



Constant Modulus Blind Adaptive Beam forming Based on Advanced Unscented Kalman Filtering

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ABSTRACT: Blind adaptive beam forming is getting appreciated for its various applications in contemporary communication systems where sources are statistically dependent or independent that are allowed to formulate new algorithms. Qualitative performance and time complexity are the main issues. In this paper, we propose a technique for constant modulus signals applying basic non-negative matrix factorization (BNMF) in blind adaptive beam forming environment. We compared the existing Unscented Kalman Filter based Constant Modulus Algorithm (UKF-CMA) with proposed NMF-UKF-CMA algorithm. We see there is a better improvement of sensor array gain, signal to interference plus noise ratio (SINR) and mean squared deviation (MSD) as the noise variance and the array size increase with reduced computational complexity with the UKF-CMA.

Keywords: Blind Adaptive Beam forming, NMF-UKF-CMA, Performance Comparison, UKF-CMA, MSD, SINR

(I) Introduction

Adaptive blind beamforming plays an important role in the contemporary communication systems where it constantly tributes to the enhancement of the signals that tend to be received or transmitted. Adaptive beamforming is achieved through varying the tap weights assigned to each antenna at every time instant applying signal processing algorithm. The weights are adjusted such that maximum array sensor gain is obtained with minimal amount of residual error. On processing the beamforming signals, the computational complexity depends on the algorithm which works upon the signals. The recent UKF-CMA algorithm for blind beamforming application works quite well compared to other beamforming techniques such as Least Mean Squared-Constant Modulus Algorithm (LMS-CMA) and Recursive Least Mean Squared-Constant Modulus Algorithm (RLS-CMA) with higher computational complexity [1].

The UKF-CMA algorithm enabled in Gaussian conditions converges to optimal solution when measurement noise is considered. However, UKF-CMA with process noise results in sub-optimal solution [2] [3]. The CM criterion is incorporated into Weiner filter through which adaptability is achieved [2]. Generally, Constant Modulus (CM)

cost functions with quadratic nature are very sensitive to array tap weights and can be minimized using Stochastic Gradient Descent methods (SGD) and the stability of SGD methods relatively depends on the step-size selected and thus results in slow rate of convergence [2].

An approximation of various CM algorithms is proposed. The computational cost of the Lagrangian formulated beamforming methods is higher over the regularized beam forming methods [4]. In unscented transform, the choice of sigma points is controlled by λ , which in turn linearises equal to the second order Gauss filter that results in optimal convergence of the solution [3] [5]. A new discriminate based non-negative matrix factorization algorithm is proposed for facial image characterization problems where discriminate analysis is based on the classification features [6]. A variant of NMF algorithm is proposed for blind source separation where it is a promising solution for spectral unmixing in hyper-spectral image processing and feature extraction [7]. Different methods of initialization are studied for NMF algorithm, where initialization plays an important role since decomposition is non-convex with many local minima [8]. Non-Negative Matrix Factorization Algorithm Non-Negative Matrix Factorization (NMF), a relatively novel technique for dimensionality reduction, has been in the growing fast since its origin. It incorporates the non-negativity constraint and thus achieves the parts-based representation as well as enhancing the construe of the problem correspondingly [9] [10]. Some new algorithms for NMF are proposed for blind source separation application when sources are statistically dependent by imposing constraints to the matrix [11]. Multichannel NMF decomposition algorithms are proposed for blind audio source separation. More variants of NMF algorithms for blind sources separation techniques can be found in [12]-[14]. An extensive survey of NMF algorithms can be seen in [15]. In rectangular matrix, the solution is normally iterative and the steps normally require a $s \times b \times \min(s, b)$. In NMF, we make sure that the complexity is reduced to $s \times t \times b$, where t is the rank of the matrix. This is achieved by factoring the matrix, as a product of 2 matrices, where first matrix acts as a set of basis

vectors and other is positive definite. In quadratic problems, the coefficient matrix has to be positive-definite which is not true in general case, NMF forces the coefficient matrix to be positive-definite that results in closed-form solution.

