



Design of a Novel Reversible ALU using an Enhanced Carry Look-Ahead Adder

*K.JYOTHI

**Md.ASIM IQBAL

*M.TECH Dept Of ECE, KAKATHIYA UNIVERSITY OF ENGINEERING AND TECHNOLOGY

**Asst. prof Dept of ECE, KAKATHIYA UNIVERSITY OF ENGINEERING AND TECHNOLOGY

Abstract—Reversible logic is gaining significant consideration as the potential logic design style for implementation in modern nanotechnology and quantum computing with minimal impact on physical entropy. Recent advances in reversible logic allow schemes for computer architectures using improved quantum computer algorithms. Significant contributions have been made in the literature towards the design of reversible logic gate structures and arithmetic units, however, there are not many efforts directed towards the design of reversible ALUs. In this work, a novel programmable reversible logic gate is presented and verified, and its implementation in the design of a reversible Arithmetic Logic Unit is demonstrated. Then, reversible implementations of ripple-carry, carry-select and Kogge-Stone carry

look-ahead adders are analyzed and compared. Next, implementations of the Kogge-Stone adder with sparsity-4, 8 and 16 were designed, verified and compared. The enhanced sparsity-4 Kogge-Stone adder with ripple-carry adders was selected as the best design, and its implemented in the design of a 32-bit arithmetic logic unit is demonstrated.

Index Terms – Arithmetic Logic Unit; Carry Look-Ahead Adder; Carry-Select Adder; Emerging Technologies; Low Power; Nanotechnology; Reversible Logic; Ripple-Carry Adder; Quantum Computing

I.

INTRODUCTION

LOGICAL computing devices without a bijection between input and output states were demonstrated by Landauer to require a

minimal heat generation of $kT \ln(2)$ joules of energy per computing cycle [1]. This resulting dissipated heat also causes noise in the remaining circuitry, which results in computing errors. Bennett showed that the dissipated energy directly correlated to the number of lost bits, and that computers can be logically reversible, maintain their simplicity and provide accurate calculations at practical speeds [2]. Resultantly, a new paradigm in computer design arose with the goal of reducing the entropy increase and subsequent energy dissipation. Such a logical structure must possess the same number of inputs and outputs and a one-to-one mapping between the input and output states. Any device designed to these constraints is known as a *reversible logic device*.

Section II, we outline the goals of programmable reversible logic design and some fundamental logic gates. In Section III, a novel 5×5 programmable MG gate is proposed that may be utilized in an arithmetic logic unit requiring the calculation of AND, NAND, OR, NOR, XOR and XNOR results. In Section IV, previous work in programmable reversible arithmetic logic units is reviewed, and the proposed MG gate is implemented in the

design of a novel programmable arithmetic logic unit. In Section V, reversible adder designs are presented for a reversible ripple carry adder, reversible carry-select adder and reversible carry look-ahead adder. A reversible implementation of the carry logic presented in the Kogge-Stone is proposed and verified. In Section VII, the methods of quantum cost reduction through combining ripple-carry, carry-select and carry lookahead implementations of the reversible adder are analyzed, and the most effective ALU in terms of quantum cost and delay is selected.

II.

REVERSIBLE LOGIC

A. Programmable Reversible Design Goals

The three major design goals of reversible logic are as follows. First, minimization of the quantum cost – the number of 1×1 and 2×2 reversible calculations necessary to generate the logical output [6] - will reduce the device's computational complexity. Second, minimization of the delay - the logical depth of the device [7] – will improve the throughput of the device. Third, reduction of the ancillary inputs and garbage outputs - inputs and outputs not implemented in the design of the gate and

only serve to maintain reversibility of the device – will improve the design

space require to implement the logic A programmable reversible logic gate is defined in [8]

as a logic structure which possesses a bijection between input and output states and an equal number of inputs and outputs wherein a subset of the inputs are fixed select lines, and a fixed subset of the outputs produce guaranteed logical calculations.. An ideal programmable reversible logic gate with j inputs and outputs has a quantity of fixed select inputs m , fixed select outputs n , data inputs d and propagated outputs p such that $|m + n| = |d + p|$ [8]. In addition, an ideal programmable reversible logic gate with m select inputs may produce at maximum 2^m logical calculations on the n logical outputs [8].

