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A Study on Methods of Contour Integration Of Complex Analysis

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ABSTRACT:

Complex analysis is considered as a powerful tool in solving problems in mathematics, physics, and engineering. In the mathematical field of complex analysis, contour integration is a method of evaluating certain integrals along paths in the complex plane. Contour integration is closely related to the calculus of residues, a method of complex analysis. One use for contour integrals is the evaluation of integrals along the real line that are not readily found by using only real variable methods. Contour integration methods include:

-Direct integration of a complex –valued function along a curve in the complex plane (a contour)

-Application of the Cauchy integral formula

-Application of the residue theorem.

One method can be used, or a combination of these methods or various limiting processes, for the purpose of finding these integrals or sums.

INTRODUCTION:

Cauchy is considered as a principal founder of complex function theory. However, the brilliant Swiss mathematician Euler (1707-1783) also took an important role in the field of complex analysis. The symbol i for $\sqrt{-1}$ was first used by Euler, who also introduced π as the ratio of the length of a circumference of a circle to its diameter and e as a base for the natural logarithms, respectively. He also developed one of the

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Volume 04 Issue 03
March 2017

most useful formulas in mathematics. That is Euler's formula $e^{i\theta} = \cos\theta + \sin\theta$, which can be derived from De Moivre's theorem [1].

Letting $\theta = \pi$ in Euler's formula, we obtain the equation $e^{i\pi} + 1 = 0$. The equation, $e^{i\pi} + 1 = 0$, contains what some mathematicians believe as the most significant numbers in all of mathematics, i.e. . e, π, i And the two integers o and 1. It also gave the remarkable result $i^i = e^{-\frac{\pi}{2}}$, which means that an imaginary power of an imaginary number can be a real number , and he showed that the system of complex numbers is closed under the elementary transcendental operations. Moreover, Euler was the first person to have used complex analysis methods for trying to prove Fermat's last theorem, and he actually proved the impossibility of integer solutions of $x^3 + y^3 = z^3$. Euler's contribution to complex analysis is significant. His work laid the foundation from which another mathematical giant, Cauchy, developed the full scale theory of complex analysis and its applications.

Complex analysis is essentially the study of complex numbers, their derivatives, and integrals with many other properties. On the other hand, number theory is concerned with the study of the properties of the natural numbers. These two fields of study seen to be unrelated to each other. However, in number theory, it is sometimes impossible to prove or solve some problems without the use complex analysis. The same is true in many problems in applied mathematics, physics and engineering. In a calculus course, the exact value of some real –valued definite integrals cannot be computed by finding its anti-derivative because some anti-derivative is impossible to find in terms of elementary functions. In these cases, the integrals are only calculated numerically.

Amazingly, complex analysis enables us to evaluate many of these integrals. One important application of complex analysis is to find the values of real definite integrals by calculating contour integrals in the complex plane [2].

A real definite integral is an integral $\int_a^b f(x)dx$, where $f(x)$ is a real-valued function with real numbers a and b. Here, a general definite integral is taken in the complex plane, and we have $\int_c f(z)dz$ where c is a contour and z is a complex variable. Contour integration provides a method for evaluating integrals by investigating singularities of the function in domains of the complex plane. The theory of the functions of a real variable had been developed by Lagrange, but the theory of functions of a complex variable had been achieved by the efforts of Cauchy.

The French mathematician Augustin-Louis Cauchy (1789-1857) brought great contributions to mathematics, providing foundations for mathematical analysis, establishing the limit concept and general theory of convergence, and defining the definite integral as the limit of a Sum.

Cauchy also devised the first systematic theory of complex numbers. Especially, his best-known work in complex function theory provides the Cauchy integral theorem as a powerful tool in analysis. Moreover, his discovery of the calculus of resides is extremely valuable and its applications are marvelous because it can be applied to the evaluation of definite integrals, the summation of series , solving ordinary and partial differential equations , difference and algebraic equations, to the theory of symmetric functions , and moreover to mathematical physics.

In the resume, Cauchy gave his definition of the derivative as a limit and defined the definite integral as the limit of a sum. As a consequence, Weirstrass and Riemann extended his work on complex function theory, and also Riemann and Lebesgue improved on his definition of the integral. In his book, he also retained the mean-value theorem much as Lagrange had derived it. Unlike Lagrange, Cauchy used tools from integral calculus to obtain Taylor's theorem [3].

