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Implementation of Hadamard Transform (HT) for Multiple-Rate Codes based on Block Markov Superposition Transmission

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ABSTRACT

Hadamard Transform (HT) as over the twofold field gives a characteristic approach to actualize numerous rate codes (alluded to as HT-coset codes), where the code length N = 2p is settled however the code measurement K can be shifted from 1 to N − 1 by altering the arrangement of solidified bits. Superior to anything HT we can actualize Fast Walsh-Hadamard Transform for era of HT–coset codes. The HT-coset codes, including Reed-Muller (RM) codes and polar codes as average cases, can share a couple of encoder and decoder with usage many-sided quality of request O(N log N). In any case, to ensure that all codes with assigned rates perform well, HT-coset coding for the most part requires an adequately huge code length, which thusly causes challenges in the assurance of which bits are better to be solidified. In this paper, we propose to transmit short HT-coset codes in the purported piece Markov superposition transmission (BMST) way. At the transmitter, signs are spatially coupled through superposition, bringing about long codes. At the recipient, these coupled signs are recouped by a sliding-window iterative delicate progressive cancelation unraveling calculation. In particular, the execution around or beneath the bit-mistake rate (BER) of 10−5 can be anticipated by a straightforward genie-helped bring down bound. Both these limits and reenactment comes about demonstrate that the BMST of short HT-coset codes performs well (inside one dB far from the relating Shannon limits) in an extensive variety of code rates.

INTRODUCTION

In helpful correspondence structures, it is a great part of the time required to complete a couple codes with different code rates. This is fundamental for remote correspondence systems to execute the adaptable coded adjust, which can let down the power outage probability of a single code and along these lines allows more capable usage of the uncommon transmission limit resources. It is then alluring to complete codes with each and every differing rate of eagerness with a few encoder and decoder. Rate-great (RC) codes are a class of various rate codes, which are typically worked from a mother code by the use of code modifying frameworks, for instance, shortening information bits and enlarging or puncturing

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equity check bits, as consolidated. The rate-great codes generally have assorted code lengths for different code rates. For a couple of uses, the code lengths can be changed in a wide range. For example, Raptor codes (with length the length of required) were enhanced by arranging the degree profiles to extend the ordinary throughput. In 2009, Casado et al proposed different rate codes with settled code length by solidifying lines of the fairness check network of a mother code. In 2012, Liu et al fabricated different rate nonbinary low-thickness fairness check (LDPC) codes with settled square length by using higher demand Galois fields for codes of lower rates [11]. But settled length various rate codes are not suitable to cross breed customized go over request (HARQ) with incremental redundancies, they may find applications in some unique circumstances.

Settled length different rate codes can be associated with the circumstances where a confining basic is constrained on the physical layer. This happens, for instance, when orthogonal frequency division modulation (OFDM) with a settled number of subcarriers is used. In this circumstance, less information bits should be encoded to keep up the unwavering quality essential when the direct is in a low signal-to-noise ratio (SNR) level. Settled length distinctive rate codes may in like manner find applications in blast memory structures, where as far as possible diminishes with ages yet the amount of memory cells keeps un-changed.

The more settled flash memory, which has encountered a significant number of erase cycles, has a degraded farthest point and requires a code of lower rate; while the "more youthful" blast memory has a greater breaking point and backings a code of higher rate. In this circumstance, settled length various rate codes with a comparative combine of encoder and decoder are favored. Square Markov superposition transmission (BMST) is an advancement of enormous convolutional codes from short codes (suggested as major codes), which has an average execution over the binary-input additive white Gaussian noise channel (BI-AWGNC). In it has been pointed out that any short code with snappy encoding count and Softe in Soft out (SISO) translating figuring can be picked as the fundamental code. A sliding-window decoding (SWD) figuring with a tunable unraveling delay, as like the pipeline message-passing interpreting count of the LDPC convolutional codes, can be completed to disentangle the BMST system. Most importantly, the execution of the SWD count in the low oversight rate locale can be expected by an essential genie-cut down bound.

In this paper, the BMST is utilized to develop another class of numerous rate codes with settled code length. To start with, we propose to develop short various rate codes by adjusting the arrangement of solidified bits in Hadamard transform (HT), resulting in HT-coset codes. This group of codes, which have length N = 2p for some p > 0 and measurement K ranging from 1 to N − 1, can be executed by the utilization of a couple of encoder and decoder with multifaceted nature of request O(N log N). Such a couple of encoder and decoder were initially shown using typical diagram for Reed-Muller (RM) codes by Forney. The HT-coset codes considered in this paper are likewise firmly identified with the polar codes, in any case, channel polarization assumes a less imperative part here in light of the fact that the considered codes will be short (ordinarily N ≤ 16). At that point, to enhance the execution of the short HT-coset codes, we propose to transmit short HT-coset codes in the BMST way.

The resulting framework alluded to as a BMST-HT framework for comfort, takes as the fundamental code a Cartesian result of short HT-coset codes and henceforth has encoding/decoding many-sided quality linearly growing with the transmission piece length. Also, the bound together structure of the HT-coset codes can fundamentally disentangle the equipment execution. Reproduction comes about demonstrate that the BMST-HT frameworks perform well (within one dB far from the corresponding Shannon limits) in an extensive variety of code rates. Whatever remains of this paper is composed as takes after. In Section II, we show the encoding and decoding calculation for the numerous rate HT-coset codes. In Section III, we build BMST-HT frameworks with a general plan methodology and present the encoding and decoding calculation for the different rate BMST-HT frameworks. Segment IV closes this paper.

FULL-DUPLEXCASE

Relay channel

Vander Meulen first proposes the idea of hand-off channel. Cover and El Gamal additionally outline a few vital procedures for transfer channels, including decipher forward, pack forward, and blended disentangle forward and pack forward. A variety of the disentangle forward plan is incomplete interpret forward, in which the Transfer just translates a part of the message from the source and advances it to the destination instead of decoding the entire message. Kramer, Gastpar and Gupta extend these plans to the numerous hub hand-off systems and propose a few rate locales in light of unravel forward, pack forward and blended techniques. Lim, Kim, El Gamal and Chung propose another plan called Noise Network coding (NNC) in view of pack forward relaying. These hand-off coding procedures have been broadly connected in different channels. For instance Liang and Kramer concentrate the hand-off communicate channel using rate splitting, square Markov encoding and halfway interpret forward relaying.

Interference channel

Carleial first presents the obstruction channel and proposes inward and external limits and in addition limit comes about for a few exceptional cases. Sato concentrates the limit with respect to the Gaussian impedance divert with solid obstruction. Han and Kobayashi propose the notable Han-Kobayashi coding strategy in utilizing rate part at the transmitters and joint translating at the recipients, which to date accomplishes the biggest rate locale for the obstruction channel. Chong, Motani, Garg and El Gamal propose a variation plot in light of superposition coding, which accomplishes a similar rate area as the first Han-Kobayashi conspire yet has less assistant irregular factors and subsequently decreases the encoding and translating complexities.