Figure 1 describes about the flow of the algorithm. The algorithm can be given as, Initialize U_0 V_0 , and $m = 0$ for

$$\begin{aligned} U_{m+1} &\leftarrow U_m \circ \begin{bmatrix} ZV_m \\ U_m V_m^H V_m \end{bmatrix} \\ V_{m+1} &\leftarrow V_m \circ \begin{bmatrix} Z^H U_{m+1} \\ V_m U_{m+1}^H U_{m+1} \end{bmatrix} \\ m &\leftarrow m + 1 \\ \text{end} \end{aligned}$$

Where $Z \in \mathbb{C}^{s \times b}$, $U_m \in \mathbb{C}_+^{s \times t}$, $V_m \in \mathbb{C}_+^{b \times t}$ are non-negative matrices and the reduced rank t is given by $t < \min(s, b)$ where $(s, b, t) \in \mathbb{R}_+$. In this paper, we have reduced the computational complexity of UKF-CMA algorithm by reducing dimensionality of the matrix computation, which is achieved through the non-negative matrix factorization. Note: Notations followed in the paper are bold small letters are vector. Capital letters are matrix.

(II) Beam forming Model

Consider a linear array of size L of uniform spacing $d \leq \lambda/2$ and n is the number of source signals (interference and desired signals). The signal output of an adaptive beam former is represented as [1],

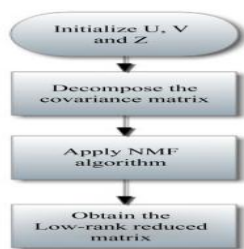


Figure 1. Flowchart of NMF-UKF-CMA algorithm.

$$z_m = \mathbf{u}_m^H \mathbf{w} \tag{1}$$

The input signal vector $\mathbf{u}_m \in \mathbb{C}^{L \times 1}$ as,

$$\mathbf{u}_m = \mathbf{D} \mathbf{s}_m + \mathbf{n}_m \tag{2}$$

The Constant Modulus (CM) cost function for adaptive beamforming problem can be formulated as

$$\min_{\mathbf{w}} J_{p,q}(\mathbf{w}) = \left[|\mathbf{u}_m^H \mathbf{w}|^p - \zeta \right]^q \tag{3}$$

Where $p > 0$, $q > 0$ and ζ is the signal modulus of the desired signal s_m , which is a known a priori. As stated, the optimization problem is non-convex and non-linear.

(III). Algorithm Formulation

The constant modulus criterion in (3) assumes that the unknown system model f_m for the input signal \mathbf{u}_m is equal to the constant modulus of the desired signal ζ in (5).

$$\mathbf{f} \leftarrow \mathbf{f}_{m|m-1} \tag{4}$$

$$\zeta \leftarrow \left| \mathbf{u}_m^H \mathbf{f}_{m|m-1} \right|^p \tag{5}$$

The final state space model is obtained by incorporating process noise \mathbf{q}_m . Since initial received signal is unknown, so we take it as noise \mathbf{v}_m adding to the model in (7). Applying the non-linearity

$$\tilde{\mathbf{f}}_m \leftarrow \mathbf{A}_m \tilde{\mathbf{f}}_{m|m-1} + \mathbf{q}_{m|m-1} \tag{6}$$

$$\zeta \leftarrow \tilde{\mathbf{f}}_{m|m-1} (L + 1) + \mathbf{v}_m \tag{7}$$

$$\tilde{\mathbf{z}}_m \leftarrow \mathbf{g}(\tilde{\mathbf{f}}_{m|m-1}) \tag{8}$$

$$\mathbf{A}_m \leftarrow \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{u}_m & \mathbf{0} \end{bmatrix} \quad \mathbf{f}_m \leftarrow \begin{bmatrix} \mathbf{f}_{m-1} \\ \mathbf{z}_{m-1} \end{bmatrix}$$