Cauchy also did more with convergence properties for Taylor's series. In the '' cours d'analyse '', he had formulated the Cauchy criterion independently of Bolzano early in 1817. In his writings, Cauchy proved and often utilized the ratio, root, and integral tests, thereby establishing the first general theory of convergence. Among those great works, the importance of Cauchy's residue theory cannot be underestimated.

CONTOUR INTEGRAL

We turn now to integrals of complex-valued functions f of the complex variable z. Such an integral is defined in terms of the values $f(z)$ along a given contour C, extending from a point $z = z_1$ to a point $z = z_2$ in the complex plane. It is, therefore, a line integral; and its value depends, in general, on the contour C as well as on the function f. It is written

$$\int_c f(z)dz \quad or \int_{z_1}^{z_2} f(z)\, dz,$$

The latter notation often being used when the value of the integral is independent of the choice of the contour taken between two fixed end points. While the integral may be defined directly as the limit of a sum, we choose to define it in terms of a definite integral . Suppose that the equation

$z = z(t)\ (a \leq t \leq b)$ (1.2.5.1)

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Volume 04 Issue 03
March 2017

Represents a contour C, extending from a point $z_1 = z(a)$ to a point $z_2 = z(b)$. We assume that $f[z(t)]$ is piecewise continuous on the interval $a \leq t \leq b$ and refer to the function $f(z)$as being piecewise continuous on C [4]. We then define the line integral, or contour integral, off along C in terms of the parameter t:

$$\int_c f(z)\, dz = \int_a^b f[z(t)]\, z'(t) dt. \, (1.2.5.2)$$

Note that since C is a contour, $z'(t)$ is also piecewise continuous on $a \leq t \leq b$; and so the existence of integral (1.2.5.2) is ensured. The value of a contour integral is invariant under a change in the representation of its contour.

ANALYTIC FUNCTIONS

We are now ready to introduce the concept of an analytic function. A function f of the complex variable z is analytic at a point z_0 if it has a derivative at each point in some neighborhood of z_0. It follows that if f is analytic at a point z_0, it must be analytic at each point in some neighborhood of z_0. A function f is analytic in an open set if it has a derivative everywhere in that set. If we should speak of a function f that is analytic in a set S which is not open, it is to be understood that f is analytic in an open set containing S.

Note that the function $f(z) = \frac{1}{z}$ is analytic at each nonzero point in the finite plane. But the function $f(z) = |z|^2$ is not analytic at any point since its derivativeexists only at $z = 0$ and not throughout any neighborhood. An entire function is a function that is analytic at each point in the entire finite plane. Since the derivative of a polynomial exists everywhere, it follows that every polynomial is an entire function. If a function f fails to be analytic at a point z_0 but is analytic at some point in every neighborhood of z_0, then z_0 is called a singular point, or singularity, of f. The point $z = 0$ is evidently a singular point of the function $f(z) = \frac{1}{z}$. The function $f(z) = |z|^2$, on the other hand, has no singular points since it is nowhereanalytic.

SERIES (LAURENT SERIES)

We turn now to Taylor's theorem, which is one of the most important results of this section.

Theorem.

Supposethatafunction f isanalyticthroughoutadisk $|z - z_0| < R_0$, centered at z_0 and with radius R_0. Then $f(z)$has the power series representation

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n \; (|z - z_0| < R_0), (1.2.7.1)$$

where

$$a_n = \frac{f(n)(z_0)}{n!} \; (n = 0,1,2,\ldots). (1.2.7.2)$$

That is, series (1.2.7.1) converges to $f(z)$when z lies in the stated open disk.