B. Fundamental Logic Gates There are three types of fundamental 2*2 reversible logic gates. First, the square-root-of-not gates utilize the unitary operators to produce reversible logic calculations. The Controlled-V and the Controlled-V+ gates are the two Types of square-root-of-not gates. In both of these gates, when the

control input is 0, the second input is propagated to

the output. The corresponding unitary operator is propagated to the second output when the control input is 1, where the unitary operation is $V = \frac{i+1}{2} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$ for the Controlled-V

gate and $V^+ = \frac{1}{i+1} \begin{pmatrix} 1 & -1/i \\ i & 1 \end{pmatrix}$ for the Controlled-V+ gate. When two Controlled-V gates are activated in series, they act as an inverter. The same holds for two Controlled-V+ gates in series. When a Controlled-V and Controlled-V+ gate are activated in series, they act as an identity. The second type of fundamental 2*2 reversible-logic gate is the Feynman gate, or the Controlled-Not gate. Proposed in [9] by Feynman, it is configured such that its outputs states correlate to the input states in the following manner: $P = A$ and $Q = A \oplus B$. The resulting value of thesecond output corresponds to the result of a conventional XOR gate. Since fanout is expressively forbidden in reversible logic, since a fanout has one input and two outputs, the Feynman gate may be used to duplicate a signal when B is equal to 0. Its quantum configuration is shown in Fig 2.

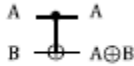


Fig 1: Quantum Representation of Feynman gate

The third type of fundamental 2*2 reversible logic gate is the integrated qubit gate. This gate is implemented with a Feynman gate with either a Controlled-V or Controlled V+ gate. The XOR output of the Feynman gate is used as the control signal for the

Controlled-V or V+ gate it is coupled with. The quantum cost of the integrated qubit gate is 1 and its worst-case delay is 1. The quantum configuration of these gates are shown below in Fig. 3.

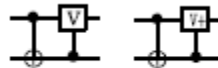


Fig 2: Quantum Representations of Integrated Qubit Gates

There are three 3*3 fundamental reversible logic gates. The first was proposed in [10] by Fredkin and Toffoli. The Fredkin gate's outputs states map to the inputs as follows:
 $P = A, Q = A'B \oplus AC$ and $R = AB \oplus A'C$
 Therefore, the outputs serve as a multiplexed output of the two data inputs based on the control input. It is realized using 2 Feynman gates, a Controlled-V gate

and two integrated qubit gates. Toffoli proposed the second fundamental 3*3 reversible logic gate in [11]. The output states of the Toffoli gate map to the inputs in this manner: $P = A, Q = B$ and $R = AB \oplus C$. The quantum cost is 5 and the worst-case delay is 5. The quantum representation is shown below in Fig. 5.

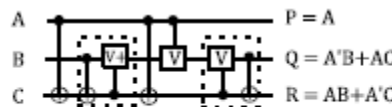


Fig 3: Quantum Representation of Fredkin gate

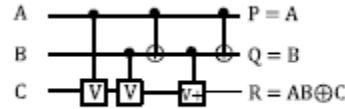


Fig 4: Quantum Representation of Toffoli gate

The 3*3 Peres gate was proposed by Peres in [12]. The Peres gate has a quantum cost of 4 and a worst-case delay of 3. The

quantum representation is shown in Fig. 6. The output states map to the inputs in this manner: $P = A, Q = A \oplus B$ and $R = AB \oplus C$.

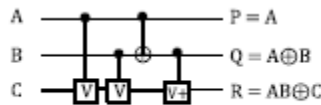


Fig 5: Quantum Representation of Peres gate

The HNG gate, also designed by Hagarparast and Navi and proposed in [13], is a full adder with inputs A,B,C,D producing

outputs $P = A, Q = B, R = (A \oplus B) \oplus C,$ and $S = (A \oplus B)C \oplus (AB \oplus D),$ and is shown in Fig. 6.

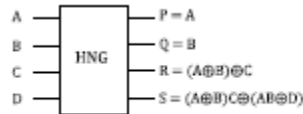


Fig. 6: Hagarparast-Navi (HNG) Gate

The Peres And-Or (PAOG) gate, proposed in [8], produces the outputs $P = A, Q = A \oplus B, R = AB \oplus C$ and $S = (AB \oplus C) \oplus ((A \oplus B) \oplus D)$. Fig. 7 shows the quantum representation of the PAOG gate. This gate is an extension of the Peres gate

for ALU realization. When the PAOG is utilized as a programmable reversible logic gate with two select inputs, it will calculate four logical calculations on those two logical outputs: AND, NAND, NOR and OR.