Cognitive interference channel

The intellectual obstruction channel is another firmly related channel, which assumes a signifi-cannot part in enhancing range productivity. Devroye, Mitran and Tarokh first propose the idea and give an achievable rate locale in light of joining Gelfand-Pinsker coding with Han-Kobayashi plot. They concentrate both the genie-supported (non-causal) and the non genie-helped (causal) cases. Maric, Yates and Kramer decide the limit area for the channel with exceptionally solid obstruction. Wu, Vishwanath and Arapostathis decide the limit district for the powerless impedance case. Other coding plans for the psychological obstruction direct can be seen. Jovicic and Viswanath dissect the Gaussian subjective channel and give the biggest rate for the intellectual client under the limitation that the essential client encounters

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no rate debasement and utilizations single-client decoder. Rini, Tuninetti and Devroye additionally propose a few new internal, external limits and limit comes about in view of rate spitting, superposition coding, a communicate channel-like binning plan and Gelfand-Pinsker coding.

A vital strategy utilized as a part of all subjective coding is the binning system proposed by Gelfand and Pinsker. In Gelfand-Pinsker binning, the condition of the channel is known at the information, however obscure at the yield. Marton proposes the twofold binning plan and applies it to the communicate channel. Kim, Sutivong and Cover additionally examine Gelfand-Pinsker binning to permit the translating of a piece of state data at the goal at a diminished data rate. Costa applies Gelfand-Pinsker binning to the Gaussian channel and proposes the outstanding messy paper coding (DPC) conspire, which accomplishes an indistinguishable rate from if the channel is without obstruction. An astonishing element of DPC binning is that the transmit flag is autonomous of the state.

Hadamard Matrix

The *Kronecker product* of two matrices $\mathbf{A} = [a_{ij}]_{m\times n}$ and $\mathbf{B} = [b_{ij}]_{k\times l}$ is defined as

$$\mathbf{A}\otimes\mathbf{B} \triangleq \begin{bmatrix} a_{11}\mathbf{B} & \cdots & a_{1n}\mathbf{B} \\ \cdots & \cdots & \cdots \\ a_{m1}\mathbf{B} & \cdots & a_{mn}\mathbf{B} \end{bmatrix}_{mk\times nl}$$

In general, $\mathbf{A}\otimes\mathbf{B} \neq \mathbf{B}\otimes\mathbf{A}$.

The *Hadamard Matrix* can be extensively defined recursively as below:

$$\mathbf{H}_1 \triangleq \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Obviously $\mathbf{H}^T = \mathbf{H}$ is rean and symmetric. Here are two examples for $n=2$ and $n=3$:

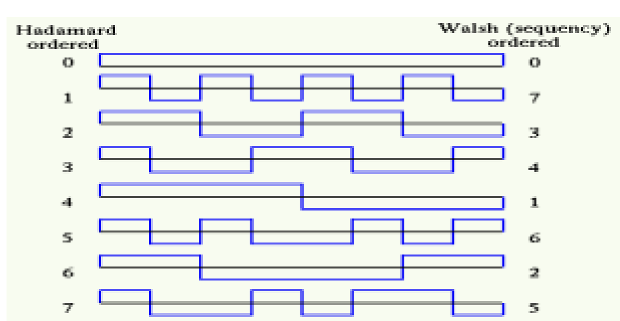
$$\mathbf{H}_2 = \mathbf{H}_1\otimes\mathbf{H}_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_1 \\ \mathbf{H}_1 & -\mathbf{H}_1 \end{bmatrix} = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\mathbf{H}_3 = \mathbf{H}_1\otimes\mathbf{H}_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{H}_2 & \mathbf{H}_2 \\ \mathbf{H}_2 & -\mathbf{H}_2 \end{bmatrix} = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}\begin{matrix} 0 & 0 \\ 1 & 7 \\ 2 & 3 \\ 3 & 4 \\ 4 & 1 \\ 5 & 6 \\ 6 & 2 \\ 7 & 5 \end{matrix}$$

Note that H_n is $N = 2^n$ by $N = 2^n$ matrix.

The primary section on the privilege of the framework H_3 is for the indecies of the lines, and the second segment speaks to the sequency (the quantity of zero-intersections or sign changes) in every line. Sequency is like yet not quite the same as recurrence as in it quantifies the rate of progress of non-periodical signs.

The rows of the matrix can be considered as the $N = 2^3$ samples of the following waveforms:



The Hadamard Matrix can likewise be acquired by characterizing its component in the k[th] push and m[th] section of as

$$h[k,m] = (-1)^{\sum_{i=0}^{n-1} k_i m_i} = \prod_{i=0}^{n-1} (-1)^{k_i m_i} = h[m,k] \quad (k,m = 0,1,\cdots$$

Where

$$k = \sum_{i=0}^{n-1} k_i 2^i = (k_{n-1}k_{n-2}\cdots k_1 k_0)_2 \quad (k$$

$$m = \sum_{i=0}^{n-1} m_i 2^i = (m_{n-1}m_{n-2}\cdots m_1 m_0)_2 \quad (m_i = 0,1)$$

i.e., $(k_{n-1}k_{n-2}\cdots k_1 k_0)_2$ and $(m_{n-1}m_{n-2}\cdots m_1 m_0)_2$ are the binary representations of k and m, respectively. Obviously, $n = log_2 N$.

We can show the Hadamard matrix $\mathbf{H}$ is orthogonal by induction. First, it is obvious that $\mathbf{H}_1$ is orthogonal:

$$\mathbf{H}_1^T\mathbf{H}_1 = \mathbf{H}_1\mathbf{H}_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & \\ 1 & - \end{bmatrix}$$

Next we assume $\mathbf{H}_{n-1}$ is orthogonal, i.e.,

$$\mathbf{H}_{n-1}^T\mathbf{H}_{n-1} = \mathbf{H}_{n-1}\mathbf{H}_{n-1} = \mathbf{I}_{2^{n-1}\times 2^{n-1}}$$

Finally we show that $\mathbf{H}_n = \mathbf{H}_1 \otimes \mathbf{H}_{n-1}$ is also orthogonal:

$$\mathbf{H}_n^T\mathbf{H}_n =] = \frac{1}{2}\begin{bmatrix} \mathbf{H}_{n-1} & \mathbf{H}_{n-1} \\ \mathbf{H}_{n-1} & -\mathbf{H}_{n-1} \end{bmatrix}\begin{bmatrix} \mathbf{H}_{n-1} & \mathbf{H}_{n-1} \\ \mathbf{H}_{n-1} & -\mathbf{H}_{n-1} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \mathbf{H}_{n-1}\mathbf{H}_{n-1} + \mathbf{H}_{n-1}\mathbf{H}_{n-1} & \mathbf{H}_{n-1}\mathbf{H}_{n-1} - \mathbf{H}_{n-1}\mathbf{H}_{n-1} \\ \mathbf{H}_{n-1}\mathbf{H}_{n-1} - \mathbf{H}_{n-1}\mathbf{H}_{n-1} & \mathbf{H}_{n-1}\mathbf{H}_{n-1} + \mathbf{H}_{n-1}\mathbf{H}_{n-1} \end{bmatrix} = \mathbf{I}_{N\times N}$$

