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A Study on TwoTypes of Two Dimensional Line Integral Problems

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Abstract

In this paper, wesolve two types of two dimensional line integrals by using a complex integral formula. Moreover, we provide some examples to do calculation practically. The method adopted in this study is to find solutions through manual calculations and verify the solutions using Maple.This method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking.

Key Words:line integrals; complex integral formula; Maple

1. Introduction

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research.

In calculus and engineering mathematics, there are many methods to deal with the integral problems which including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. In this article, we study the followingtwo types of two dimensional line integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int_\gamma \left[\exp(x)\left(\cos y \ln\sqrt{x^2+y^2} - \sin y \tan^{-1}\left(\frac{y}{x}\right) \right) dx - \exp(x)\left(\sin y \ln\sqrt{x^2+y^2} + \cos y \tan^{-1}\left(\frac{y}{x}\right) \right) dy \right], \quad (1)$$

$$\int_\gamma \left[\exp(x)\left(\sin y \ln\sqrt{x^2+y^2} + \cos y \tan^{-1}\left(\frac{y}{x}\right) \right) dx + \exp(x)\left(\cos y \ln\sqrt{x^2+y^2} - \sin y \tan^{-1}\left(\frac{y}{x}\right) \right) dy \right], \quad (2)$$

where γ is a piecewise smooth curve in R^2 defined by $\gamma(t) = (x(t), y(t))$, $t \in [t_1, t_2]$

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, and $x(t) > 0$ .The two types of lineintegralscan be determined by using a complex integral formula; these are the main results of this paper (i.e., Theorems1 and 2).Adams et al. [1], Nyblom [2], and Oster [3] provided some methods to solve the integral problems. On the other hand, Yu [4-27], Yu and Chen [28], and Yu and Sheu [29-31]used some techniques, for example, complex power series, integration term by term theorem, Parseval's theorem, area mean value theorem, and generalized Cauchy integral formula to solve some types of integrals. In this article, we propose some examples to demonstrate the manual calculations, andverifythe results using Maple.In addition, two examples are used to demonstrate the proposed calculations.The research methods adopted in this study is to find solutions through manual calculations and verify these solutions by using Maple.

2. Methods and Results

In the following, some definitions and formulas used in this paper are introduced.

2.1 Definition:

The complex logarithmic function $\ln z$ is defined by $\ln z = \ln|z| + i\phi$, where $i = \sqrt{-1}$, z is a complex number, ϕ is a real number, $z = |z| \cdot \exp(i\phi)$, and $-\pi < \phi \le \pi$.

2.2 Formulas:

2.2.1 Euler's formula:

$\exp(i\theta) = \cos\theta + i\sin\theta$, where θ is any real number.

2.2.2 DeMoivre's formula**:**

$(\cos\theta + i\sin\theta)^p = \cos p\theta + i\sin p\theta$, where p is any integer, and θ is any real number.

To derive the main results in this article, we need the following two lemmas, and the second one is the complex integral formula used in this paper.

Lemma 1 *If x, y are real numbers with $x > 0$, then*

$$\ln(x+iy) = \ln\sqrt{x^2+y^2} + i\tan^{-1}\left(\frac{y}{x}\right). \quad (3)$$

Proof $\ln(x+iy)$

$$= \ln\left[\sqrt{x^2+y^2}\left(\frac{x}{\sqrt{x^2+y^2}} + i\frac{y}{\sqrt{x^2+y^2}}\right)\right]$$

$$= \ln\sqrt{x^2+y^2} + \ln\left(\frac{x}{\sqrt{x^2+y^2}} + i\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$= \ln\sqrt{x^2+y^2} + i\tan^{-1}\left(\frac{y}{x}\right).$$

(by Definition 2.1) q.e.d.

Lemma 2 *If z is a complex number and $z \neq 0$, then*

$$\int \exp(z)\cdot \ln z dz$$

$$= [\exp(z)-1]\ln z - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}z^n + C, \quad (4)$$

where C is a constant.

Proof $\int \exp(z)\cdot \ln z dz$

$$= \exp(z)\cdot \ln z - \int \frac{\exp(z)}{z}dz$$

(by integration by parts)

$$= \exp(z)\cdot \ln z - \int \sum_{n=0}^{\infty}\frac{1}{n!}z^{n-1}dz$$

$$= [\exp(z)-1]\cdot \ln z - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}z^n + C. \text{ q.e.d.}$$

Next, the solution of line integral (1) can be obtained below.