This is the expansion of $f(z)$into a Taylor series about the point z_0. It is the familiar Taylor series from calculus, adapted to functions of a complex variable. With the agreement that $f(0)(z_0) = f(z_0)$ and $0! = 1$, series (1.2.7.1) can, of course, be written

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z - z_0) + \frac{f''(z_0)}{2!}(z - z_0)^2 + \cdot \; (|z - z_0) < R_0). (1.2.7.3)$$

Any function which is analytic at a point z_0 must have a Taylor series about z_0. For, if f is analytic at z_0, it is analytic throughout some neighborhood $|z - z_0| < \varepsilon$ of that point ; and ε may serve as the value of R_0in the statement of Taylor's theorem. Also, if f is entire,R_0 can be chosen arbitrarily large; and the condition of validity becomes $|z -$

$z_0| < \infty$. The series then converges to $f(z)$ at each point z in the finite plane. When it is known that f is analytic everywhere inside a circle centered at z_0, convergence of its Taylor series about z_0 to $f(z)$ for each point z within that circle is ensured; no test for the convergence of the series is even required. In fact, according to Taylor's theorem, the series converges to $f(z)$ within the circle about z_0 whose radius is the distance from z_0 to the nearest point $z_1$at which f fails to be analytic. we shall find that this is actually the largest circle centered at z_0 such that the series converges to $f(z)$ for allz interior to it. In the following section [5], we shall first prove Taylor's theorem when $z_0 = 0$, in which case f is assumed to be analytic throughout a disk $|z| < R_0$ and series (1.7.2.1) becomes a Maclaurin series:

$$f(z) = \sum_{n=0}^{\infty}(\frac{f^{(n)}(0)}{n!})\ z_n\ (|z| < R_0).(1.7.2.4)$$

LAURENT SERIES

If a function f fails to be analytic at a pointz_0, one cannot apply Taylor's theorem at that point. It is often possible, however, to find a series representation for $f(z)$ involving both positive and negative powers of $z - z_0$. We now present the theory of such representations, and we begin with Laurent's theorem.

Theorem

Suppose that a function f is analytic throughout an annular domain

$R_1 < |z - z_0| < R_2$, centered at z_0, and let C denote any positively oriented simple closed contour around z_0 and lying in that domain . Then, at each point in the domain, $f(z)$has the series representation

$$f(z) = \sum_{n=0}^{\infty} a_n\,(z - z_0)^n + \sum_{n=0}^{\infty} b_n\,(z - z_0)^n (R_1 < |z - z_0| < R_2),$$

Where

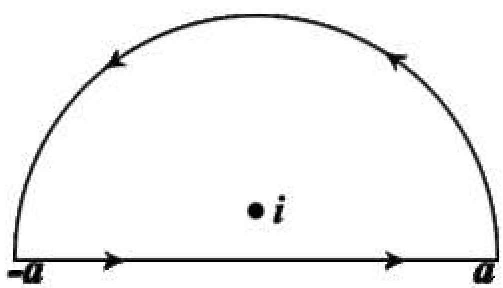
$$a_n = \frac{1}{2\pi i}\int_c \left(f(z)dz/(z-z_0)^{n+1}\right)\ (n = 0,1,2,\ldots)\ and$$

$$b_n = \frac{1}{2\pi i}\int_c \left(f(z)dz/(z-z_0)^{-n+1}\right)\ (n = 1,2,\ldots).$$

CAUCHY DISTRIBUTION

The integral

$$\int_{-\infty}^{\infty} \frac{e^{itx}}{x^2+1}\,dx$$



(which arises in probability theory as a scalar multiple of the characteristic function of the Cauchy distribution) resists the techniques of elementary calculus. We will evaluate it by expressing it as a limit of contour integrals along the contour C that goes along the real line from $-a$ to a and then counterclockwise along a semicircle centered at 0 from a to $-a$. Take a to be greater than 1, so that the imaginaryunit i is enclosed within the curve. The contour integral is

$$\int_C \frac{e^{itz}}{z^2+1}\,dz.$$

Since e^{itz} is an entire function (having no singularities at any point in the complex plane), this function has singularities only where the denominator $z^2 + 1$ is zero. Since $z^2 + 1 = (z+i)(z-i)$, that happens only where $z = i$ or $z = -i$. Only one of those points is in the region bounded by this contour [6]. The residue of $f(z)$ at $z = i$ is

$$\lim_{z\to i}(z-i)f(z)=\lim_{z\to i}(z-i)\frac{e^{itz}}{z^2+1}=\lim_{z\to i}(z-i)\frac{e^{itz}}{(z-i)(z+i)}=\lim_{z\to i}\frac{e^{itz}}{z+i}=\frac{e^{-t}}{2i}.$$