Where $\mathbf{A}_m \in \mathbb{C}^{(L+1) \times (L+1)}$, $\mathbf{f}_m \in \mathbb{C}^{(L+1) \times 1}$ and $\mathbf{q}_m \in \mathbb{C}^{(L+1) \times 1}$ is the process noise. In (10) $\tilde{\mathbf{z}}_m \approx \tilde{\mathbf{z}}$ is approximated by non-negative matrix factorization

$$\tilde{\mathbf{z}} \leftarrow \mathbf{U}_m \mathbf{V}_m^H \tag{9}$$

$$\mathbf{U}_m \mathbf{V}_m^H \rightarrow \tilde{\mathbf{z}}_m \tag{10}$$

Where

$\tilde{\mathbf{z}}_m \in \mathbb{C}^{s \times b}$, $\mathbf{U}_m \in \mathbb{C}_+^{s \times t}$, $\mathbf{V}_m \in \mathbb{C}_+^{b \times t}$ are non-negative matrices and the reduced rank t is given by $t < \min(s, b)$ where $(s, b, t) \in \mathbb{R}_+$. In the algorithm formulation, we ignore the process noise \mathbf{q}_m on including leads to suboptimal solution.

(V). Proposed NMF-UKF-CMA Algorithm

The proposed NMF-UKF-CMA algorithm is as follows,

Input: $u_m, w_m, L, \alpha, \beta, \kappa, \lambda, w_m, w_c$

Initialize: $\alpha \leftarrow 1, \beta \leftarrow 2, \kappa \leftarrow 0, n_w \leftarrow L+1, \lambda \leftarrow \alpha^2(n_w + \kappa - 1) - n_w + 2,$

$w_m \leftarrow \frac{1}{2(n_w + \lambda)} \text{ones}(n_w, n_w), w_c \leftarrow w_m, w_m(1) \leftarrow \frac{\lambda}{\lambda + n_w},$

$w_c(1) \leftarrow \frac{\lambda}{(\lambda + n_w)} + 1 - \alpha^2 + \beta$ then initialize the unscented Kalman filter with

$\hat{w}_0 \leftarrow [1; 0], P_0 \leftarrow 1/n_w I$ of size $n_w \times n_w,$

for $m \geq 1$

do

Compute and update

- Extract the sigma points $W_{m|m-1} \in \mathbb{C}^{n_w \times 2n_w+1}$ as

$$W_{m|m-1} \leftarrow \begin{bmatrix} \hat{w}_{m-1} & \hat{w}_{m-1} + \sqrt{(n_w + \lambda)} P_{m-1} & \hat{w}_{m-1} - \sqrt{(n_w + \lambda)} P_{m-1} \end{bmatrix} \quad (11)$$

Where $w \in \mathbb{R}^{n_w \times 1}$ is an initial weight vector.

- Extract matrix A_m for the input signal u_m as

$$A_m \leftarrow \begin{bmatrix} I & 0 \\ u_m & 0 \end{bmatrix}$$

And then get the sigma points $W_{m|m-1}^- \in \mathbb{C}^{n_w \times 2n_w+1}$

For the updated state as

$$W_{m|m-1}^- \leftarrow A_m W_{m|m-1} \quad (12)$$

- Extract the posteriori estimate $\hat{w}_{m|m-1}^- \in \mathbb{C}^{n_w \times 1}$ as

$$\hat{w}_m^- \leftarrow \sum_{j=1}^{2n_w+1} w_m^j W_{m|m-1}^{-j} \quad (13)$$

Where j denotes the j -th column vector for $W_{m|m-1}^-$ and j —the element for vector $w_m \in \mathbb{C}^{n_w \times n_w}$.

- Extract the sigma priori covariance $P_m^- \in \mathbb{C}^{n_w \times n_w}$

$$P_m^- \leftarrow \sum_{j=1}^{2n_w+1} w_c^j (W_{m|m-1}^{-j} - w_m^-) (W_{m|m-1}^{-j} - w_m^-)^H \quad (14)$$

Where j is the j -th column vector for w_m^- and j -th element for vector w_c .