In summary, the Hadamard matrix $\mathbf{H}$ is real, symmetric, and orthogonal:

$$\mathbf{H} = \mathbf{H}^* = \mathbf{H}^T = \mathbf{H}^{-1}$$

The Walsh-Hadamard Transform (Hadamard Ordered)

As any orthogonal (unitary) matrix can be used to define a discrete orthogonal (unitary) transform, we define a Walsh-Hadamard transform of Hadamard order (WHT_h) as

$$\begin{cases} \mathbf{X} = \mathbf{Hx} & (forward) \\ \mathbf{x} = \mathbf{HX} & (inverse) \end{cases}$$

These are the forward and inverse WHT_h transform pair. Note that the forward and inverse transforms are identical!

Here $\mathbf{x} = [x[0], x[1], \cdots, x[N-1]]^T$ and $\mathbf{X} = [X[0], X[1], \cdots, X[N-1]]^T$ are the signal and spectrum vectors, respectively. The kth element of the transform can also be written as

$$X[k] = \sum_{m=0}^{N-1} h[k,m]x[m] = \sum_{m=0}^{N-1} x[m] \prod_{i=0}^{n-1} (-1)^{m_i k_i}$$

The complexity of WHT is $O(N^2)$. Similar to FFT algorithm, we can derive a fast WHT algorithm with complexity of $O(Nlog_2 N)$.

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \mathbf{H}_2 \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} + \mathbf{H}_2 \begin{bmatrix} x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \mathbf{H}_2 \begin{bmatrix} x_1[0] \\ x_1[1] \\ x_1[2] \\ x_1[3] \end{bmatrix}$$

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We will assume $n=3$ and $N=2^n=8$ in the following derivation. An $N=8$ point WHT_h of signal $x[m]$ is defined as

$$x_1[i] \triangleq x[i]+x[i+4] \quad (i=0,\cdots,3)$$

$$\mathbf{X}=\begin{bmatrix} X[0] \\ \vdots \\ X[3] \\ X[4] \\ \vdots \\ X[7] \end{bmatrix} = \mathbf{H}_3\mathbf{x} = \begin{bmatrix} \mathbf{H}_2 & \mathbf{H}_2 \\ \mathbf{H}_2 & -\mathbf{H}_2 \end{bmatrix} \begin{bmatrix} x[0] \\ \vdots \\ x[3] \\ x[4] \\ \vdots \\ x[7] \end{bmatrix}$$

Here for simplicity, we ignored the coefficient $1/\sqrt{2}$ This equation can be separated into two parts. The first half of the X vector can be obtained as

Where

The second half of the X vector can be obtained as

$$\begin{bmatrix} X[4] \\ X[5] \\ X[6] \\ X[7] \end{bmatrix} = \mathbf{H}_2 \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix} - \mathbf{H}_2 \begin{bmatrix} x[4] \\ x[5] \\ x[6] \\ x[7] \end{bmatrix} = \mathbf{H}_2 \begin{bmatrix} x_1[4] \\ x_1[5] \\ x_1[6] \\ x_1[7] \end{bmatrix}$$

Where

$$x_1[i+4] \triangleq x[i]-x[i+4] \quad (i=0,\cdots,3)$$

What we have done is converting a WHT of size $N=8$ into two $WHTs$ of size $N/2=4$. Continuing this process recursively, we can rewrite Eq. (1) as the following (similar process for Eq. (3))

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_1 \\ \mathbf{H}_1 & -\mathbf{H}_1 \end{bmatrix} \begin{bmatrix} x_1[0] \\ x_1[1] \\ x_1[2] \\ x_1[3] \end{bmatrix}$$

This equation can again be separated into two halves. The first half is

$$\begin{bmatrix} X[0] \\ X[1] \end{bmatrix} = \mathbf{H}_1 \begin{bmatrix} x_1[0] \\ x_1[1] \end{bmatrix} + \mathbf{H}_1 \begin{bmatrix} x_1[2] \\ x_1[3] \end{bmatrix} = \mathbf{H}_1 \begin{bmatrix} x_2[0] \\ x_2[1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_2[0] \\ x_2[1] \end{bmatrix} = \begin{bmatrix} x_2[0]+x_2[1] \\ x_2[0]-x_2[1] \end{bmatrix}$$

Where

$$x_2[i] \triangleq x_1[i]+x_1[i+2] \quad (i=0,1)$$

The second half is

$$\begin{bmatrix} X[2] \\ X[3] \end{bmatrix} = \mathbf{H}_1 \begin{bmatrix} x_1[0] \\ x_1[1] \end{bmatrix} - \mathbf{H}_1 \begin{bmatrix} x_1[2] \\ x_1[3] \end{bmatrix} = \mathbf{H}_1 \begin{bmatrix} x_2[2] \\ x_2[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_2[2] \\ x_2[3] \end{bmatrix} = \begin{bmatrix} x_2[2]+x_2[3] \\ x_2[2]-x_2[3] \end{bmatrix}$$

Where

$$x_2[i+2] \triangleq x_1[i]-x_1[i+2] \quad (i=0,1)$$

(3)

$X[4]$ through $X[7]$ of the second half can be obtained similarly.

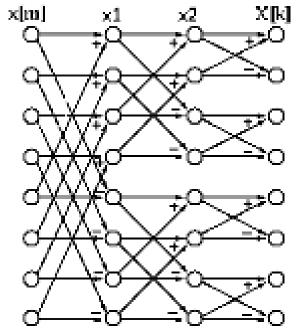
$$X[0]=x_2[0]+x_2[1]$$

(4)

and

$$X[1]=x_2[0]-x_2[1]$$

Summarizing the above steps of Equations (2), (4), (6), (8), (9) and (10), we get the Fast WHT algorithm as illustrated below.



Sequency Ordered Walsh-Hadamard Matrix

In order for the elements in the spectrum $\mathbf{X}=[X[0],X[1],\cdots,X[N-1]]^T$ to represent different sequency components contained in the signal in a low-to-high order, we can re-order the rows (or columns) of the Hadamard matrix H according to their sequencies.