According to the residue theorem, then, we have

$$\int_C f(z)\,dz=(2\pi i)\operatorname{Res}_{z=i}f(z)=2\pi i\frac{e^{-t}}{2i}=\pi e^{-t}.$$

The contour C may be split into a "straight" part and a curved arc, so that

$$\int_{\text{straight}}+\int_{\text{arc}}=\pi e^{-t},$$

and thus

$$\int_{-a}^{a}=\pi e^{-t}-\int_{\text{arc}}.$$

It can be shown that if $t>0$ then

$$\int_{\text{arc}}\frac{e^{itz}}{z^2+1}\,dz\to 0\text{ as }a\to\infty.$$

Therefore if $t>0$ then

$$\int_{-\infty}^{\infty}\frac{e^{itx}}{x^2+1}\,dx=\pi e^{-t}.$$

A similar argument with an arc that winds around $-i$ rather than i shows that

if $t<0$ then

$$\int_{-\infty}^{\infty}\frac{e^{itx}}{x^2+1}\,dx=\pi e^{t},$$

and finally we have this:

$$\int_{-\infty}^{\infty} \frac{e^{itx}}{x^2+1}\,dx = \pi e^{-|t|}.$$

(If $t = 0$ then the integral yields immediately to real-valued calculus methods and its value is π.)

If we call the arc of the semicircle *Arc*, we need to show that the integral over *Arc* tends to zero as $a \to \infty$ — using the estimation lemma

$$\left|\int_{\mathrm{Arc}} f(z)\,dz\right| \le ML$$

where M is an upper bound on $|f(z)|$ along the Arc and L the length of *Arc*. Now,

$$\left|\int_{\mathrm{Arc}} f(z)\,dz\right| \le \frac{a\pi}{(a^2-1)^2} \to 0 \text{ as } a \to \infty.$$

So

$$\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2}\,dx = \int_{-\infty}^{\infty} f(z)\,dz = \lim_{a\to+\infty}\int_{-a}^{a} f(z)\,dz = \frac{\pi}{2}. \quad \square$$

APPLICATION TO CALCULATE THE SUMS

In 1831, Cauchy announced the theorem that an analytic function of a complex variable $w = f(z)$ can be expanded about a point $z = z_0$ in a power series that is convergent for all values of z writin a circle having z_0 as center with a radius. $r > 0$ From this time on, the use of infinite series became an essential part of the theory of functions of both real and complex variables.

The residue theorem can also calculate some sums endless. Is a function g having for each residue integer n equal to n-th general term of an infinite sum S and a set E of residues corresponding to other points. Suppose that the integral of this function along a lace γ correctable infinitely large is zero [7]. There is then the residue theorem:

$$\int_\gamma g(z)\;\mathrm{d}z = 2i\pi\left[S + \sum_{z_k\in E}\operatorname{Res}(g;z_k)\right] = 0.$$

Therefore, one can express the infinite sum by another sum (usually finite) residue:

$$S = -\sum_{z_k \in E} \operatorname{Res}(g; z_k).$$

Statements below provide more general examples of cases where this method is applicable:

the sum of the "first type

$$\sum f(n)$$

the sum of the "second type

$$\sum (-1)^n f(n)$$

First type

Let the evaluation of the following sum:

$$S = \sum_{-\infty, n \notin E}^{\infty} f(n)$$

with f having a set E of singularity . Suppose that the following conditions are respected :

There exist $M, R > 0$ and $\alpha > 1$ such that $|f(z)| \leq \frac{M}{|z|^\alpha}$ for all z complex of modulus greater than or equal to R.

Then , we have :

$$\sum_{-\infty, n \notin E}^{\infty} |f(n)| < +\infty$$

And

$$\sum_{-\infty, n \notin E}^{\infty} f(n) = -\sum_{z_k \in E} \operatorname{Res}\left(f(z)\pi\cot(\pi z); z_k\right).$$

Proof

We have

$$\sum_{n \geq R, n \notin E} |f(n)| \leq M \sum_{n \geq R, n \notin E} \frac{1}{|z|^\alpha}.$$

By using the integral test of convergence we observe that this sum converge .

