are added up at the scatterers, which may break the independence between the received replicas, during the double scattering process. An example on this point is the keyhole channel $(nR, 1, nT)$, where only $\min(nR, nT)$, independent fading paths exist. Clearly, the double-scattering process has imposed some kind of correlation to the received replicas. When the scattering condition is poor (i.e., $nT < nR + nT - 1$), the correlation imposed is so strict, that only a (small) portion of the independent replicas can be extracted. Also when the scattering condition is good (i.e., $nT \geq nR + nT - 1$), the correlation is bland and can be removed. In that case, the maximum diversity gain $nR \times nT$. is achieved. To see the impact of the scatterer on the diversity gain, we fix

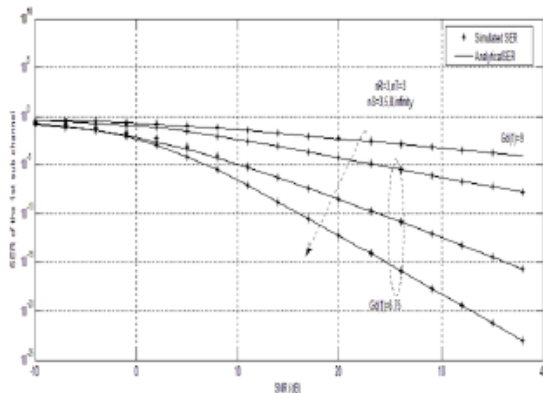


Fig.2. SER of the strongest sub-stream in four double-scattering channels: $(3,3,3), (3,5,3), (3,8,3)$ & $(3, \infty, 3)$

average SER of the strongest sub-stream is plotted in Fig. 2. In the $(3, 3, 3)$ case, we observe a diversity gain of 6.75, which is exactly the same as we expected from (18). In the remaining cases, we notice that, once the number of scatterer is greater than 5, adding more scatterers into the channel will not change the diversity gain. This verifies our earlier analysis that the diversity gain reaches its upper bound whenever $nS \geq nR + nT - 1$.

Also from (18) we observe that the diversity gain is independent of the order of (nR, nS, nT) . i.e., $(nR, nS, nT), (nR, nT, nS), (nT, nR, nS), (nT, nS, nR)$, and $(nS, nR, nT), (nS, nT, nR)$, are indeed equivalent. This is an extension to the results of [4, Theo. 2] and [3, Theo. 4], where they showed that interchanging nR with nT would not change the diversity gain of the Rayleigh/ Rician channel. Besides this rotational symmetry, we also see that the diversity gain of the k -th sub-stream in a (nR, nS, nT) channel is indeed equivalent to that of the first sub-stream in a $(nR+1-k, nT+1-k, nT+1-k)$ channel. This indicates that reducing the sub-stream index by 1 is equivalent, by reducing the number of transmit antenna; receive antenna and scatterers all by one in the sense of diversity gain.

B. Coding Gain

According to Theorem 2, the coding gain of the k MIMO- MB sub-stream is ($k=1...L$)

$$G_d(k) = \left[\frac{ak\sqrt{b}}{2\sqrt{\pi}} \sum_{i \in S_k} C_{k,i} (-1)^i (\ln \rho + \ln \phi k)^i \times \int_0^\infty x^{G_d(k)-1/2} e^{-bkx} (\ln x)^i dx \right]^{-1/G_d(k)} \quad (19)$$

where the expression simplifies to

$$G_d(k) = b_k \left(\frac{ak C_{k,0} \Gamma[G(k) + \frac{1}{2}]}{2\sqrt{\pi}} \right)^{-1/G_d(k)}$$

if and only if $n_R + n_T + n_T + 1 - k \leq 2 \max(n_R, n_S, n_T)$.