The conversion of a given sequency s into the corresponding index number k in Hadamard order is a three-step process:

1. represent s in binary form:

$$s=(s_{n-1}s_{n-2}\cdots s_1 s_0)_2=\sum_{i=0}^{n-1} s_i 2^i$$

2. convert this binary form to Gray code:

$$g_i = s_i \oplus s_{i+1} \quad (i=0,\cdots,n-1)$$

where $\oplus$ represents exclusive or and $s_n=0$ by definition.

3. bit-reverse g_i's to get k_i's:

$$k_i = g_{n-1-i} = s_{n-1-i} \oplus s_{n-i}$$

Now k can be found as

$$k=(k_{n-1}k_{n-2}\cdots k_1 k_0)_2=\sum_{i=0}^{n-1} s_{n-1-i}\oplus s_{n-i} 2^i=\sum_{j=0}^{n-1} s_j \oplus s_{j+1} 2^{n-1-j}$$

where $j=n-1-i$ or equivalently $i=n-1-j$.

For example, $n=log_2 N=log_2 8=3$, we have

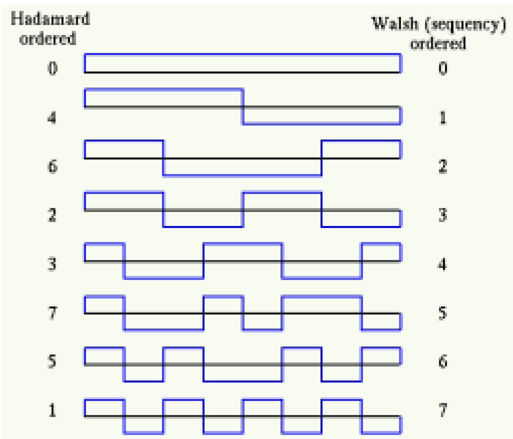
s	0	1	2	3	4	5	6	7
binary	000	001	010	011	100	101	110	111
Gray code	000	001	011	010	110	111	101	100
bit-reverse	000	100	110	010	011	111	101	001
k	0	4	6	2	3	7	5	1

Now the sequency-ordered or Walsh-ordered Walsh-Hadamard matrix can be obtained as

$$\mathbf{W}=\frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{matrix} 0 & 0 \\ 1 & 4 \\ 2 & 6 \\ 3 & 2 \\ 4 & 3 \\ 5 & 7 \\ 6 & 5 \\ 7 & 1 \end{matrix}$$

The principal section on the privilege of the grid is for the sequency of the comparing line, which is the list for the sequency-requested matrix, and the second segment is the list of the Hadamard requested. We see that this lattice is still symmetric:

$$w[k,m] = w[m,k]$$



Fast Walsh-Hadamard Transform (Sequency Ordered)

The sequency-ordered Walsh-Hadamard transform (WHT_w also called Walsh ordered WHT) can be obtained by first carrying out the fast WHT_h and then reordering the components $X[k]$ as shown above. Alternatively, we car the following fast WHT_w directly with better efficiency.

The sequency ordered WHT of $x[m] =$ can also be defined as

$$X[k] = \sum_{m=0}^{N-1} w[k,m]x[m] = \sum_{m=0}^{N-1} x[m] \prod_{i=0}^{n-1} (-1)^{(k_i+k_{i+1})m_{n-}}$$

where $N = 2^n$, $k_n \triangleq 0$, $X[k] =$ exponent of -1 represents the conversion from sequency ordering to Hadamard ordering (binary-to-Gray code conversion and bit-reversal conversion).

In the following, we assume $n = 3, \quad N = 2^3 = 8$, and we represent m and k in binary form as, $(m_2 m_1 m_0)_2$ and $(k_2 k_1 k_0)_2$ respectively, i.e.,

$$m = \sum_{i=0}^{n-1} m_i 2^i = 4m_2 + 2m_1 + m_0 \qquad (m_i = 0,1)$$

$$k = \sum_{i=0}^{n-1} k_i 2^i = 4k_2 + 2k_1 + k_0 \qquad (k_i = 0,1)$$

As the first step of the algorithm, we rearrange the order of the samples $x[m]$ by bit-reversal to get

$$x_0[4m_0 + 2m_1 + m_2] \triangleq x[4m_2 + 2m_1 + m_0] \qquad m = 0$$

We also define $l_i = m_{n-1-i}$. Now the WHT_w can be written as

$$\sum_{m_2=0}^{1}\sum_{m_1=0}^{1}\sum_{m_0=0}^{1} x_0[4m_0 + 2m_1 + m_2] \prod_{i=0}^{2} (-1)^{(k_i+k_{i+1})m_{n-1-i}}$$

$$\sum_{l_0=0}^{1}\sum_{l_1=0}^{1}\sum_{l_2=0}^{1} x_0[4l_2 + 2l_1 + l_0] \prod_{i=0}^{2} (-1)^{(k_i+k_{i+1})l_i}$$

Expanding the 3rd summation into two terms, we get

$$\sum_{l_0=0}^{1}\sum_{l_1=0}^{1}\prod_{i=0}^{1} (-1)^{(k_i+k_{i+1})l_i} [x_0[2l_1 + l_0] + (-1)^{k_2+k_3} x_0[4 + 2l_1 + l_0]]$$

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$$= \sum_{l_0=0}^{1}\sum_{l_1=0}^{1}\prod_{i=0}^{1}(-1)^{(k_i+k_{i+1})l_i} x_1[4k_2+2l_1+$$

where $k_3 \triangleq 0$ and x_1 is defined as

$$x_1[4k_2+2l_1+l_0] \triangleq x_0[2l_1+l_0]+(-1)^{k_2+k_3}x_0[4+2l_1+l_0]$$

Expanding the 2nd summation into two terms, we get

$$= \sum_{l_0=0}^{1}(-1)^{(k_i+k_{i+1})l_0}[x_1[4k_2+l_0]+(-1)^{k_1+k_2}x_1[4k_2+2+l_0]]$$

$$= \sum_{l_0=0}^{1}(-1)^{(k_i+k_{i+1})l_0}x_2[4k_2+2k_1+m_0]$$

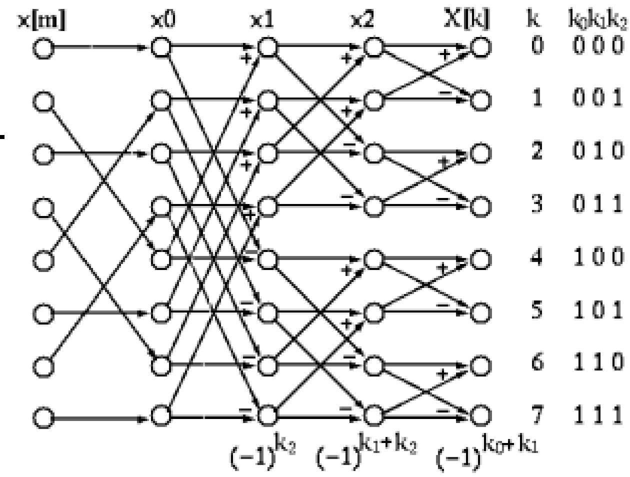
where x_2 is defined as

$$+2k_1+l_0] \triangleq x_1[4k_2+l_0]+(-1)^{k_1+k_2}x_1[4k_2+2+l_0$$

Finally, expanding the 1st summation into two terms, we have

$$X[k]=x_2[4k_2+2k_1]+(-1)^{k_0+k_1}x_2[4k_2+2k_1+1]$$

Summarizing the above steps, we get the fast WHT_w algorithm composed of the bit-reversal and the three equations (11), (12), and (13), as illustrated below:



nple Example

ider a signal vector of N=4 elements (samples):

$$\mathbf{x}=[x_0,x_1,x_2,x_3]^T=[0,1,2,3]^T$$

The corresponding 4 by 4 WHT matrix (Hadamard ordered) is:

$$\mathbf{H}_h=\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{bmatrix}$$

The rows (or colums) of this matrix can be reordered according to their sequency by the following mapping:

sequency order	0	1	2	3
binary	00	01	10	11
Gray code	00	01	11	10
bit-reverse	00	10	11	01
Hadamard order	0	2	3	1

to get the sequency ordered (also called Walsh ordered) matrix

$$\mathbf{H}_w=\frac{1}{2}\begin{bmatrix}1&1&1&1\\1&1&-1&-1\\1&-1&-1&1\\1&-1&1&-1\end{bmatrix}=\frac{1}{2}[h_0,h_1,h_2,h_3]$$

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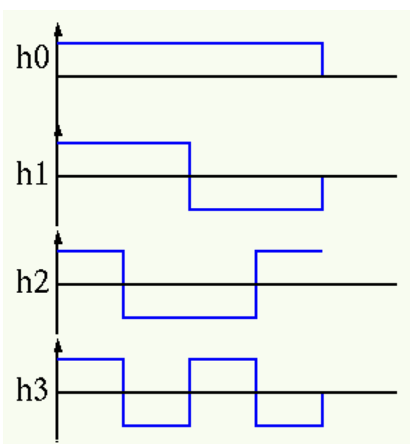
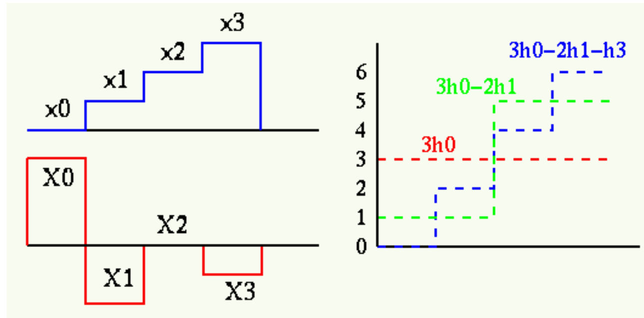
where $\mathbf{h}_i$ is the ithcolum (or row) vector of the symmetric matrix $\mathbf{H}_w^T = \mathbf{H}_w$ representing a square wave of sequency i (with i zero-crossings). Now the sequency spectrum of the signal can be found as

$$\mathbf{X} = \mathbf{H}_w\mathbf{x} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} =$$

and the inverse transform (note $H_w^{-1} = H_w$) represents the signal vector as a linear combination of a set of square waves of different sequencies:

$$\mathbf{x} = \mathbf{H}_w\mathbf{X} = \frac{1}{2}[h_0, h_1, h_2, h_3]\begin{bmatrix} 3 \\ -2 \\ 0 \\ -1 \end{bmatrix} = \frac{3h_0 - 2h_1 -}{2}$$





We can also verify that indeed the inverse transform will produce the original signal from its spectrum:

$$\mathbf{x} = \mathbf{H}_w\mathbf{X} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

III. BMSTOFSHORTHT-COSETCODES

A. ChoiceofEncodingMemory

We have exhibited that Hadamard change can be utilized to develop a group of codes with steady length N and adaptable measurement K. This group of codes can be executed by a similar match of encoder and decoder however with flexible sources of info. Notwithstanding, the exhibitions are far from as far as possible, as confirm by the development case with N = 8. One

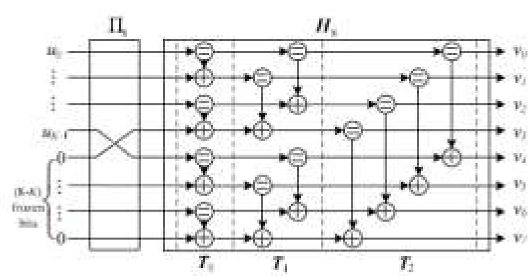


Fig.3: The encoding process for HT-coset codes with N=8.

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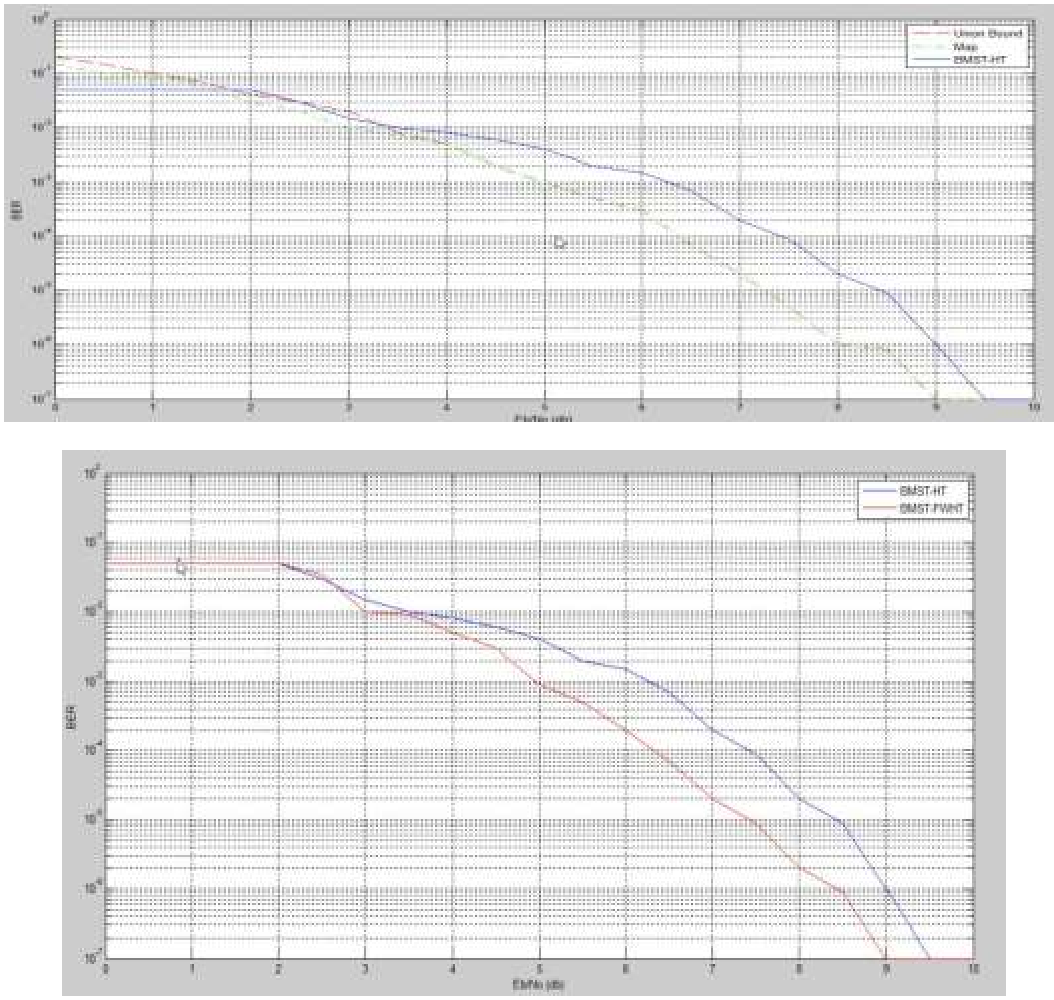