We use the same argument to prove that the sum

$$\sum_{n \leq -R, n \notin E} |f(n)|$$

Converge:

As we avoid the set E of singularity of f in the sum , we have only

$$\sum_{n \geq -R, n \notin E}^{n \leq R} |f(n)| < +\infty$$

(finite sum of bonded terms) and finally:

$$\sum_{-\infty, n\notin E}^{\infty} |f(n)| < +\infty.$$

We have to find a function g which its residues are $\{f(n), n \in \mathbb{Z}\}$.

Suppose that $g(z) = f(z)\varphi(z)$

It must then the function φ has a simple pole of residue 1 at each integer .

A function having this property is given by:

$$\varphi(z) = \pi \frac{\cos(\pi z)}{\sin(\pi z)} = \pi \cot(\pi z).$$

Then ,$\sin(\pi z)$ has a simple zero for each integer z and

$$\text{Res}\left(\pi \frac{\cos(\pi z)}{\sin(\pi z)}; n\right) = \frac{\pi \cos(\pi n)}{\pi \sin'(\pi n)} = \frac{\cos(\pi n)}{\cos(\pi n)} = 1$$

Where we have used the residue formula for the fraction having simple zero to the denominator .

Take for the contour the circle center at the origin and of radius $R = N + 0.5$ with $N \in \mathbb{N}$ and the increment of one half proving that we avoid the pole located on $\pm N$.

At the limit [8], the residue theorem give:

$$\lim_{N\to\infty} \int_{C(0,R)} f(z)\pi \cot(\pi z)\, \mathrm{d}z = 2\pi i \lim_{N\to\infty} \left[\sum_{-N, n\notin E}^{N} f(n) + \sum_{z_k \in E} \text{Res}\left(f(z)\pi \cot(\pi z); z_k\right)\right].$$

It remains us now to prove that that limit is null to get the result we want.

By using the estimation lemma, we have:

$$L = \lim_{N\to\infty} \left| \int_{C(0,R)} f(z)\pi \cot(\pi z)\ \mathrm{d}z \right| \le \lim_{N\to\infty} \left(2\pi R \cdot \max_{|z|=R} \left|\pi f(Re^{it}) \cot(\pi Re^{it})\right|\right).$$

The modulus of the function cot is bounded by a some constant $K > 0$ on the contour because we avoid the integers of the real axes of the choice of contour , the right member of the inequality below is majored by

$$L \leq \lim_{N \to \infty} \frac{2\pi RMK}{R^\alpha} = 0$$

Where we have used the reason that > 1 . as the limit is as well zero , the result is proved.

Second type

Let the calculus of the following sum:

$$S = \sum_{-\infty, n \notin E}^{\infty} (-1)^n f(n)$$

With f having a set E of isolated singularity .

Suppose that f satisfy at the same condition that for the sum of the first type :

There exist

$$M, R > 0, \alpha > 1$$

Such that

$$|f(z)| \leq \frac{M}{|z|^\alpha}$$

For all complex z of the modulus greater than or equal to R.

Then, the sum converge absolutely and we have:

$$\sum_{\infty, n \notin E}^{\infty} (-1)^n f(n) = -\sum_{z_k \in E} \operatorname{Res}\left(f(z)\pi \csc(\pi z); z_k\right).$$

Proof

The proof of identic to the type , it enough to prove that the function

$\pi \csc(\pi z)$

Has as residue

$\{(-1)^n; n \in \mathbb{Z}\}$.

We have

$$\csc(\pi z) = \frac{1}{\sin(\pi z)}$$

With a simple pole at each integer point.

The residue of the fraction having a simple zero at the denominator is given by:

$$\text{Res}\left(\frac{\pi}{\sin(\pi z)}; n\right) = \frac{\pi}{\sin'(n\pi)} = \frac{\pi}{\pi\cos(n\pi)} = (-1)^n$$

Which complete the proof.

