
**Analysis of Finite Elements Based On the Vibration and Stability of the Rotating Shaft System
Functionally Classified Under The Thermal Environment**

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ABSTARCT:

The present work deals with the study of vibration and stability analyses of functionally graded (FG) spinning shaft system under thermal environment using three noded beam element based on Timoshenko beam theory (TBT). Temperature field is assumed to be a uniform distribution over the shaft surface and varied in radial direction only. Material properties are assumed to be temperature dependent and graded in radial direction according to power law gradation and exponential law gradation respectively. In the present analysis, the mixture of Aluminum Oxide (Al₂O₃) and Stainless Steel (SUS304) is considered as FG material where metal contain (SUS304) is decreasing towards the outer diameter of shaft. The FG shafts are modeled as a Timoshenko beam by mounting discrete isotropic rigid disks on it and supported by flexible bearings that are modeled with viscous dampers and springs. Based on first order shear deformation

(FOSD) beam theory with transverse shear deformation, rotary inertia, gyroscopic effect, strain and kinetic energy of shafts are derived by adopting three-dimensional constitutive relations of material. The derivation of governing equation of motion is obtained using Hamilton's principle and solutions are obtained by three-node finite element (FE) with four degrees of freedom (DOF) per node. . In this work the effects of both internal viscous and hysteretic damping have also been incorporated in the finite element model. A complete code has been developed using MATLAB program and validated with the existing results available in literatures. The analysis of numerical results reveals that temperature field and power law gradient index have a significance role on the materials properties (such as Young modulus, Poisson ratio, modulus of rigidity, coefficient of thermal expansion etc.) of FG shaft. Various results have also been obtained such as Campbell diagram, stability speed limit (SLS),

damping ratio and time responses for FG shaft due unbalance masses and also compared with conventional steel shaft. It has been found that the responses of the FG spinning shaft are significantly influenced by radial thickness, power law gradient index and internal (viscous and hysteretic) damping and temperature dependent material properties. The obtained results also show that the advantages of FG shaft over conventional steel shaft.

Keywords: Power law gradient index; Functionally graded shaft; Temperature dependent material properties; Viscous and hysteretic damping; Rotor-Bearing-shaft system; Finite element method; Campbell diagram; Damping ratio; stability speed limit (SLS)

INTRODCUTION:

Composite materials and structures are more and more frequently used in advanced engineering fields mainly because of their high stiffness-to-weight ratio that is particularly favorable. However the main downside of composite materials is represented by the weakness of interfaces between adjacent layers known as delimitation phenomena that may lead to structural failure. To partially overcome these problems, a new class of materials named Functionally Graded Materials

(FGMs) has recently been proposed whose various material properties vary through the radial and thickness direction in a continuous manner and thus free from interface weakness. The gradation of material properties reduces thermal stresses, residual stresses, and stress concentrations. A functionally graded structure is defined as, those in which the volume fractions of two or more materials are varied continuously as a function of position along certain dimension (typically the radius and thickness) of the structure to achieve a require function. FGMs can provide designers with tailored material response and exceptional performance in thermal environments. For example, the Space Shuttle utilizes ceramic tiles as thermal protection from heat generated during re-entry into the Earth's atmosphere. An FGM composed of ceramic on the outside surface and metal on the inside surface. Due to high strength, high stiffness, and low density characteristics, FGMs rotor shafts have been sought as new potential candidates for replacement of the conventional metallic shafts in many application areas for the design of rotating mechanical components such as, driveshaft for helicopters and cars and jet engine, commercial and military rotating machines, aerospace and space vehicles etc. In Rotor-dynamic applications,

