Porosity, Permeability and Darcy's Law

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Abstract:

During recent years, the theory of fluids flowing through porous media has received great attention because of its important applications in the flow of oil through porous rocks, the flow of liquids through ion exchange beds, the extraction of energy in geothermal regions and so on. Importance of such studies in rock and heavy oil recovery attracted several researchers and they carried out the investigations of different stability problems through porous medium. The physical properties of the comets meteorites and interplanetary dust strongly suggest the significance of effects of porosity in astrophysical context. A porous medium is characterized by two properties- porosity and permeability. Darcy introduced, for the first time, permeability in terms of measurable quantities

Keywords: porosity, permeability, pressure gradient, incompressible fluid, laminar, Reynolds number.

Introduction

Pores are void spaces imbedded in a material. These may be connected or non-connected, distributed more or less frequently in either a regular or a random manner in the material. Inter-connected pores are called effective pores while the non-interconnected are the ineffective pores. By ineffective pores, we mean the pores through which the fluid cannot pass. This may be either due to surface tension caused by fine holes or the holes may not be inter-connected so that they do not affect the flow directly through such ineffective pores but affect the compressibility of the medium. In describing instability of flow phenomena in a porous medium, we consider the interconnected pores since those pores affect the flow. Permeability is that property of a porous material, which characterizes the ease with which a fluid flows through the material by an applied pressure gradient. In other words, the permeability is the fluid conductivity of the porous material.

For the horizontal flow of an incompressible fluid through a porous material of length I in the direction of the flow and cross-sectional area A, the Permeability of the material is defined as

$$k = \frac{\overline{Q}\mu}{A\left(\frac{\Delta P}{1}\right)}$$

Where $\overline{Q}$ is the volumetric flow rate, μ is the viscosity of the fluid and Δp applied pressure difference across the length of the porous bed.

Darcy's Law:

The theory of flows through porous media is largely based on a simple and interesting law obtained by classical experiments performed by Darcy while experimenting with the flow of water through sand filter. Darcy concluded that the rate of percolation of water through that filter bed is directly proportional to the cross-sectional area of the filter bed and the total force impressed on it and inversely proportional to the thickness of the bed. Fig 1 is a rough sketch of the Darcy's experiment. Here the horizontal plane areas bound a homogeneous filter of height L. The fluid enters the bed between the points 1 and 2, at the top of the bed at volumetric flow rate Q. If open manometer tubes are attached at the upper and lower boundaries of the filter bed, the difference between the pressure heads or the hydraulic head can be measured from the same datum point z = 0. Clearly $h_1 - h_2$ is the difference between the fluid heads of the inlet and outlet faces of the bed. Therefore, the Darcy's Law provides $Q = \frac{-CA\,(h_2 - h_1)}{L}$, where C is the constant of proportionality depending upon the properties of the fluid and of the porous medium and the minus sign indicates that the flow is in the opposite direction of h increasing.

In order to find the influence of porosity and the influence of the liquid separately, Nutting had defined the constant C as

$$C = \frac{\kappa}{\mu} \text{ so that } \frac{Q}{A} = U = -\frac{\kappa\,(h_2 - h_1)}{\mu L}$$

where μ is the fluid viscosity, κ is the specific permeability of the medium and U is called the seepage velocity.

This is to be kept in mind that this study of seepage is based on a statistical method so that the real flow of fluid in the pores is replaced by an imaginary seepage flow of that fluid. In terms of the pressure gradient dp/dx, the Darcy's law may be written as

$$U = \frac{-\kappa}{\mu}\frac{dp}{dx}$$

In the branch of civil engineering dealing with the seepage of water through dams, or of agriculture engineering concerned with the seepage of water out of the canals or ditches, or in any phase of homogeneous fluid flow through porous medium in which gravity is the predominating driving agency, the pressure gradient in Darcy's law may be expressed in terms of hydraulic gradient

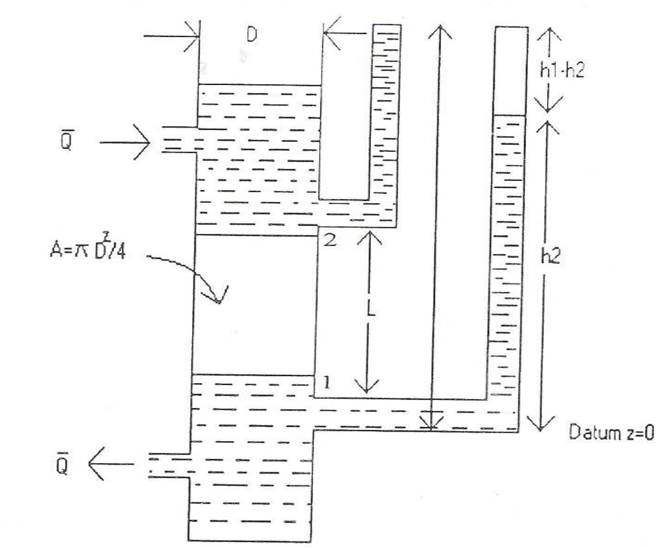

Fig. 1

$$\frac{dh}{dx}$$

where h is the fluid head and the permeability k is replaced by

$$\bar{\kappa} = \frac{\kappa \rho g}{\mu}$$

The Darcy's law as stated above is restricted to porous media in which the flow is necessarily a one-dimensioned flow. The generalized Darcy's law for three-dimensional flow, for the case when the gravity force is acting on the fluid affecting the velocity just as the pressure gradient, is given as $\frac{\mu \mathrm{U}}{\kappa} = \nabla p - gp\bar{\lambda}$, where $\mathrm{U} = (u_x, u_y\ u_z)$, and $\bar{\lambda} = (0, 0, 1)$.

Although Darcy's law is empirical, **Dewist** showed the equivalence of Darcy's law to the Navier-Stoke's equation. It was shown that the terms μ(U/k) calculated from Darcy's law and known to be Darcy's drag force or Darcy's resistance, replaces the usual viscous term $\mu\ \nabla^2 q$ in the equation of motion.

The validity of Darcy's law has been tested on many occasions and is shown to be valid for a wide range of flows. However, the use of Darcy's law is restricted to the cases in which the flow is stream lined or laminar. In the laminar flow, the individual fluid particles in a pore flow along paths roughly parallel to the walls of the pores. The laminar flow region is at low flow rates, where the inertial effects are negligible. The laminar flow region breaks down at sufficiently high flow rates. If the velocities in the pores become considerably high, the inertial forces may becomes commensurable with the frictional force. The linear relationship between the velocity and forces than no longer holds and Darcy's law loses its applicability. The range of flow rates for which laminar flow exists is defined in terms of the Reynolds number R. At a certain value of the Reynolds number, Darcy's law will fail. It is established by researchers that Darcy's law applies only for the values of R less than one. For high Reynolds numbers, the inertia force becomes high and the Darcy's law loses its applicability.

Another restriction on the use of Darcy's law applies to the flow of gases at low pressures. In such cases a different value of permeability and hence a modified form of Darcy's law is applied.

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