

“INSTABILITY OF SOLUTE PARTICLES”

By

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Abstract:

The thermosolutal stability is investigated by taking the fluid to be viscous, electrically conducting and heterogeneous, confined between two free boundaries under the action of vertical magnetic field. The study of thermosolutal convection in porous medium in a heterogeneous fluid is of great importance. It finds many applications in biomechanics. It is found that in this case principle of exchange of stabilities is valid and thus the marginal state is a stationary state. Both the suspended particles and the permeability of the medium have destabilizing character. It is found that in this case the marginal state is oscillatory in character and the stable solute gradient has stabilizing effect. The effect of suspended particles and thermal Rayleigh number greater than or equal to solute Rayleigh number is also studied. Further, the effect of suspended particles is to destabilize the layer.

Keywords: Rayleigh number, solute, biomechanics, oscillatory character .

We consider the stability of an incompressible, electrically conducting, viscous, density stratified fluid layer in a porous medium in the presence of a uniform magnetic field and let fluid is rotating about axes $(0, 0, \hat{z})$ with solute concentration X . The fluid is to be taken statically non-homogeneous confined between two horizontal boundaries and heated from below. Let T_0 and T_1 ($T_0 > T_1$) denote the uniform temperatures at the lower and upper boundaries, respectively.

we consider the marginal state of the system. We thus take $\omega_r = 0$ and so express $\omega = i\omega_1, \omega_1$ is real and represent the oscillatory character of the perturbations. we have

$$\frac{Ra^2}{A_0 \cdot iP_r \omega_1} = A_0(A \cdot i\omega_1) \cdot \frac{Qa_x^2 A_0}{A_0 \cdot iP_1 \omega_1} \cdot i \frac{R_2 a^2}{P_r \omega_1} \cdot \frac{R_1 a^2}{\omega_1 A_0 \cdot iP_r \omega_1} \cdot \frac{T \omega_1^2}{[(A_0 \cdot iP_1 \omega_1)(A \cdot i\omega_1) \cdot Qa_x^2]}$$

Taking the real and imaginary parts of the above equation, we have the real part as:

$$\frac{Ra^2 A_0}{(A_0^2 \cdot P_r^2 \omega_1^2)} = A_0 A \cdot \frac{Qa_x^2 A_0}{(A_0^2 \cdot P_1^2 \omega_1^2)} \cdot \frac{R_1 a^2 \omega_1 A_0}{(\omega_1^2 A_0^2 \cdot P_r^2 \omega_1^2)} \cdot \frac{T \omega_1^2 (A_0 A \cdot Qa_x^2 \cdot P_1 \omega_1^2)}{(A_0 A \cdot Qa_x^2 \cdot P_1 \omega_1^2)^2 \cdot \omega_1^2 (A_0 \cdot A P_1)^2}$$

and the imaginary part:

$$\begin{aligned} & \frac{Qa_x^2 A_0 P_1}{A_0^2 \cdot P_1^2 \omega_1^2} \cdot \frac{R_1 a^2 P_r}{\omega_1^2 A_0^2 \cdot P_r^2 \omega_1^2} \cdot A_0 \cdot \frac{R P_r a^2}{A_0^2 \cdot P_r^2 \omega_1^2} \\ & \cdot \frac{T \omega_1^2 (A_0 \cdot P_1 A_1)}{(A_0 A_1 \cdot Qa_x^2 \cdot P_1 \omega_1^2)^2 \cdot \omega_1^2 (A_0 \cdot P_1 A_1)^2} \frac{R_2 a^2}{P_r} \end{aligned}$$

After eliminating the Rayleigh number R from above equations, we find the frequency of oscillation at the marginal state and the Rayleigh number is given and its minimum value is called the critical Rayleigh number corresponding with critical wave number a_c and the frequency ω_c .

Let us say the non-oscillatory modes exist for which ω_1 is zero and $\omega = \omega_2, \omega_3$ is real. Hence substituting $\omega = \omega_2$ in equation, we get

$$B_0 \omega_2^7 \cdot B_1 \omega_2^6 \cdot B_2 \omega_2^5 \cdot B_3 \omega_2^4 \cdot B_4 \omega_2^3 \cdot B_5 \omega_2^2 \cdot B_6 \omega_2 \cdot B_7 = 0$$

where,

$$B_0 = A_0 P_r^3 P_1^2$$

$$B_1 = A_0 P_r^2 P_1 [A_0 P_1 (1 + \omega \cdot 2 P_r) + 2 P_r (A_0 \cdot P_1 B)]$$

$$\begin{aligned} B_2 = & A_0 P_r^3 \left[P_1^2 (\omega A_0 \cdot P_r^2 A_1^2) + 2 P_1 P_r \omega A_0 (1 + \omega) \frac{(A_0 \cdot P_1 A_1)}{3} \right. \\ & \left. \cdot P_r (2 A_0 A_1 \cdot Qa_x^2) \right] \cdot A_0^2 P_r^2 \left[P_1^2 P_r^2 (R_1 \cdot R_2 \cdot R) a^2 \cdot T \omega_1^2 P_1 P_r^3 \right] \end{aligned}$$

