Modified Ratio Estimator-1

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Abstract

The present study proposes improved ratio estimator by utilizing the estimator of Ray and Singh (1981) in simple random sampling. We calculate Mean square error (MSE) and compared it with other existing estimators. Theoretical result is supported by a numerical illustration.

Keywords: *Ratio type estimator Simple random sampling, Mean square error, Auxiliary variable, Efficiency;*

1. INTRODUCTION

The classical ratio estimate for the population mean $\bar{Y}$ of variate y is defined by

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X} \tag{1.1}$$

where it is assumed that the population mean $\bar{X}$ of auxiliary variate x is known .Here $\bar{y}$ is the sample mean of variate of interest and $\bar{x}$ is the sample mean of auxiliary variate .From (1.1), we have

MSE of classical ratio estimate is given as:

$$\text{MSE}(\bar{y}_r) \cong \frac{1-f}{n}(R^2S_x^2 - 2RS_{xy} + S_y^2) \tag{1.2}$$

Where f=n\N, n is the sample size and N is the population size, $R = \frac{\bar{Y}}{\bar{X}}$ is the population ratio ; S_x^2 is the population variance of auxiliary variable and S_Y^2 is the population variance of variable of interest.

When the population coefficient of variation of auxiliary variate C_x is known , Sisodia and Dwivedi (1981) suggested a modified ratio estimator for $\bar{y}$ as

$$\bar{y}_{SD} = \bar{y}\frac{\bar{X}+C_X}{\bar{x}+C_X} = \frac{\bar{Y}}{\bar{x}_{SD}}\bar{X}_{SD} = \hat{R}_{SD}\bar{X}_{SD} \tag{1.3}$$

From (1.3), $\bar{x}_{SD} = \bar{x} + C_{X;}\bar{X}_{SD} = \bar{X} + C_X; \hat{R}_{SD} = \frac{\bar{Y}}{\bar{x}_{SD}} and R_{SD} = \frac{\bar{Y}}{\bar{X}_{SD}}$

MSE of this estimate is given

$$\text{MSE } (\overline{Y}_{SD}) \cong \frac{1-f}{n}\overline{Y}^{2}[C_Y^2 + C_X^2\alpha(\alpha - 2K)] \qquad (1.4)$$

Where C_Y is the population coefficient of variation of variate of interest,

$$\alpha = \frac{\overline{X}}{\overline{X} + C_X} \quad \text{And} \quad K = \rho\frac{C_y}{C_x}$$

Where ρ is the correlation coefficient between auxiliary and study variable in the population.

Motivated by Sisodia and Dwivedi (1981), Singh and Kakran (1993) developed ratio type estimator for $\overline{Y}$ as

$$\overline{Y}_{SK} = \overline{Y}\frac{\overline{X} + \beta_2}{\overline{x} + \beta_2}(X) = \frac{\overline{y}}{\overline{x}_{SK}}\overline{X}_{SK} = \hat{R}_{SK}\overline{X}_{SK} \qquad (1.5)$$

Where $\beta_2(x)$I s the population coefficient of kurtosis of auxiliary variable .From (1.5); $\overline{x}_{SK} = \overline{x} + \beta_2(x); \overline{X} + \beta_2(x); \hat{R}_{SK} = \frac{\overline{y}}{\overline{x}_{SK}}; R_{SK} = \frac{\overline{Y}}{\overline{X}_{SK}}$

Similar to (1.4);MSE of this estimator is given as

$$\text{MSE}(\ \overline{y}_{SK}) \cong \frac{1-f}{n}\overline{Y}^{2}[C_Y^2 + C_X^2\,\delta(\delta - 2K)] \qquad (1.6)$$

Where $\delta = \frac{\overline{X}}{\overline{X} + \beta_2(X)}$

Motivated by Singh and Kakran (1981), we introduced measure of skewness in existing estimator, then our proposed estimator takes the following form,

$$\overline{Y}_{AS2} = \overline{Y}\frac{\overline{X} + B_1(x)}{\overline{x} + B_1(x)} = \frac{\overline{Y}}{\overline{x}_{AS2}}\overline{X}_{AS2} = R_{AS2}\overline{X}_{AS2} \qquad (1.7)$$

where $\beta_1(x)$ is the measure of skewness

$$\overline{x}_{AS2} = \overline{x} + B_1(x); \overline{X}_{AS2} = \overline{X} + B_1(x); \hat{R}_{AS2} = \frac{\overline{Y}}{\overline{x}_{AS2}}; R = \frac{\overline{Y}}{\overline{X}_{AS2}}$$