Theorem 1 *If $\gamma : [t_1, t_2] \to R^2$ is a piecewise smooth curve in R^2 defined by $\gamma(t) = (x(t), y(t))$ with $x(t) > 0$,*

$t \in [t_1, t_2]$, *and let*

$$F(x,y) = [\exp(x)\cos y - 1]\cdot \ln\sqrt{x^2+y^2} - \exp(x)\sin y \tan^{-1}\left(\frac{y}{x}\right) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(\sqrt{x^2+y^2})^n \cos\left[n\tan^{-1}\left(\frac{y}{x}\right)\right],$$

then

$$\int_\gamma \left[\exp(x)\begin{pmatrix}\cos y \ln\sqrt{x^2+y^2} \\ -\sin y \tan^{-1}\left(\frac{y}{x}\right)\end{pmatrix}dx - \exp(x)\begin{pmatrix}\sin y \ln\sqrt{x^2+y^2} \\ +\cos y \tan^{-1}\left(\frac{y}{x}\right)\end{pmatrix}dy\right]$$

$$= F(x(t_2), y(t_2)) - F(x(t_1), y(t_1)). \tag{5}$$

Proof Let $z = x + iy$ in Eq. (4), then

$$\int \exp(x+iy)\ln(x+iy)d(x+iy) = [\exp(x+iy)-1]\cdot\ln(x+iy) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(x+iy)^n + C.$$

It follows from Euler's formula that

$$\int\left[\begin{matrix}\exp(x)\cdot(\cos y + i\sin y)\cdot \\ \left[\ln\sqrt{x^2+y^2} + i\tan^{-1}\left(\frac{y}{x}\right)\right]\end{matrix}\right](dx+idy)$$

$$= [\exp(x+iy)-1]\cdot\ln(x+iy) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(x+iy)^n + C. \tag{6}$$

Using Lemma 1, DeMoivre's formula, and the real parts of both sides of Eq. (6) are equal, we obtain

$$\int\left[\exp(x)\begin{pmatrix}\cos y \ln\sqrt{x^2+y^2} \\ -\sin y \tan^{-1}\left(\frac{y}{x}\right)\end{pmatrix}dx - \exp(x)\begin{pmatrix}\sin y \ln\sqrt{x^2+y^2} \\ +\cos y \tan^{-1}\left(\frac{y}{x}\right)\end{pmatrix}dy\right]$$

$$= [\exp(x)\cos y - 1]\cdot\ln\sqrt{x^2+y^2} - \exp(x)\sin y\tan^{-1}\left(\frac{y}{x}\right) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(\sqrt{x^2+y^2})^n\cos\left[n\tan^{-1}\left(\frac{y}{x}\right)\right] + C.$$

Thus, the desired result holds.q.e.d.

In the following, the solution of line integral (2) can be determined.

Theorem 2 *If the assumptions are the same as Theorem1, and let*

$$G(x,y) = \exp(x)\sin y\ln\sqrt{x^2+y^2} + [\exp(x)\cos y - 1]\tan^{-1}\left(\frac{y}{x}\right) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}(\sqrt{x^2+y^2})^n\sin\left[n\tan^{-1}\left(\frac{y}{x}\right)\right],$$

then

$$\int_\gamma\left[\exp(x)\begin{pmatrix}\sin y\ln\sqrt{x^2+y^2} \\ +\cos y\tan^{-1}\left(\frac{y}{x}\right)\end{pmatrix}dx + \exp(x)\begin{pmatrix}\cos y\ln\sqrt{x^2+y^2} \\ -\sin y\tan^{-1}\left(\frac{y}{x}\right)\end{pmatrix}dy\right],$$

$$= G(x(t_2), y(t_2)) - G(x(t_1), y(t_1)). \tag{7}$$

Proof Using the imaginary parts of both sides of Eq. (6) are equal, we obtain the desired result. q.e.d.

3. Examples

For the two dimensional line integral problems discussed in this paper, two examples are provided and we use Theorems 1 and 2 to determine their solutions. In addition, Maple is used to calculate the approximations of some line integrals and their solutions for verifying our answers.

Example 1 Let $\gamma:[1,2]\to R^2$ be a piecewise smooth curve defined by $\gamma(t)=(2t,3t)$, then using Theorem 1 yields

$$\int_\gamma \left[\exp(x)\left(\begin{array}{l} \cos y \ln\sqrt{x^2+y^2} \\ -\sin y \tan^{-1}\left(\frac{y}{x}\right) \end{array}\right) dx - \exp(x)\left(\begin{array}{l} \sin y \ln\sqrt{x^2+y^2} \\ +\cos y \tan^{-1}\left(\frac{y}{x}\right) \end{array}\right) dy \right]$$

$$= [\exp(4)\cos 6-1]\cdot \ln\sqrt{52} - [\exp(2)\cos 3-1]\cdot \ln\sqrt{13} - [\exp(4)\sin 6-\exp(2)\sin 3]\cdot \tan^{-1}\left(\frac{3}{2}\right) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}[(\sqrt{52})^n-(\sqrt{13})^n]\cos\left[n\tan^{-1}\left(\frac{3}{2}\right)\right].$$