$$\begin{aligned}
 B_3 & \square A_0 P_r \{ A_0 P_r (1 \cdot \bar{\Delta}) [A_0^2 \cdot P_1^2 A^2 \cdot 2 P_1 (2 A_0 A \cdot Q a_x^2)] \\
 & \cdot 2 \bar{\Delta} A_0^2 P (A_0 \cdot P_1 A) \cdot 2 P_r^2 (A_0 A \cdot Q a_x^2) (A_0 \cdot P_1 A) \} \\
 & \cdot R_2 a^2 P_1 P_r [2 A_0 P_r \cdot P_1 (A_0 (1 \cdot \bar{\Delta}) \cdot A P_r)] \\
 & \cdot R_1 a^2 P_r [P_1 P_r (A_0 \cdot A) \cdot A_0 (P_r \cdot \bar{\Delta} P_1^2)] \cdot T \bar{\Delta}^2 P_r^2 A_0 (P_r \\
 & \cdot (1 \cdot \bar{\Delta}) P_1) \cdot R a^2 P_r [P_1 P_r (A_0 \cdot A) \cdot A_0 (P_r \cdot \bar{\Delta} P_1^2)] \\
 B_4 & \square A_0 P_r \{ P_r^2 (A_0 A \cdot Q a_x^2)^2 \cdot \bar{\Delta} A_0^2 [A_0^2 \cdot P_1^2 A^2 \cdot 2 P_1 (Q a_x^2 \cdot 2 A_0 A)] \\
 & \cdot 2 (A_0 A \cdot Q a_x^2) (A_0 \cdot P_1 A) A_0 P_r (1 \cdot \bar{\Delta}) \} \cdot R_2 a^2 \{ A_0^2 (\bar{\Delta} P_r \cdot P_r \cdot \bar{\Delta} P_1) P_1 \\
 & \cdot (A_0 \cdot P_1 A) (P_r \cdot (1 \cdot \bar{\Delta}) P_1) A_0 P_r \cdot P_1 P_r^2 (A_0 A \cdot Q a_x^2) \} \\
 & \cdot R_1 a^2 P_r \{ A_0^2 P_1 \cdot (P_r \cdot P_1) A_0 (A_0 \cdot P_1 A) \cdot P_1 P_r (A_0 A \cdot Q a_x^2) \} \\
 & \cdot R a^2 P_r \{ \bar{\Delta} A_0^2 P_1 \cdot (A_0 \cdot P_1 A) (P_r \cdot \bar{\Delta} P_1) A_0 \cdot T \bar{\Delta}^2 P_r A_0^2 (\bar{\Delta} P_1 \cdot (1 \cdot \bar{\Delta}) P_r) \} \\
 B_5 & \square R_1 a^2 P_r \{ (A_0 A \cdot Q a_x^2) (P_r \cdot P_1) A_0 \cdot A_0^2 (A_0 \cdot P_1 A) \} \cdot T \bar{\Delta}^2 P_r \bar{\Delta} A_0^3 \\
 & \cdot R a^2 P_r \{ (P_r \cdot \bar{\Delta} P_1) (A_0 A \cdot Q a_x^2) A_0 \cdot \bar{\Delta} A_0^2 (A_0 \cdot P_1 A) \} \\
 & \cdot A_0 R_2 a^2 \{ (A_0 A \cdot Q a_x^2) (P_r \cdot (1 \cdot \bar{\Delta}) P_1) P_r \cdot (A_0 \cdot P_1 A) \\
 & ((1 \cdot \bar{\Delta}) P_r \cdot \bar{\Delta} P_1) A_0 \cdot \bar{\Delta} A_0^2 P_1 \} \cdot A_0 P_r \{ 2 \bar{\Delta} A_0^2 (A_0 A \cdot Q a_x^2) (A_0 \cdot P_1 A) \\
 & \cdot (A_0 A \cdot Q a_x^2) (1 \cdot \bar{\Delta}) A_0 P_r \} \\
 B_6 & \square A_0^3 P_r \bar{\Delta} (A_0 A \cdot Q a_x^2)^2 \cdot R_2 a^2 A_0^2 \{ (A_0 A \cdot Q a_x^2) (\bar{\Delta} P_1 \cdot (1 \cdot \bar{\Delta}) P_r) \\
 & \cdot \bar{\Delta} A_0 (A_0 \cdot P_1 A) \} \cdot R_1 a^2 P_r A_0^2 (A_0 A \cdot Q a_x^2) \cdot R a^2 P_r \bar{\Delta} A_0^2 A_3 \\
 B_7 & \square \cdot R_2 a^2 \bar{\Delta} A_0^3 (A_0 A \cdot Q a_x^2)
 \end{aligned}$$

$$A_0 \square \bar{\Delta}^2 \cdot a^2 \text{ and ,}$$

$$A_1 \square A_0 \cdot B$$

- (i) Since $R_2 \square 0$ then $B_7 \square 0$, and the product of all seven roots is positive, therefore at least one root is positive, these making the system is unstable. Therefore, we conclude that non-oscillatory modes are unstable.

- (ii) If $R_2 \neq 0$, then $B_7 \neq 0$ and product of all roots is negative, hence either all roots are negative or at least one root is positive, thus both negative or positive roots are possible.

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