Fig. 4. The performance of the HT coset code with K = 4 and N = 8 under the SISO decoding algorithm (Algorithms 2 and 4) with an iteration number J = 3. The union bound and the performance curve of the MAP decoding are also plotted.

Conceivable approach to enhance the execution is to develop the request of the Hadamard change. As N turns out to be adequately extensive, for any given measurement K, we have opportunity to choose dynamic set to approach the channel limit. The trouble lies in the decision of the dynamic set when N turns out to be expansive. Here we propose to join the BMST with the HT-coset codes. As pointed out in any short code can be inserted into the BMST framework to acquire additional coding increase the low BER area. The basic parameter for BMST is the encoding memory m, which predicts the additional coding increase of 10 log10(m+1) dB over AWGN channels. Given this anticipated additional coding increase, we can utilize the accompanying deterministic system to discover the encoding memory mkrequired by the BMST-Ht sytem (N=2p, p>1) to approach as far as possible at an objective BER $_{p\ target}$.

AGeneralProcedureofDeterminingtheEncodingMemoriesfortheBMST-HTSystems

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TABLEII

THEMEMORYREQUIREDFOREACHCODE RATEUSINGTHEBMSTOFHT-COSETCODESWITHN=8TOAPPROACH THESHANNONLIMITATTHEBEROF10^{-5}

Rate $R = K/8$	1/8	2/8	3/8	4/8	5/8	6/8	7/8
γ_K^* (dB)	-1.2	-0.8	-0.3	0.2	0.8	1.6	2.9
γ_K (dB)	9.6	9.8	8.4	7.7	8.9	8.6	8.2
Gap $\gamma_K - \gamma_K^*$ (dB)	10.8	10.6	8.7	7.5	8.1	7.0	5.3
Memory m_K	11	10	6	5	5	4	2

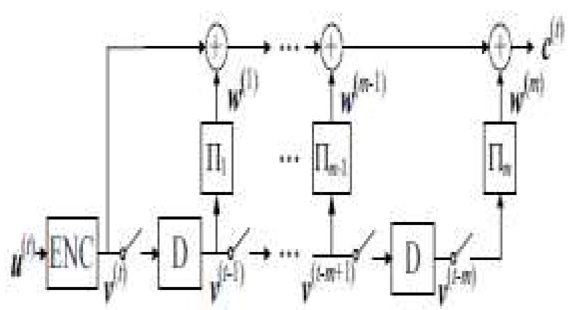

Fig. 5. The encoding diagram of the BMST-HT system with a maximum memory m.

• For K = 1, 2, … ,N− 1, determine the encoding memory m_K with the following steps.

1) From the union bound of the HT-coset code with information bit K, find the required $E_b/N_0 = \Gamma_K$ to achieve the target BER P_{target}.

2) Find the Shannon limit for the code rate K/N, denoted by γ_K^*.

3) Determine the encoding memory by $10 \log_{10} (m + 1) \geq \Gamma_K - \gamma_K^*$. That is,

$$m_K = \left\lfloor 10^{\frac{\gamma_K - \gamma_K^*}{10}} - 1 \right\rceil, \qquad (7)$$

where[x]stands for the integer that is closest to x.

Table II shows the memory required for each code rate using the BMST of HT-coset codes With N = 8 to approach the Shannon limit at a target BER of $P_{target} = 10^{-5}$.

B. Encoding of BMST-HT System

A BMST-HT framework is built by taking the encoder for the HT-coset codes as the essential encoder, which acknowledges as info a double data grouping of length BK and conveys as yield a parallel coded arrangement of length BN. All the more correctly, the fundamental code [n, k] is a B-crease Cartesian result of [N,K], where n = BN and k = BK. To approach the limit,

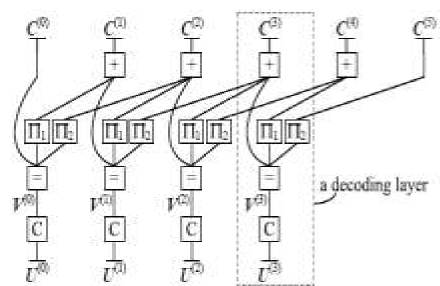

Fig. 6. The normal graph of the BMST-HT system with L = 4 and m = 2.

D.AConstructionExample(Continued)

We continue the construction example given in Section II-C. We take the B-fold Cartesian product $\mathscr{C}[8,K]^B (1 \leq K \leq 7)$ as the basic code, where B =1250.The memory required for each