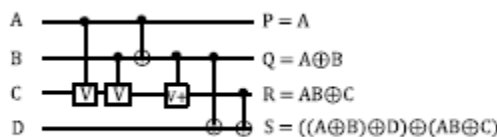
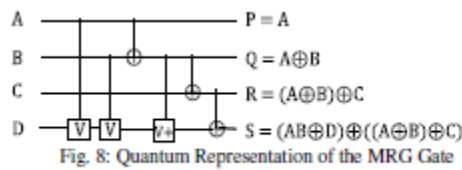


Fig. 7: Quantum Representation of the PAOG

The Morrison-Ranganathan (MRG) gate, proposed in [8], has a quantum cost of 6, since it consists of three XOR gates, 2 Controller-V and one Controller-V+ gate. The worst-case delay of the MRG gate is 6. The quantum representation of the MRG

gate is shown in Fig. 8 below. When the MRG is utilized as a programmable reversible logic gate with two select inputs, it will calculate four logical calculations on those two logical outputs: OR, NOR, XOR and XNOR.



III. PROPOSED 5*5 PROGRAMMABLE REVERSIBLE MG GATE

Next, we propose the design of a 5*5 programmable reversible logic gate structure utilized in the implementation of an ALU. Fig. 9 shows the block diagram of the MG, and the logical calculations based on the

programmable inputs are presented in Table 2. The cost of the MG is 7, and the worst-case delay is 7. The design for the programmable MG was verified and simulated using VHDL in Xilinx 12.4.

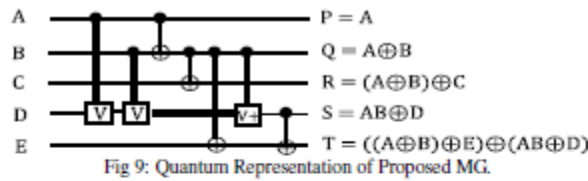


TABLE II
 MG PROGRAMMABLE INPUTS AND LOGICAL OUTPUTS

C	D	E	R	S	T
0	0	0	$A \oplus B$	AB	$A + B$
0	0	1	$A \oplus B$	AB	$(A + B)'$
0	1	0	$A \oplus B$	$(AB)'$	$(A + B)'$
0	1	1	$A \oplus B$	$(AB)'$	$A + B$
1	0	0	$A = B$	AB	$A + B$
1	0	1	$A = B$	AB	$(A + B)'$
1	1	0	$A = B$	$(AB)'$	$(A + B)'$
1	1	1	$A = B$	$(AB)'$	$A + B$

IV. MODIFIED ALU DESIGN WITH MG GATE

is 24, and the worst-case delay is 16. For n-bit ALU devices, an addition cost of 2 is incurred per bit in order to propagate S1 and S2 to other bits. Therefore, the total cost for an n-bit ALU is $26n-2$. The MG gate is utilized in the implementation of a novel arithmetic logic unit based on those proposed in [8]. The ALU, in addition to producing Two 1-bit ALUs were proposed in [8]. The first utilizes the MRG gate and HNG gate to produce six logical calculations: ADD, SUB, XOR, XNOR, OR and NOR. The ALU has 8 inputs and 8 outputs. The inputs consist of three data inputs (A, B and Cin) and five fixed input

select lines. The eight outputs are: A, S0, S3 and S4 propagated to the output, A _\$B, SUM, Cout, Overflow and Result. The cost of this 1-bit ALU the same logical calculations as the MG, is able to perform addition and subtraction by utilizing the HNG gate and store less-than operation. The cost an n-bit ALU is $37n-3$ and had a worstcase delay of $4n+13$. The proposed ALU is shown in Fig. 10, and the logical results based on the input opcodes are presented in Table 3. The design for the novel one-bit ALU was verified and simulated using VHDL in Xilinx 12.4.

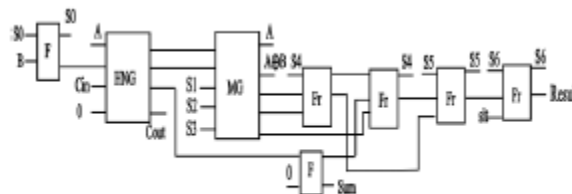


Fig. 10: Reversible ALU with MG and HNG gates

TABLE III
 ALU OPCODES AND LOGICAL RESULT FOR FIG. 9

S6	S5	S4	S3	S2	S1	S0	Result
0	0	0	0	0	0	0	ADD
0	0	0	0	0	0	1	SUB
0	0	1	0	0	0	0	$A + B$
0	0	1	1	0	0	0	$(A + B)'$
0	1	0	0	0	0	0	$A \oplus B$
0	1	0	0	0	1	0	$A = B$
0	1	1	0	0	0	0	AB
0	1	1	0	1	0	0	$(AB)'$
1	0	0	0	0	0	0	SLT