- Extract the sigma points $Z_{m|m-1}^- \in \mathbb{C}^{n_w \times 2n_w+1}$ through non-linear function as

$$Z_{m|m-1}^- \leftarrow g(W_{m|m-1}^{-j}) \quad (15)$$

Where $g(\cdot) \leftarrow |\cdot|^p$ for each element of the j -th column vector for $W_{m|m-1}^-$ for $j \leftarrow 1, 2, \dots, 2n_w+1$.

- The output sigma points are approximated using non-negative matrix factorization algorithm as $\tilde{Z}_{m|m-1}^- \in \mathbb{C}^{n_w \times 2n_w+1}$

$$\tilde{Z}_{m|m-1}^- \leftarrow U_{m|m-1} V_{m|m-1}^H \quad (16)$$

Where $U_m \in \mathbb{C}_+^{n_w \times n_w}, V_m \in \mathbb{C}_+^{2n_w+1 \times n_w}$ are non-negative matrices and reduced rank

$$\tilde{z}_{m|m-1}^- \leftarrow \sum_{j=1}^{2n_w+1} w_m^j \tilde{Z}_{m|m-1}^{-j} \quad (17)$$

- The obtained cross covariance matrix $C_{mm} \in \mathbb{C}^{n_w \times 1}$ as

$$C_{mm} \leftarrow \sum_{j=1}^{2n_w+1} w_c^j (W_{m|m-1}^{-j} - w_m^-) (\tilde{Z}_{m|m-1}^{-j} - \tilde{z}_{m|m-1}^-)^H \quad (18)$$

- The obtained auto covariance R_{mm} as

$$R_{mm} \leftarrow \sum_{j=1}^{2n_w+1} w_c^j (\tilde{Z}_{m|m-1}^{-j} - \tilde{z}_m^-) (\tilde{Z}_{m|m-1}^{-j} - \tilde{z}_m^-)^H + \sigma_v^2 \quad (19)$$

$$\sigma_v^2 = \frac{1}{n_w - 1} \sum_{j=1}^{n_w-1} \tilde{z}_{m|m-1}^{-j}$$

Where

- Now apply the Kalman innovation matrix and the update formulas as

$$K_m \leftarrow C_{mm} R_{mm}^{-1} \quad (20)$$

$$\hat{w}_m \leftarrow \hat{w}_m^- + K_m (\zeta - \tilde{z}_m^-)^* \quad (21)$$

$$P_m \leftarrow P_{m|m-1}^- - K_m R_{mm} K_m^{-1} \quad (22)$$

- Update the optimal weight vector $w \leftarrow w_m^- (1 : n_w - 1, 1)$. end

5 SIMULATION RESULTS

In this section, we compare the performance of the proposed UKF-CMA algorithm with the LMS-CMA [6] and RLS-CMA [3] algorithms in linear adaptive beamforming application. We consider a uniform linear array of 60 sensors i.e., with spacing $d = \lambda/2$ and the number of remote sources m to be 4. The desired signal was a minimum shift keying (MSK) signal with unity modulus and the magnitude of the interference signals were set equal to the amplitudes of Gaussian noise signal with unity variance and their phases were set equal to a uniformly distributed noise signal in the range of $-\pi$ to π .

The direction of arrival of the desired signal was set to 10 and for the interference signals they were set as -30° , -45° , and 25° , respectively. We set $p=1$ since in the simulations in [3], [4], this lead LMS-CMA and RLS-CMA to achieve the highest signal-to-interference-plus-noise ratio (SINR). In addition to SINR, we used sensor array gain [3] and the mean square deviation (MSD), where $s_{1,k}$ is the desired signal component in , to assess the performance of all the algorithms. All curves were obtained by averaging the ensemble of 500 independent

In Experiment 1 we set the variance of the white Gaussian measurement noise signal to 0.1. The SINR, array gain, and MSD plots obtained are shown in Fig. 2. In Experiment 2 we set the variance of the white Gaussian measurement noise signal to 0.0316. The SINR, array gain, and MSD plots obtained are illustrated in Fig. 3.