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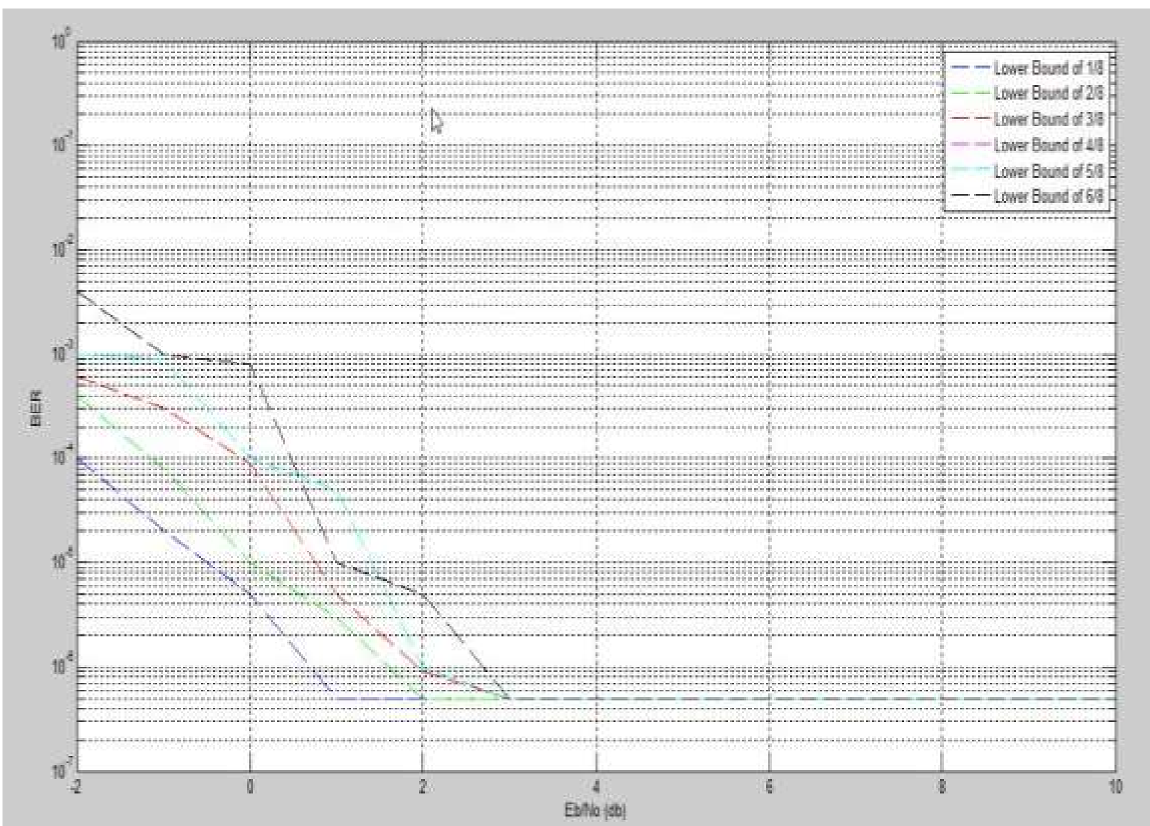
K to approach the corresponding Shann onlimit at the BER of 10^{-5} is specified in TableII. Hence we need an encoder with a maximum memory m=11. The SISO decoding algorithm (Algorithms 2 and 4) with an iteration number J=3 is used to implement the SISO decoding algorithm for the basic code. The iterative sliding-window decoding algorithm for the BMST-HT system is performed with a maximum iteration number of 18, where the entropy- based early stopping criterion is used with athreshold of 10^{-5}. Simulation results with L=1000 for encoding are shown in Fig.7, where the decoding delay is specified as $d_K=2m_K$ forcoderate K/N. Also shown in Fig.7 are the genie-aided lower bounds, which are obtained by shifting the corresponding performance curves of the basic codes to left by $10\log_{10}(1+m_K)$ dB. We can see that the performances of the BMST-HT system match well with the respective genie-aided lower bounds in the low BER region for all considered code rates. To evaluate the bandwidth efficiency of the BMST-HT system, we plot the required SNR to achieve the BER of 10^{-5} against the code rate in Fig.8. We can see that the BMST-HT system achieves the BER of 10^{-5} with in oned B from the Shannon limit for all considered code.



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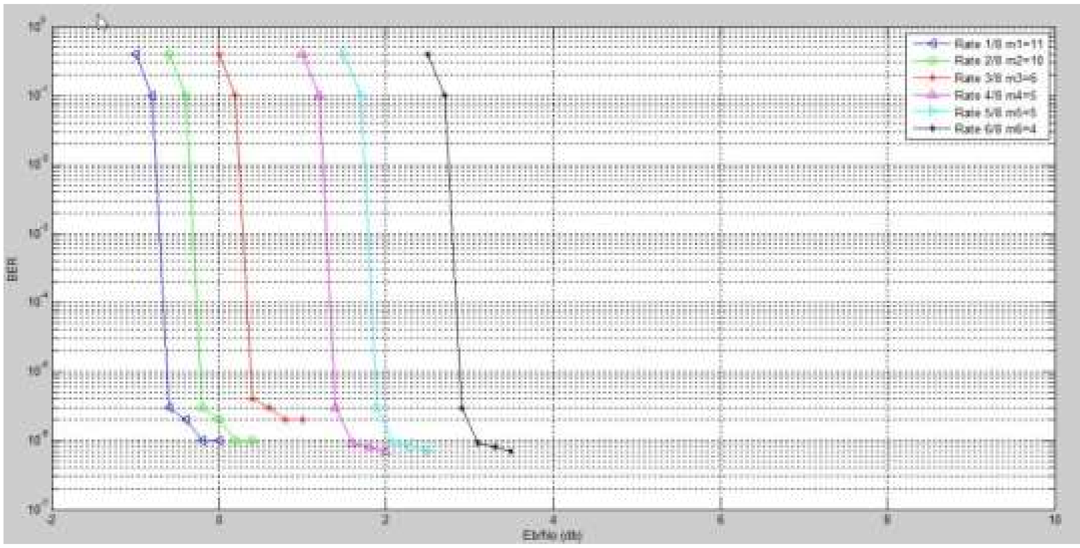

Figure 7: Performance of the BMST-HT system using the Cartesian products of the HT-coset codes $[8,K]^{1250}(1\leq K\leq 7)$ with L=1000.

The sliding-window decoding algorithm is performed with a maximum iteration number $I_{max}=18$ and a decoding delay $d_K=2m_K$ for the code of rate K/8 rates. However, the performance gap of the code is slightly larger. This is due to the sub-optimality of Algorithm2, as mentioned previously. If an optimal (locally) SISO algorithm (the MAP decoding) is used for the code, the performance can be improved, as marked in Fig.8 by the cross×.

E. Further Discussions

We have also constructed a BMST-HT system using the B-fold Cartesian product $C[16,K]^B$ $(1\leq K\leq 15)$ as the basic code, where B=625. The required encoding memories can be determined following the procedure described in Section III-A, which are shown in Table III. The required SNR to achieve the BER of 10^{-5} is plotted against the code rate in Fig.9. We can see that the BMST-HT system also has a BER of 10^{-5} with in oned B away from the Shannon limits for all considered code rates. Using HT-coset codes with larger N, we can implement codes with finer code rates. However, this simple approach can only construct codes with rates of form $K/2^p$. In practice, other code rates (say1/3and4/5 in the standard of Long Term Evolution (LTE)) are required. In this case, we can construct the basic code by combining several HT-coset codes with different rates. Taking rate1/3 as an example, we can use the Cartesian product of the HT-coset codes $[[4,1]\times[4,1]\times[4,2]]^B$ as the basic code.