SUMMATION OF SERIES USING COMPLEX VARIABLES

Another way to sum infinite series involves the use of two special complex functions, namelywhere $f(z)$ is any function with a finite number of poles at

$z_1, z_2, .. z_N$within the complex plane and cot(B z) and csc(Bz) have the interesting property that they have simple poles at all the intergers $n = -4, ..., 0, ... +4$ along the real z axis. One knows from Cauchy's residue theorem that the closed line contour enclosing all the poles of functions $F(z)$ and $G(z)$ equals 2 πi times the sum of the residues. If we now demand that both F(z) and G(z) vanish on a rectangular contour enclosing all the poles , where again the z_n refers to the location of the N poles of $f(z)$. In deriving these results we have made use of the well known result that the residue for first order poles of $g(z)/h(z)$ at the zeros of $h(z)$ is simply $g(zn)/$

$h'(zn)$. Let's demonstrate this summation approach for several classical examples. Look first at the function $f(k) = \frac{1}{a^2+k^2}$ where $f(z)$ has poles at $z_1 = ia$ and [9]

$z_2 = -ia$. Plugging into the first residue formula above, we have

$$\sum_{k=-\infty}^{+\infty} \frac{1}{k^2+a^2} = -\frac{\pi\cot(ia\pi)}{2ia} - \frac{\pi\cot(-ia\pi)}{-2ia} = \frac{\pi}{a}\coth(\pi a)$$

or, noting the even symmetry of the quotient $cot(\pi a)/a$, that

$$\sum_{k=1}^{+\infty} \frac{1}{k^2+a^2} = -\frac{1}{2a^2} + \frac{\pi}{2a}\coth(\pi a)$$

If one takes the limit as a approaches zero(done by using the series expansions about $a = 0$ for cosine and sine plus application of the geometric series) the famous result of Euler that the sum of the reciprocal of the square of all positive integers is equal to $\pi^2/6$is obtained.

As the next example look at $f(k) = (1/(k^\wedge 2m)$.

Here we have just a single $2m$ th order pole at $z = 0$ and one finds

$\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$, $\sum_{k=1}^{\infty} \frac{1}{k^6} = \frac{\pi^6}{945}$, $\sum_{k=1}^{\infty} \frac{1}{k^8} = \frac{\pi^8}{9450}$

for$m = 2, 3$ and 4, respectively.

Next we look at a series with alternating signs. For the following case we get

$$\sum_{k=-\infty}^{\infty} \frac{(-1)^k}{(k^2+a^2)} = -Res\left[\frac{\pi\csc(\pi z))}{z^2+a^2}, z = \pm ia\right] = \frac{\pi}{\operatorname{asinh}(\pi a)}$$

which allows one to state that

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$$\frac{1}{5}-\frac{1}{10}+\frac{1}{17}+\cdots.\frac{(-1)^k}{1+k^2}=\frac{\pi}{2\sinh(\pi)}$$

when a=1.

All of the above examples have involved even functions $f(k)$. One now asks what about odd functions such as $(k)=1/k^3$? Although the above residue formulas do not apply to odd functions, a modification is possible as we now show. Consider the function $H(z)=\pi\, sec(\pi z)f(z)$, where $f(z)$ is now an odd function, and then make a closed line contour integration of $H(z)$ about the rectangular contour with corners at $(N+1/2)(1+i)$, $(N+1/2)(-1+i)$,

$(N+1/2)(-1-i)$, and $(N+1/2)(1-i)$.

This leads towhere the left integral vanishes as N goes to infinity, the residue for $H(0)$ becomes $\pi^3/2$ when $f(z)=1/z^3$, and the residues $H(n+1/2)$ become $1/\left[n+\frac{1}{2}\right)^3 sin(\pi(n+1/2))]$ .We thus have the interesting result that-[10]

$$\frac{\pi^3}{32}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^3}$$
$$=\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\cdots$$

Evaluate the following integral by using residue method

$$I=\int_{-\infty}^{+\infty}\frac{dx}{x^2+a^2}\,,(a>0)$$

Solution

The function has 2 simple poles $p1{,}2=\pm ia$.

One of these 2 pole is inside the upper plan, we have then $I=2i\,\pi\, Res\,(\,,a)$

With

$$Res\,(f,ia)=\lim_{z\to ia}\frac{z-i\,a}{z^2+a^2}=\frac{1}{2ia}$$

Therefore $I=2i\pi\times\frac{1}{2ia}$

$$I=\frac{\pi}{a}$$

CONCLUSION:

In this project we will apply the powerful technique of Contour integral in the complex plane to evaluate some improper integrals. These integrals are very difficult to tackle with the regular calculus techniques of real variables. We are going to use the integration along a branch cut, and the residue theorem, plus the proper choice of contours, to solve interesting integrals.

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