

Non-Linear and Multi-Domain Non-Separable Two-Dimensional Representation Based on the Representation of the Mean Cell

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Abstract The aim of this paper is to construct a new nonlinear and nonseparable multiscale representation of piecewise continuous bidimensional functions. This representation is based on the definition of a linear projection and a nonlinear prediction operator, which locally adapts to the function to be represented. This adaptivity of the prediction operator proves to be very interesting for image encoding in that it enables a considerable reduction in the number of significant coefficients compared with other representations. Applications of this new nonlinear multiscale representation to image compression and super-resolution conclude this paper.

I. INTRODUCTION

FOR the last few decades, research has been carried out to improve multiscale image representation by departing from traditional linear tensor product (bi)orthogonal wavelet

representations. Despite the fact that these representations are known not to be optimal, in terms of the number of non zero detail coefficients they generate, they are supported by powerful encoders such as EZW or EBCOT making them very efficient when applied to image compression. Nevertheless, the fact that wavelet representations generate too many detail coefficients has motivated new research toward more compact representations. For instance:

- Frames having some anisotropic directional selectivity, such as curvelets and contourlets
- Bandlets based on tensor products of wavelet bases combined with locally adapted edge operators.
- Edgeprint approximations that are computed, in the vicinity of an edge,

according to a wedge function which locally fits the image.

In all these approaches, and in order to take into account the presence of an edge, the multiscale structure is changed. Here, we introduce a new type of nonlinear multiscale representation based on cell-average discretization that accurately represents edges using a reduced number of detail coefficients when compared with wavelet transforms but maintains the same quadtree structure as wavelet transform approaches. The main difference with respect to wavelet representations is that the detail coefficients are computed by means of a local and nonlinear prediction operator. The new nonlinear multiscale representation (NMR) we introduce in this paper is based on cell-average discretization, and is close to the essentially non oscillatory edge adapted (ENO-EA) method previously discussed in [1]. The paper is organised as follows, first we recall the general framework for NMR in the cell-average discretization context.

II. HARTEN'S NONLINEAR MULTISCALE REPRESENTATION

A. Harten introduced in a strategy to construct NMRs

based on two discrete interscale operators, called projection and prediction operators respectively and denoted by P_{j-1}^j and P_j^{j-1} in the sequel. Assuming an image is some function v defined on $[0, 1]^2$ and v_j is its approximation on the grid $(2^{-j}k_1, 2^{-j}k_2)$, $0 \leq k_1, k_2 \leq 2^j - 1$, you first define a linear projection operator P_{j-1}^j acting from the fine to coarse levels, i.e., $v_{j-1} = P_{j-1}^j v_j$. In the cell-average framework, this operator is completely characterized since v_j is a rescaled version of a local cellaverage of v computed as:

$$v_j(k) = \frac{1}{\Delta x \Delta y} \int_{C_j(k)} v(x, y) dx dy, \quad (1)$$

with $C_j(k) = [2^{-j}k_1, 2^{-j}(k_1 + 1)] \times [2^{-j}k_2, 2^{-j}(k_2 + 1)]$, and where $k = (k_1, k_2)$. In what follows, $C_j(k)$ will be called a cell. From this, one infers that the projection operator reads: $v_{j-1}(k) = \frac{1}{4} (v_j(k) + v_j(k + e_1) + v_j(k + e_2) + v_j(k + e_1 + e_2))$, (2)

where e_1 and e_2 are unit vectors oriented to the right and upward, respectively. The prediction operator P_j^{j-1} acts from the coarse to fine levels by computing an 'approximation' \hat{v}_j of v_j from v_{j-1} , i.e. $\hat{v}_j = P_j^{j-1} v_{j-1}$. This operator may be nonlinear. In addition, it is assumed that these operators satisfy the following consistency property:

$$P_j^{j-1} P_{j-1}^j = I, \quad (3)$$

i.e. the projection of \hat{v}^j coincides with v^{j-1} :

$$v_k^{j-1} = \frac{1}{4} (\hat{v}_{2k}^j + \hat{v}_{2k+e_1}^j + \hat{v}_{2k+e_2}^j + \hat{v}_{2k+e_1+e_2}^j).$$

The prediction error $e^j := v^j - \hat{v}^j$ satisfies, from (2) and (5),

$$P_{j-1}^j e^j = P_{j-1}^j v^j - P_{j-1}^j \hat{v}^j = v^{j-1} - v^{j-1} = 0.$$

Hence $e^j \in \text{Ker}(P_{j-1}^j)$ and, using a basis E of this kernel, one writes e^j in a non-redundant way to obtain the *detail coefficients* d^{j-1} , satisfying $e^j = E d^{j-1}$. Thus v^j is equivalent to (v^{j-1}, d^{j-1}) . In practice, this non-redundancy means the size of the data is preserved through decomposition. Assuming the size of the original image is $2^J \times 2^J$ and iterating the proposed nonlinear procedure from the initial data v^J , we obtain its NMR

$$\mathcal{M}v^J = (v^0, d^0, \dots, d^{J-1}).$$

$$v(x, y) = A\chi_{\{y \geq h(x)\}}(x, y) + B\chi_{\{y < h(x)\}}(x, y), \quad (6)$$

with $h(x) = mx + n$, $\chi_C(x, y)$ the indicator function of C , and A and B some constants.

The edge detection mechanism makes use of the 1D cost functions whose descriptions follow:

$$H_k^{j-1} := |v_k^{j-1} - v_{k-e_1}^{j-1}| + |v_{k+e_1}^{j-1} - v_k^{j-1}| \quad (7)$$

$$V_k^{j-1} := |v_k^{j-1} - v_{k-e_2}^{j-1}| + |v_{k+e_2}^{j-1} - v_k^{j-1}|. \quad (8)$$

For each k , one defines:

$$l_{h,k} = \underset{l}{\operatorname{argmin}} \left\{ H_{k+le_1}^{j-1}, l \in \{-1, 0, 1\} \right\} \quad (9)$$

$$l_{v,k} = \underset{l}{\operatorname{argmin}} \left\{ V_{k+le_2}^{j-1}, l \in \{-1, 0, 1\} \right\}. \quad (10)$$

III. EDGE DETECTION

To begin, we consider edge detection at level $j-1$ since the prediction at level j is based only on the information available at level $j-1$. In this section, we consider step-edges, modeled by straight lines separating regions with constant gray level, that is, on a cell $C_{j-1,k}$ containing an edge, the function v is assumed to have the form $v(x, y) = A\chi_{\{y \geq h(x)\}}(x, y) + B\chi_{\{y < h(x)\}}(x, y)$, (6) with $h(x) = mx + n$, $\chi_C(x, y)$ the indicator function of C , and A and B some constants.

The edge detection mechanism makes use of the 1D cost functions whose descriptions follow:

IV. CONCLUSION

In this paper, we have derived a new type of nonlinear multiscale representation based on a nonlinear prediction operator in a cell-average framework. As the structure follows the same quadtree structure as the orthogonal wavelet transform, a compression algorithm such as EZW can be applied to the nonlinear multiscale representation. We noted significant improvement in terms of compression performance when compared with linear multiscale representations. Another application of the proposed nonlinear representation is super-resolution, for which we have shown that accurate reconstruction of piecewise regular images can be achieved by using an approximation of the image at a coarse resolution level.

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