composites have been demonstrated both numerically and experimentally. Accompanied by the development of many new advance composite materials and various mathematical models of rotor-shafts were also developed by researchers. 1.1 Background and Importance of Rotor Dynamic Rotor dynamics has a remarkable history, largely due to the interplay between its theory and its practice. Rotor dynamics is a specialized branch of applied mechanics concerned with the behavior and diagnosis of rotating structures. It is commonly used to analyze the behavior of structures ranging from jet engines and steam turbines to auto engines and computer disk storage. Basic level of Rotor Dynamic is concerned with rotor and stator. Rotor is a rotating part of a mechanical device or structures supported by bearings and influenced by internal phenomena that rotate freely about an axis fixed in space. Engineering components concerned with the subject of rotor dynamics are rotors in machines, especially of turbines, generators, motors, compressors, blowers, alternators, pumps, brakes, distributors and the like. Rotor provides with materials bearings to constrain their spin axis in a more or rigid way to a fixed position in space, are referred to as fixed rotor (which consider spin speed is constant), where as those that are not

considered in any way are referred free rotor (which consider spin speed is governed by conservation of angular momentum). In operation Rotors have a great deal of rotational energy and a small amount of vibrational (bending, axial and torsional) energy. In Rotor Dynamics field William John Macquorn Rankine (1869) performed the first analysis of a spinning shaft. He chose a two-degrees-of-freedom model consisted of a rigid mass whirling in an orbit, with elastic spring acting in the radial direction. He defined the whirling speed of the shaft but he can be shown that beyond this whirling speed the radial deflection of Rankine's model increases without limit and this speed is called threshold speed for the divergent instability. In 1883 Swedish engineer Carl Gustaf Patrik de Laval developed a single-stage steam impulse turbine for marine applications and succeeded in its operation at 42,000 rpm. He first used a rigid rotor, but latter used a flexible rotor and shown that it was possible to operate above critical speed by operating at a rotational speed about seven times the critical speed. In 1895, Stanley Dunkerley published a study of the vibration of shafts loaded by pulleys. The first sentence of his paper reads, "It is well known that every shaft, however nearly balanced, when driven at a particular speed, bends, and, unless the

amount of deflection is limited, might even break, although at higher speeds the shaft again runs true. This particular speed or 'critical speed' depends on the manner in which the shaft is supported, its size and modulus of elasticity, and the sizes, weights, and positions of any pulleys it carries." In 1895 German civil engineer August Foppl who showed that an alternate rotor model exhibited a stable solution above Rankine's whirling speed. In England W. Kerr (1916) published experimental evidence that a second "critical speed" existed and it was obvious to all that a second critical speed could only be attained by the safe traversal of the first critical speed. In 1918 Ludwig Prandtl was the first to study a Jeffcott rotor with a noncircular cross-section. In 1919, Henry Jeffcott modeled a simple rotor to study the flexural behavior of rotors and dynamic behavior in "The lateral vibration of loaded shafts in the neighborhood of a whirling speed-the effect of want of balance," It is often referred to as Jeffcott rotor. But Jeffcott's analytical model the disk did not wobble. As a result, the angular velocity vector and the angular momentum vector were collinear and no gyroscopic moments were generated.

LITERATURE REVIEW:

Numerous research works have been done in the field of modeling and analysis of functionally graded (FG) structures. Some important works based on the different analysis have been presented in the following sections. Advanced composite materials offer numerous superior properties over metallic materials, like high specific strength and high specific stiffness. This has resulted in the extensive use of laminated composite materials in aircraft, spacecraft and space structures. In an effort to develop the super heat resistant materials, Koizumi [1] first proposed the concept of FGM. These materials are microscopically heterogeneous and are typically made from isotropic components, such as metals and ceramics. After the concept of FGMs was set by the Japanese school of material science and confirmation of their potentials several branches of research originated and are still being broadened by research groups all over the world. 2.2 Composite Materials Structure Zinberg and Symonds [2] investigated the model of rotating anisotropic shafts and compared the critical speeds of composite shaft over aluminum alloy shaft by using equivalent modulus beam theory (EMBT). Nelson et al. [3] and Nelson [4] contributed a substantial improvement in the computational analysis of rotating shafts incorporating the effect of