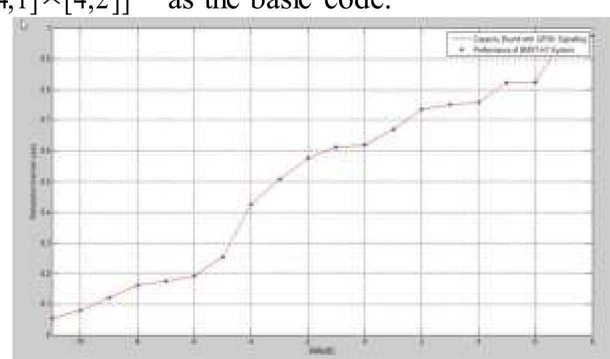

Fig. 8. The required SNR for the BMST-HT system using the Cartesian products of HT-coset codes $[8,K]^{1250}(1\leq K\leq 7)$to achieve the BER of

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10^{-5} with BPSK signaling over AWGN channels

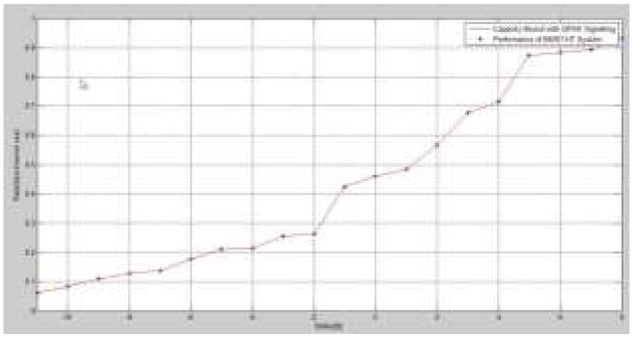

Fig.9.The required SNR for the BMST-HT system using the Cartesian products of HT-coset codes $[16,K]^{625}(1\leq K\leq 15)$ to achieve the BER of 10^{-5} with BPSK signaling over AWGN channels.

CONCLUSIONS

In this paper, we have proposed a new class of multiple-rate codes by embedding the Fast Walsh-Hadamard transform (FWHT) coset codes in to the block Markov super position transmission (BMST) system, resulting in the BMST-FWHT system. The implementation complexity of the system is linear in the code length, while the performance in the low error rate region can be predicted. The simulation results show that the BMST-FWHT system can approach the Shannon limit with in oned B at the BER of 10^{-6} for a wide range of code rates.

TABLE III

THE MEMORY REQUIRED FOR EACH CODE RATE USING THE BMST OF HT-COSET CODES WITH N = 16 TO APPROACH THE SHANNON LIMIT AT THE BER OF 10^{-5}

$R = K/16$	γ_K^* (dB)	γ_K (dB)	$\gamma_K - \gamma_K^*$ (dB)	m_K
1/16	-1.4	9.6	11.0	12
2/16	-1.2	9.8	11.0	12
3/16	-1.0	8.4	9.4	8
4/16	-0.8	7.7	8.5	6
5/16	-0.6	7.4	8.0	5
6/16	-0.3	8.1	8.4	6
7/16	-0.1	7.9	8.0	5
8/16	0.2	7.6	7.4	4
9/16	0.5	7.4	6.9	4
10/16	0.8	7.2	6.4	3
11/16	1.2	7.5	6.3	3
12/16	1.6	8.2	6.6	4
13/16	2.2	8.4	6.2	3
14/16	2.8	8.4	5.6	3
15/16	3.9	8.3	4.4	2

REFERENCES

1. Tse, D., & Viswanath, P. (2005). *Fundamentals of wireless communication*. Cambridge university press.

2. Alamouti, S. M., & Kallel, S. (1994). Adaptive trellis-coded multiple-phase-shift keying for Rayleigh fading channels. *IEEE Transactions on Communications*, *42*(6), 2305-2314.

3. Sun, Y., Karkooti, M., & Cavallaro, J. R. (2007, May). VLSI decoder architecture for high throughput, variable block-size and multi-rate LDPC codes. In *2007 IEEE International Symposium on Circuits and Systems* (pp. 2104-2107). IEEE.

4. Hagenauer, J. (1988). Rate-compatible punctured convolutional codes (RCPC codes) and their applications. *IEEE transactions on communications*, *36*(4), 389-400.

5. Acikel, O. F., & Ryan, W. E. (1999). Punctured turbo-codes for BPSK/QPSK channels. *IEEE Transactions on Communications*, *47*(9), 1315-1323.

6. Yazdani, M., & Banihashemi, A. H. (2004, June). On construction of rate-compatible low-density parity-check codes. In

Communications, 2004 IEEE International Conference on (Vol. 1, pp. 430-434). IEEE.

7. Ha, J., Kim, J., & McLaughlin, S. W. (2004). Rate-compatible puncturing of low-density parity-check codes. *IEEE Transactions on Information Theory*, *50*(11), 2824-2836.

8. Ha, J., Kim, J., & McLaughlin, S. W. (2004). Rate-compatible puncturing of low-density parity-check codes. *IEEE Transactions on Information Theory*, *50*(11), 2824-2836.

9. Zhang, K., Ma, X., Zhao, S., Bai, B., & Zhang, X. (2012, July). A new ensemble of rate-compatible LDPC codes. In *Information Theory Proceedings (ISIT), 2012 IEEE International Symposium on* (pp. 2536-2540). IEEE.

10. Casado, A. I. V., Weng, W. Y., Valle, S., & Wesel, R. D. (2009). Multiple-rate low-density parity-check codes with constant blocklength. *IEEE Transactions on Communications*, *57*(1), 75-83.

11. Uppal, M., Yue, G., Wang, X., & Xiong, Z. (2011). A rateless coded protocol for half-duplex wireless relay channels. *IEEE Transactions on Signal Processing*, *59*(1), 209-222.

12. Liu, L., Zhou, W., & Zhou, S. (2012). Nonbinary multiple rate QC-LDPC codes with fixed information or block bit length. *Journal of Communications and Networks*, *14*(4), 429-433.

13. Huang, X., Kavcic, A., Ma, X., Dong, G., & Zhang, T. (2013). Multilevel flash memories: Channel modeling, capacities and optimal coding rates. *International Journal on Advances in Systems and Measurements Volume 6, Number 3 & 4, 2013.*

14. Ma, X., Liang, C., Huang, K., & Zhuang, Q. (2013, July). Obtaining extra coding gain for short codes by block Markov superposition transmission. In *Information Theory Proceedings (ISIT), 2013 IEEE International Symposium on* (pp. 2054-2058). IEEE.

15. Ma, X., Liang, C., Huang, K., & Zhuang, Q. (2015). Block Markov superposition transmission: Construction of big convolutional codes from short codes. *IEEE Transactions on Information Theory*, *61*(6), 3150-3163.