As seen from Fig. 2 (a) and Fig. 3(a), the proposed UKF-CMA yields a faster convergence and a slightly higher steady-state SINR compared to LMS-CMA and RLS-CMA. The plot for LMS-CMA in Fig. 2 (b) is obtained after 60,000 iterations at which point it achieves similar SINR to the other two algorithms. From Fig. 1 (middle) and Fig. 3 (b), we note that the proposed UKF-CMA offers better attenuation in most regions away from the desired direction 10 and hence it provides more noise reduction compared to other algorithms. From Fig. 2 (c) and Fig. 3 (c), we note that UKF-CMA outperforms the other two algorithms in terms of MSD. In addition, LMS-CMA outperforms RLS-CMA because in RLS-CMA, the gradient vector (), and subsequently, relies instead of the true input signal, which results in larger phase distortion.

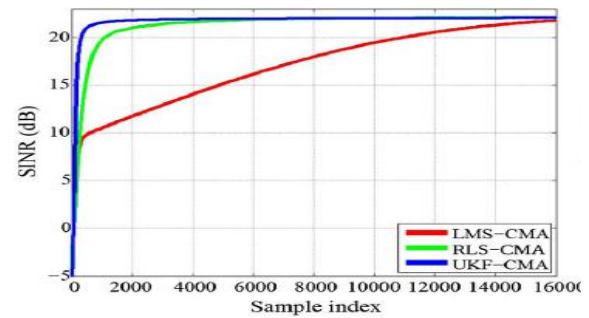


Fig.2.Experiment1 (a) SINR in dB algorithm.

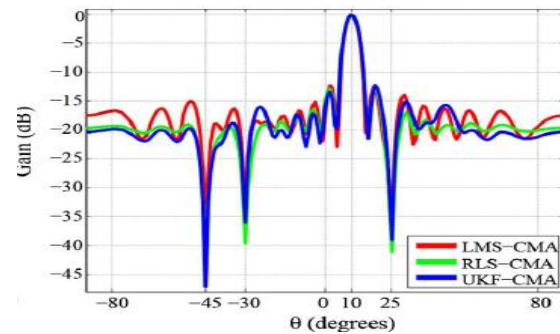


Fig.2. Experiment1 (b) Sensor array gain in dB (middle) algorithm

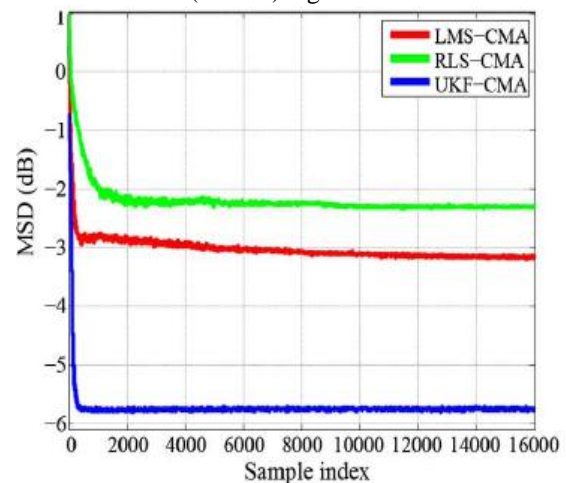


Fig 2 Experiment 1 (c) MSD in dB (right) of the algorithm

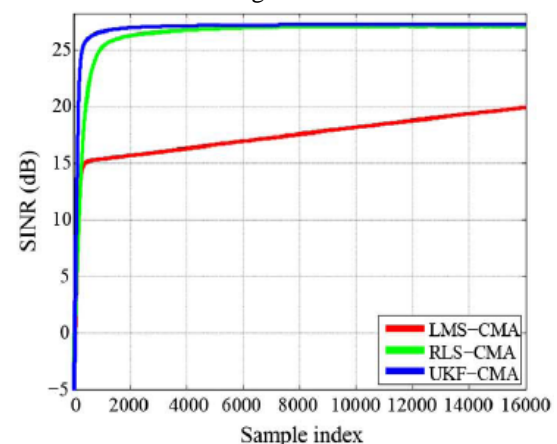


Fig.3.Experiment 2 (a) SINR in dB algorithm.