gyroscopic, rotary inertia, moment and shear deformation into the FE shafts model. Rouch et al. [5] implemented dynamics behavior of a linearly tapered circular Timoshenko beam formulation by numerical integration method to reduce the system bandwidth, system degree of freedom without a significant loss of accuracy and verify the gyroscopic effects of shafts rotation in rotor dynamic. Zorzi et al. [6] investigated the effect of constant axial torque on the dynamics, reliability, safety and survivability of rotor-bearing systems using FEM and determine the static bulking torque and critical speeds of long and short bearing. By using Bernoulli-Euler beam theory, Bert [7] developed governing equation of composite shaft, including effect of gyroscopic, bending and torsion coupling and determines critical speeds of composite shaft. Kim and Bert [8] adopted Sanders best first approximation shell theory to determine critical speeds of a rotating circular cylinder hollow shaft, containing layers of arbitrary laminated composite material. Abramovich et al. [9] employed a special orthogonality procedure which is applied to help understanding the dynamic behavior of the non-symmetrical laminated beam. Further by using Bresse-Timoshenko beam theory, Bert and Kim [10] employed Hamilton's principle to derive equations of motion of

composite shafts and to find out critical speeds. Bert and Kim [11] analyzed the dynamic instability of a composite drive shaft subjected to fluctuating torque and/ or rotational speed by using various shell theories and they investigated effect of constant torque and rotational speed. Singh and Gupta [12] implemented a Layer wise Beam Theory (LBT) with assuming a layer-wise displacement field and were extended to solve a composite rotor dynamic problem. Again Singh and Gupta [13] analyzed and compared the conventional rotor dynamic parameter like natural frequencies, critical speeds, damping factor, unbalance response (UBR) and threshold stability by using EMBT and LBT. Forrai [14] implemented a finite element model for stability analysis of self-excited bending vibrations of linear symmetrical rotor bearing system with internal damping. Chatelet et al. [15] proposed a finite element modeling to reduce the dynamic behavior of rotating structures and whole disk shaft assembly is supposed to be cyclically symmetrically. By using FOSD beam theory (continuum based Timoshenko beam theory), M. Y. Chang et al. [16] implemented the vibration behaviors of rotating laminated composite shaft model where transverse shear deformation, rotary inertia, gyroscopic effects and coupling effect are incorporated. Kapuria et al. [17]

presents static and dynamic electro-thermo-mechanical analysis of angle-ply hybrid piezoelectric beams using an efficient coupled zigzag theory. Span-to-thickness ratio, type of loading and the orientation of the angle-ply on the accuracy of the theories is investigated. Gubran et al. [18] analyzed natural frequencies of composite tubular shafts by using modified EMBT with shear deformation, rotary inertia and gyroscopic effects. Modifications take into account effects of stacking sequence and different coupling mechanisms in composite materials. By using FEM, Wang et al. [19] established a solution method for the one-dimensional transient temperature and thermal stress fields in non-homogeneous materials. Syed et al. [20] investigated simple analytical expressions for computing thermal stresses in fiber-reinforced composite beams with rectangular and hat sections due to change of temperature. By using a homogenized finite element beam model, Sino et al. [21] analyzed dynamic instability and natural frequencies of an internally damped rotating composite shaft. The influence of laminate parameters: stacking sequences, fiber orientation, transversal shear effect on natural frequencies and instability thresholds of the shaft are considered.

CONCLUSIONS The present work enables to arrive at the following important conclusions: A three noded beam finite element has been implemented for modeling and vibration analysis of the FG shaft system by incorporating both the internal viscous and hysteretic damping in the thermal environment. The temperature distribution is nonlinear along the radial direction of the crosssection of FG shaft. The material property distribution of the FG shaft has been performed very smoothly along the radial direction by accounting different temperatures and power law gradient indexes. From the comparison of various responses between steel and FG shaft, it has been found that FG shaft is more stable than the steel shaft. From the comparison of various responses of the FG shaft without and with temperatures consideration, it has been noticed that the FG shaft is more stable in case of without temperature consideration than that of with temperature consideration. The power law gradient index plays an important role in the responses (viz. Campbell diagram, damping ratio, critical speed, stability limit speed and time responses) of the FG shaft system. It is observed that the less value of temperature and the power law gradient index promotes more stable system than that of higher values of temperature and

power law gradient index. Finally, it can be concluded that the present work can be used for modeling and vibration analysis of the FG shaft system considering with or without temperature dependent material properties according to power law gradation by incorporating both internal viscous and hysteretic damping.

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