V. REVERSIBLE ADDER DESIGNS AND COMPARISON

A. Design of Reversible Ripple-Carry and Carry-Select Adders

In order to design the most efficient 32-bit reversible arithmetic logic unit, we designed and compared reversible implementation of ripple-carry, carry-select and carry lookahead adders. A reversible ripple-carry adder and a reversible carry-select adder are designed using the new ALU. The ripple-carry adder has a cost of $40n-3$ and a delay of $4n + 13$. The most-significant bit of the ALU ties the Sum output to the SLT input of the least significant bit, and the SLT input for all other bits is 0. The reversible carry-select adder uses the carry out of the first $n/2$ bits as the control signal to a Fredkin gate implemented as a multiplexer, since the next

$n/2$ bits are calculated with both a carry-in of 0 and a carry-in of 1, which requires a cost of $40(3n/2)-3$ and a delay of $2n + 19$.

B. Reversible Kogge-Stone Cumulate Logic

Next, a reversible carry look-ahead adder based on the Kogge-Stone adder is presented [9]. First, a RKS Cumulate utilized in the calculation of the carry out signal is designed and verified. The logical structure of the RKSC is shown in Fig. 11. The cost of the RKSC is 14 and it has a worst-case delay of 4. A cost and delay analysis of the presented adders in the ALU implementation is presented in Table 4.

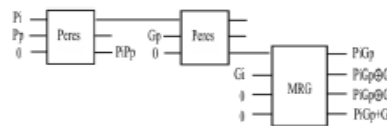


Fig. 11: Reversible Kogge-Stone Cumulate (RKSC) Logical Layout

TABLE IV
 COST AND DELAY COMPARISON OF 32-BIT REVERSIBLE ADDERS

	Ripple-Carry	Carry-Select	Kogge-Stone
Cost	192	293	2287
Delay	128	83	25

The ripple-carry adder has the lowest cost, but the highest delay. The carry look-ahead adder has a prohibitively-high cost, but the most-desirable delay.

C. Comparison of Reversible Ripple-Carry and Carry- Select Adders with Sparsity

The Kogge-Stone adder may be enhanced to reduce overhead and design complexity by generating a carry every n -bits instead of every bit, and the carry is used for the

carryin of an n -bit ripple-carry or carry-select adder. The number n is defined as sparsity. This implementation was designed and tested for sparsity-4, 8 and 16 with the corresponding nbit ripple-carry and carry-select adders. The cost and delay

comparison of each implementation is shown in Table 5.

The design for each adder presented in Table 5 was verified using VHDL in Xilinx 12.4.

TABLE V
COST AND DELAY COMPARISON OF MODIFIED CARRY LOOK-AHEAD IMPLEMENTATIONS FOR REVERSIBLE ADDERS

	Sparsity-4	Sparsity-8	Sparsity-16
Ripple-Carry	Cost: 666 Delay: 40	Cost: 462 Delay: 56	Cost: 316 Delay: 88
Carry-Select	Cost: 802 Delay: 37	Cost: 578 Delay: 45	Cost: 422 Delay: 61

VI. REVERSIBLE CARRY LOOK-AHEAD DESIGN AND IMPLEMENTATION IN A REVERSIBLE ALU

A 32-bit reversible ALU utilizing a sparsity-4 carry lookahead, adder implemented with ripple-carry has a cost of 1656 and a delay of 59. A 32-bit ALU utilizing a sparsity-8 carry look-ahead, adder implemented with carry-select has a cost of 1568 and a delay of 64. The carry-save implementation represents a 5.3% improvement in cost over the ripple-carry implementation, while the ripple-carry produces a 7.8% improvement in delay. Therefore, the modified carry look-

ahead adder with sparsity-4 implemented with 4-bit ripple-carry addition is ideal for implementation in the reversible arithmetic logic unit. A 4-bit implementation of this adder is shown in Fig. 12 below. The figure shows the propagate and generate signal logic for any adder which is not the most-significant 4-bits or the least significant 4-bits of adder. The least-significant adder is different in that the LSB does not have input propagate or generate signals, so the RKSCs are not necessary. The most significant adder is different in that it does not need to generate any future propagate or generate signals other than the initial carry.

Therefore, the HNG gates for the second, third and fourth bit which produce the propagate and generate signals are not required, nor is the RKSC for the third bit. The design for the entire 32-bit ALU was verified

and simulated using VHDL in Xilinx 12.4.