Mean square error of (1.7) can be calculated by using the taylor series expansion method .

$$\text{MSE}\left(\overline{Y}_{AS2}\right) = \frac{1-f}{n}\overline{Y}^{2}[C_Y^2 + C_X^2\delta(\delta - 2K) \qquad (1.8)$$

where

$$\delta = \frac{\overline{X}}{\overline{X} + B_1(x)} \quad \text{and} \quad \text{K} = \rho\frac{C_Y}{C_X}$$

2 . The Suggested Estimator:

Ray and Singh (1991) suggested estimator of type

$$\bar{y}_{RS} = \frac{\bar{y} + b(\bar{x}^{\alpha} - \bar{X}^{\alpha})}{\bar{x}^{\gamma}} \bar{X}^{\gamma}$$

In the above estimator, we proposed to take $\alpha = 1$ and $\gamma = 1$ then the improved estimator which is based on measure of skewness of the auxiliary variable takes the following form

$$\bar{y}_{PAS2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x} + B_1(x)} \bar{X} + B(x)_1 = \hat{R}_{PAS2} \bar{X} + B_1(x) \qquad (2.1)$$

Where b= $\frac{s_{xy}}{s_x^2}$; s_x^2 is the sample variance of auxiliary variate and s_{xy} is the sample covariance between auxiliary variable and variable of interest .From (2.1) $R_{pr} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}}$ .Note that when b=0 in (2.1),the proposed estimator turns to be traditional ratio estimate given in (1.1)

MSE of the proposed estimator can be found using Taylor series method defined as

$$h(\bar{x}, \bar{y}) \cong h(\bar{X}, \bar{Y}) + \frac{\partial h(c,d)}{\partial c|}\Big|_{x\bar{y}} (\bar{x} - \vec{X}) + \frac{\partial h(c,d)}{\partial d}\Big|_{x\bar{y}} (\bar{y} - \bar{Y}) \qquad (2.2)$$

Where $h(\bar{x}, \bar{y}) = \hat{R}_{pr}$ and $h(\bar{X}, \bar{Y}) = R$.

As per wolter(1985), equation (2.2) can be applied to the modified estimator in order to obtain MSE equation as follow ;

$$\hat{R}_{PAS2} - R \cong \frac{\partial((\bar{y} + b(\bar{X} - \bar{x})) \setminus \bar{x} + B_1(x)}{\partial \bar{x}}\Big|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{\partial((\bar{y} + b(\bar{X} - \bar{x})) \setminus \bar{x} + B_1(x)}{\partial \bar{y}}\Big|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y}) \qquad (2.3)$$

$$\cong \left(\frac{-\bar{y}^2}{(\bar{x} + B_1(x))^2} - \frac{b\bar{X}}{(\bar{x} + B_{1(}(x))^2} - \frac{b(\bar{x} + md) - b\bar{x}}{(\bar{x} + B_1(x))^2} \right) \Big|_{\bar{X},\bar{Y}} (\bar{x} - \bar{X}) + \frac{1}{(\bar{x} + B_1(x))^2} \Big|_{\bar{X},\bar{Y}} (\bar{y} - \bar{Y})$$

$$E(\hat{R}_{PAS1} - R) \cong -\left(\frac{\bar{Y}}{(\bar{X} + B_1(x))^2} + \frac{b(\bar{X} + B_1(x))}{(\bar{X} + B_1(x))^2} \right) (\bar{x} - \bar{X}) + \frac{1}{(\bar{X} + B_1(x))^2} (\bar{y} - \bar{Y}) 2 \qquad (2.4)$$