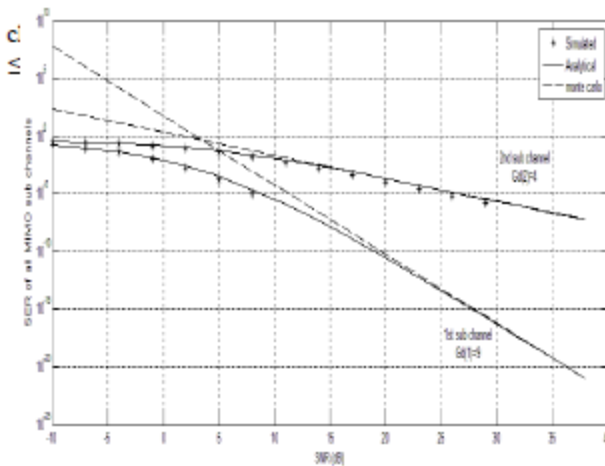


Fig.3. SER of all sub-streams in a (3, 3, 4) double-scattering channel

else, it varies logarithmically with ρ . Our result here provides a whole picture of the coding gain. To verify our more general result, we present in Fig.3 the average SER of the MIMO MB system in a (3, 3, 4) double-scattering channel. We can notice that the approximate SER results agree very well with actual curves especially in the high-SNR regime.

Given the coding gain in (19), we revisit the double scattering channel (n_R, n_S, n_T) and its Rayleigh counterpart (n_R, ∞, n_T). We notice from (19) that if and only if $n_T \geq n_R +$

n_T , the diversity gains of all the MIMO MB sub-streams are independent of the SNR. We also know that each sub-stream attains its upper-bound diversity gain whenever the number of scatterers n_S is above a certain level (i.e., $n_R + n_T - 1$). From these all, we conclude that, given the coding gain being independent of the SNR, increasing the number of scatterers will not change the diversity gain of the sub-stream. Although the diversity gain remains unchanged, the increase in the scatterer number certainly brings advantages to the coding gain, causing a horizontal (leftward) shift of the SER curve. This idea is confirmed by Fig.4, where three double-scattering channels (3,7,4), (3,9,4) and (3,∞,4) are considered. (The asymptotic SER curve of the Rayleigh faded case is computed based on [5, Eq. (34)].) In the figure, the coding gain becomes larger and larger as the scatterer number increases from 7 to 9 and infinity.

6. CONCLUSION:

We have examined the average SER performance of MIMO Multichannel Beam forming under the general double-scattering channel Model. Our results are based on two performance parameters, i.e., the diversity gain and the coding gain, which

characterized the SER of the system in the high SNR regime. In order to get analytical results on the two gains, we derived asymptotic expansions on the eigen-value distribution of the MIMO channel matrix, introducing a new method known as “Expand-Remove-Omit Method”. The asymptotic expansion was then applied to get the approximate expression for the average SER. Our results proved that the diversity gain of the double scattering channel was upper bounded by the diversity gain of the corresponding Rayleigh when the condition $n_S \geq n_R + n_T - 1$ was satisfied. Unlike conventional Rayleigh and Rician channels, where the coding gain was a constant number, we proved here that, the coding gain of the double-scattering channel was indeed a function of the SNR which becomes independent of SNR only when $n_S \geq n_R + n_T$ is satisfied.

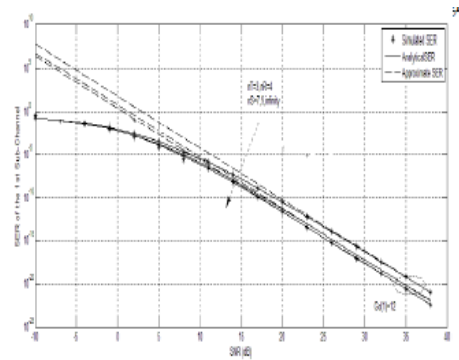


Fig.4. SER of the strongest sub-streams in three double-scattering channels: (3, 7, 4), (3, 9, 4), and (3, ∞ , 4)

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