VII.

CONCLUSION

A novel 5*5 programmable MG gate was proposed and verified that may calculate of AND, NAND, OR, NOR, XOR and XNOR depending on the inputs from the programmer. The proposed MG gate was implemented in the design of a novel programmable arithmetic logic unit, and its design compared to previous work in programmable ALU design. The novel 1-bit ALU required only minimal increase in quantum cost and delay due to the MG

design, which also allowed for increased functionality for the programmer. Next, we presented reversible implementations of ripple carry adder, reversible carry-select adder and reversible carry look-ahead adder. A reversible implementation of the carry logic presented in the Kogge-Stone was presented and verified. Using these designs, modified Kogge-Stone implementations of these adders were compared in terms of quantum cost and delay. The modified Kogge-Stone carry look-ahead adder with sparsity-4 implemented with 4-bit ripple-carry addition was determined to be the ideal design implementation in the reversible arithmetic logic unit. This adder was then implemented with the presented one-bit ALU to produce an efficient 32-bit ALU capable of addition, subtraction and all of the functions of the programmable MG.

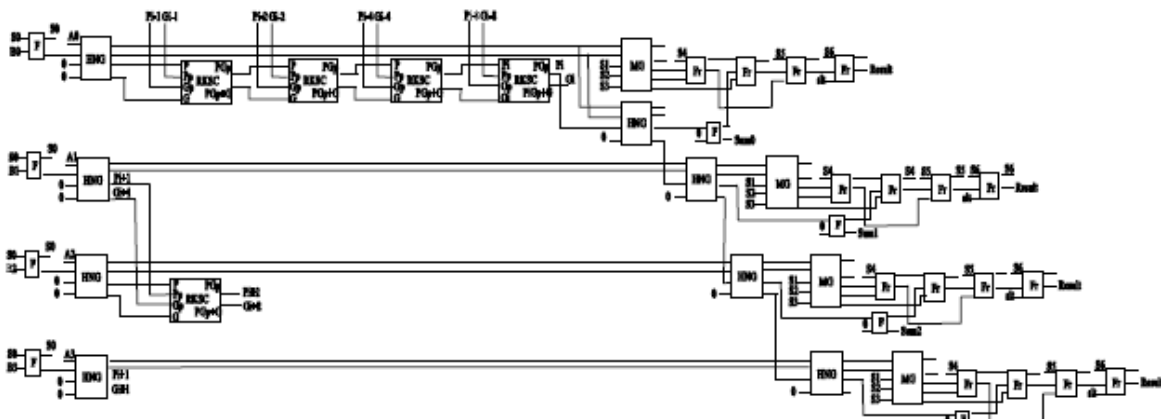


Fig. 12: Reversible 4-bit ALU with Modified Kogge-Stone Adder (Not Most Significant or Least Significant)

REFERENCES

- [1] R. Landauer, "Irreversibility and Heat Generation in the Computational Process," *IBM Journal of Research and Development*, vol. 5, 1961, pp. 183-91.
- [2] C. Bennett, "Logical Reversibility of Computation," *IBM Journal of Research and Development*, vol. 17, 1973, pp. 525-532.
- [3] R. Feynman, "Simulating Physics with Computers," *International Journal of Theoretical Physics*, 1982.
- [4] R. Feynman, "Quantum Mechanical Computers," *Foundations of Physics*, vol. 16, iss. 6, 1986,
- [5] E. Fredkin and T. Toffoli, "Conservative Logic," *International Journal of Theoretical Physics*, vol. 21, 1980, pp. 219-53.
- [6] T. Toffoli, "Reversible Computing," *Technical Report MIT/LCS/TM- 151*, 1980.
- [7] A. Peres, "Reversible Logic and Quantum Computers," *Physical Review*, vol. 32, iss. 6, 1985, pp. 3266-3267.
- [8] M. Morrison and N. Ranganathan, "Design of a Reversible ALU Based on Novel Programmable Reversible Logic Gate Structures," To Appear in the *IEEE International Symposium on VLSI*, 2011.

AUTHOR 1:-

*K.JYOTHI completed her B.Tech in Dr.PAUL RAJ ENGINEERING COLLEGE in 2014 and M.Tech completed in KAKATHIYA UNIVERSITY OF ENGINEERING AND TECHNOLOGY

AUTHOR 2:-

**Md.ASIM IQBAL working as Asst. prof in Dept of ECE, KAKATHIYA UNIVERSITY OF ENGINEERING AND TECHNOLOGY