$$E(\hat{R}_{PAS1} - R)^2 \cong \left\{ \frac{\bar{Y} + B(\bar{X} + B_1(x))}{(\bar{X} + B_1(x))^2} \right\}^2 v(\bar{x}) - \frac{2\bar{Y}B(\bar{X} + B_1(x))}{(\bar{X} + B_1(x))^3} \operatorname{cov}(\bar{x}, \bar{y}) + \frac{1}{(\bar{X} + B_1(x))} v(\bar{y}) \qquad (2.5)$$

$$\cong \frac{1}{(X + B_1(x))^2} \left\{ \frac{\bar{Y} + B(\bar{X} + B_1(x))}{(\bar{X} + B_1(x))} \right\}^2 v(\bar{x}) - \frac{2\bar{Y}B(\bar{X} + md)}{(\bar{X} + B_1(x))} \operatorname{cov}(\bar{x}, \bar{y}) + v(\bar{y})$$

$$\text{MSE}\left(\bar{Y}_{PAS1}\right) \cong (\bar{X} + md)^2 E(\hat{R}_{PAS1} - R) \qquad (2.6)$$

$$\cong \left\{\frac{\bar{Y}+B(\bar{X}+B_1(x))}{(\bar{X}+B_1(x))}\right\}^2 - 2\frac{\bar{Y}+B(\bar{X}+B_1(x))}{(\bar{X}+B_1(x))}\operatorname{cov}(\bar{x},\bar{y})+v(\bar{y})$$

$$\cong \frac{\bar{Y}^2+B(\bar{X}+B_1(x))^2}{(X+B_1(x))^2} + \frac{2\bar{Y}B(X+B_1(x))}{(X+B_1(x))^2}v(\bar{x}) - 2(\frac{\bar{Y}+B(\bar{X}+B_1(x))}{(\bar{X}+B_1(x))}\operatorname{cov}(\bar{x},\bar{y})+v(\bar{y})$$

$$\cong \frac{1-f}{n}\left\{\frac{\bar{Y}^2}{(\bar{X}+B_1(x))^2}+B^2+2\frac{\bar{Y}}{(\bar{X}+B_1(x))}\right\}S_X^2 - 2\frac{\bar{Y}}{(\bar{X}+B_1(x))}S_{XY}+2BS_{XY}+S_Y^2$$

$$\cong \frac{1-f}{n}\{R_{PAS2}{}^2 S_X^2 + B^2 S_X^2 + 2RS_X^2 - 2RS_{XY} + S_Y^2\}$$

$$MSE(\bar{Y}_{AS2}) \cong \frac{1-f}{n}\{R_{PAS2}^2 S_X^2 + S_Y^2(1-\rho^2)\} \qquad (2.7)$$

3. Efficiency comparison:

We compare the MSE of Existing estimator with the MSE of the proposed AS1 estimator using (1.2) and (2.7)

$$\text{MSE}(\bar{Y}_{AS2}) < \text{MSE}(\bar{Y}_{Existing})$$

$$\frac{\bar{Y}S_X^2}{(\bar{X}+B_1(x))^2} < \frac{\bar{Y}S_{x2}}{\bar{X}_2} \qquad (3.1)$$

It is evident from the above comparison that the condition given in (3.1) is satisfied in all situations.

4. Numerical illustration:

We have used the data of Murthy (1967) in which fixed capital is denoted by Y and output of 80 factories are denoted by X

The data statistics about the population under consideration

Data statistics: $N = 80$, $n = 20$, $\bar{X} = 51.8264$

$\bar{Y} = 11.2646$, $C_Y = 0.7505$, $C_X = 0.3542$, $\beta_2(x) = 0.06339$, $\beta_1(x) = 1.05$, $\rho = 0.9413$,

$md = 7.575$, $S_X = 18.3569$, $S_Y = 8.4563$

MSE values of ratio estimator

	Proposed type	Classical type
Sisodia – Dwivedi Singh-Kakran AS2(proposed)	0.891 0.898 **0.876**	1.29 1.32 2.93

From the numerical results, we infer that the proposed estimator is more efficient in all situations. Hence it is recommended to use the proposed estimator for future studies.

5. Conclusion:

We have derived a new ratio type estimator and obtain their MSE equations.MSE of proposed estimator is compared with each other. It is deduced that the proposed estimator is more efficient than the classical one. We hope to extend the formulation presented here to ratio estimator in stratified random sampling.

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