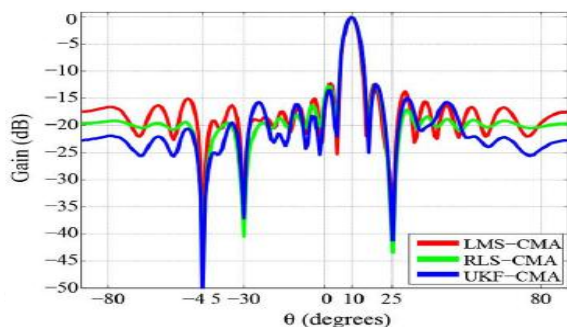


Fig.3. Experiment2 (b) Sensor array gain in dB (middle) algorithm

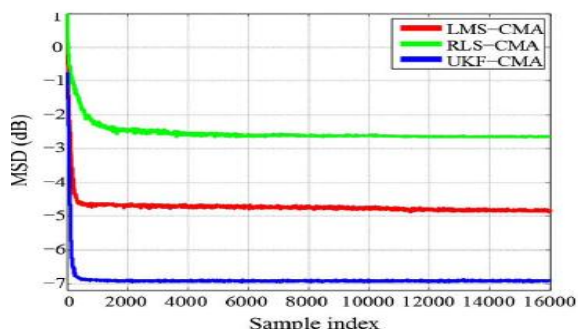


Fig 3 Experiment 2 (c) MSD in dB (right) of the algorithm

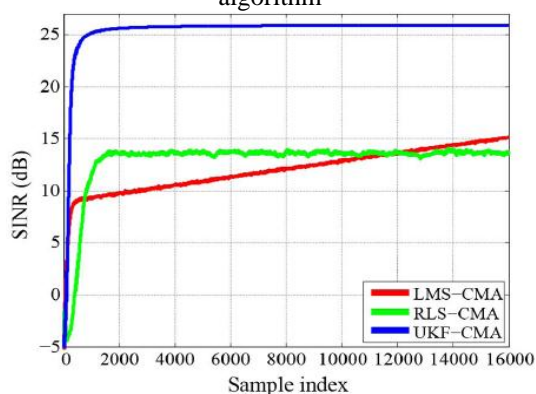


Fig 4 Experiment 3: SINR in DB

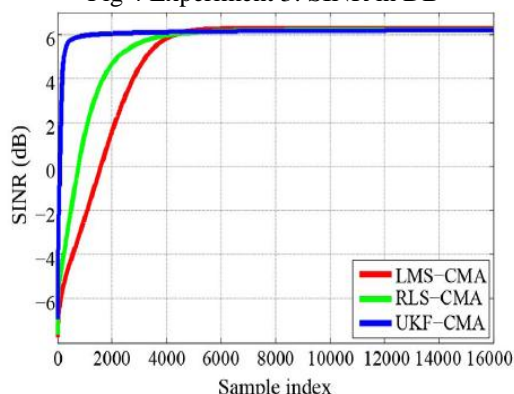


Fig 5 Experiment 4: SINR in DB

(VI) CONCLUSIONS

An unscented Kalman filter-based constant modulus algorithm for blind adaptive beamforming is developed. The proposed algorithm considers the output signal as part of the state transition equation of the Kalman filter. <http://dx.doi.org/10.1109/TKDE.2012.51>

In doing so, it turns out that no a priori information about the process noise and measurement noise covariance matrices is required and furthermore, the modulus of the output signal is not required to be differentiable with respect to the weight vector. Simulation results showed that the proposed algorithm offers improved performance compared to two other blind adaptive beamforming methods, LMS-CMA and RLS-CMA.

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