That is,

$$\int_1^2 \exp(2t)\left[\begin{array}{l} (2\cos 3t-3\sin 3t)\cdot\ln(\sqrt{13}t) \\ -\tan^{-1}\left(\frac{3}{2}\right)\cdot(3\cos 3t+2\sin 3t) \end{array}\right] dt$$

$$= [\exp(4)\cos 6-1]\cdot \ln\sqrt{52} - [\exp(2)\cos 3-1]\cdot \ln\sqrt{13} - [\exp(4)\sin 6-\exp(2)\sin 3]\cdot \tan^{-1}\left(\frac{3}{2}\right) - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}[(\sqrt{52})^n-(\sqrt{13})^n]\cos\left[n\tan^{-1}\left(\frac{3}{2}\right)\right]. \quad (8)$$

Using Maple to verify the correctness of Eq. (8) as follows:

```
>evalf(int(exp(2*t)*((2*cos(3*t)-3*sin(3*t))
*ln(sqrt(13)*t)-arctan(3/2)*(3*cos(3*t)+2*
sin(3*t))),t=1.0..2.0),18);
127.273464226178372
>evalf((exp(4)*cos(6)-1)*ln(sqrt(52))-(exp
(2)*cos(3)-1)*ln(sqrt(13))-(exp(4)*sin(6)-
exp(2)*sin(3))*arctan(3/2)-sum(1/(n*n!)*((
sqrt(52))^n-(sqrt(13))^n)*cos(n*arctan(3/2))
,n=1..infinity),18);
127.273464226178372
```

Example 2 If $\sigma:[1,3]\to R^2$ is a piecewise smooth curve defined by $\sigma(t)=(t,4t)$, then by Theorem 2, we have

$$\int_\sigma \left[\exp(x)\left(\begin{array}{l} \sin y \ln\sqrt{x^2+y^2} \\ +\cos y \tan^{-1}\left(\frac{y}{x}\right) \end{array}\right) dx + \exp(x)\left(\begin{array}{l} \cos y \ln\sqrt{x^2+y^2} \\ -\sin y \tan^{-1}\left(\frac{y}{x}\right) \end{array}\right) dy \right]$$

$$= \exp(3)\sin 12\cdot\ln\sqrt{153} - \exp(1)\sin 4\cdot\ln\sqrt{17} + [\exp(3)\cos 12-\exp(1)\cos 4]\cdot\tan^{-1}4 - \sum_{n=1}^{\infty}\frac{1}{n\cdot n!}[(\sqrt{153})^n-(\sqrt{17})^n]\sin(n\tan^{-1}4).$$

Therefore,

$$\int_1^3 \exp(t)\left[\begin{array}{l} (4\cos 4t+\sin 4t)\cdot\ln(\sqrt{17}t) \\ +\tan^{-1}4\cdot(\cos 4t-4\sin 4t) \end{array}\right] dt$$

$$= \exp(3)\sin 12 \cdot \ln\sqrt{153}$$
$$- \exp(1)\sin 4 \cdot \ln\sqrt{17}$$
$$+ [\exp(3)\cos 12 - \exp(1)\cos 4] \cdot \tan^{-1} 4$$
$$- \sum_{n=1}^{\infty} \frac{1}{n \cdot n!}[(\sqrt{153})^n - (\sqrt{17})^n]\sin(n\tan^{-1} 4). \tag{9}$$

We also employ Maple to verify the correctness of Eq. (9).

```
>evalf(int(exp(t)*((4*cos(4*t)+sin(4*t))*ln(sqrt(17)*t)+arctan(4)*(cos(4*t)-4*sin(4*t))),t=1.0..3.0),20);
                2.5588483429155503922
>evalf(exp(3)*sin(12)*ln(sqrt(153))-exp(1)*sin(4)*ln(sqrt(17))+(exp(3)*cos(12)-exp(1)*cos(4))*arctan(4)-sum(1/(n*n!)*((sqrt(153))^n-(sqrt(17))^n)*sin(n*arctan(4)),n=1..infinity),20);
                2.5588483429155503932
```

4. Conclusion

As mentioned, we mainly use a complex integral formula to solve two types of two dimensional line integrals. In fact, the applications of complex integral formulas are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. Moreover, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and use Maple to verify our answers.

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