

Fuzzy Grey Relational Analysis for the Identification of Optimum Influencing Cutting Parameters in Turning

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Abstract:

This paper presents the optimization of feed rate and tool geometry, which are influential parameters of the cutting forces in turning by using Fuzzy Grey Relational Analysis. Taguchi's L_{16} orthogonal array is considered for 3 factors 4 levels of input cutting parameters. According to the design sixteen turning experiments are performed on Aluminum workpiece with the High Speed Steel (HSS) cutting tools of different rake angles, at different feed rates by varying tool approach angle; and the cutting responses like feed force, cutting force and Radial forces are recorded. Grey-fuzzy approach is used for the study of these response characteristics and optimal combination of influential input parameters is discovered. Based on the results of verification experiments it is found that there is a significant improvement in optimization of cutting forces by using Taguchi and Fuzzy-Grey Relational Analysis.

Keywords

Taguchi's DOE, tool geometry, cutting forces, feed rate, Grey Fuzzy Relational Analysis.

1. Introduction

In metal cutting operations, determination of optimum tool geometry and cutting parameters require detailed investigations of their effects on cutting forces. The analysis of cutting forces is necessary for design and evaluation of machining parameters, tool design and calculation of power requirements etc. The cutting forces are mainly influenced by depth of cut and feed rate, respectively more than by cutting speed [1]. The tool geometry is also an important factor on cutting forces; the cutting force components are very sensitive even for the smallest changes in the cutting process. Therefore, instead of calculating the cutting force theoretically, measuring them in process by dynamometers is preferred.

Rake angle (α) is the angle contained by a plane perpendicular to the main cutting edge of the tool and is a measure of the edge in relation to the cut itself. This angle can be take positive or negative value. Positive rake angle produces higher shear angle and therefore, it leads to reduction of cutting forces, but excessive value of this angle causes tool breakage. Approaching angle (k) is the angle at which cutting tool enters and leaves the cutting zone. The chip cross-section is determined by the approaching angle. Since the main cutting edge enters and leaves cutting zone suddenly at 90° of approaching angle it is subjected to maximum loading and unloading. When the tool is fed along a line at 90° to the axis of work-piece, namely cutting action is orthogonal [2]. In oblique cutting, approaching angle is $0^\circ < k < 90^\circ$. The tool geometry is given in Figure1(a).

2. Experimental Data

The complete experimental setup is shown in Figure.2, the experimental data required for the accomplishment of experiments are mainly influential input factors and their levels, Design of experiments and experimentation procedure.

2.1. Input Factors

The influential factors and their levels are summarized in Table 1.

Table 1. Factors and factor levels.

Controllable factors	Symbol	Factor Levels			
Rake Angle (degree)	α	-7°	-3°	3°	7°
Approach Angle (degree)	k	45°	60°	75°	90°
Feed rate (mm/rev)	f	0.16	0.20	0.25	0.32

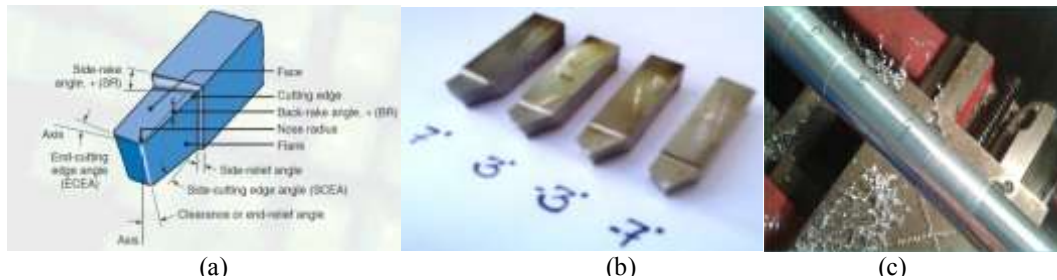


Figure.1.(a) Tool geometry; (b) Cutting tools with different rake angles; (c) Aluminum workpiece.

2.2. Design of Experiments (DOE)

For *three-factor Four-level*, a Taguchi L_{16} orthogonal array experimental design is selected, with which the total number of experiments to be performed are optimized and experimental procedure is as follows.

2.3. Experimentation procedure

According to L_{16} Taguchi experimental design sixteen turning experiments are performed on Aluminum work-piece of size $\Phi 36 \times 100$ mm with four HSS cutting tools (rake angles: -7° , -3° , 3° and 7°) at different approach angles (45° , 60° , 75° and 90°) and with different feed rates (0.16, 0.20, 0.25 and 0.32) in dry cutting conditions on PSG A141 conventional lathe [3]. The constant parameters during turning experiments are cutting speed of $v_c = 450$ rev/min and depth of cut = 1 mm. Cutting forces are measured with a *three-component compact force dynamometer* shown in Figure.2, and transferred from over serial port to the PC directly for further evaluation [4]. The cutting forces namely feed force (F_f), Cutting force (F_c) and Radial force (F_r) are recorded in Table 2.

3. Methodology

The methodology followed in this work is mainly based on Grey Relational Analysis (GRA) and Fuzzy approach on GRA is explained below. The predicted results are evaluated through Taguchi and ANOVA for determining the optimal combination of inputs.

3.1. Grey Relational analysis

The grey Relational Analysis is used to optimize multiple responses [5]. This process is done in three steps (1) Normalization, (2) Calculation of Grey Relation coefficient, and (3) Calculation of Grey Relation Grade [5, 6].

3.1.1. Normalization

Normalization is performed to prepare the basic data for the analysis where the original combination is transferred to a comparable combination. Linear normalization is usually in the range between zero

and unity is also called as the grey relational generation.

Data Pre-Processing is normally required, since the range and unit in one data sequence may differ from others. It is also necessary when the sequence scatter range is too large, or when the directions of the target in the sequences are different. The formulae are given in equations (1) and (2).

'Higher – the – Better':

$$X_i^*(k) = \frac{X_i(k) - \min X_i(k)}{\max X_i(k) - \min X_i(k)} \quad (1)$$

If 'Lower – the – better':

$$X_i^*(k) = \frac{\max X_i(k) - X_i(k)}{\max X_i(k) - \min X_i(k)} \quad (2)$$

where $X_i^*(k)$ and $X_i(k)$ are normalized data and observed data respectively for the i^{th} experiment by using k^{th} response.

3.1.2. Grey Relational Coefficient (GRC)

GRC expresses the relationship between the ideal (best) values and actual normalized values for all the combinations. GRC can be calculated using the following equation (3):

$$\xi_i(k) = \frac{\Delta_{\min} + \zeta \Delta_{\max}}{\Delta_i(k) + \zeta \Delta_{\max}} \quad (3)$$

where, $\Delta_i(k)$ is absolute value of the difference between $x_i^0(k)$ and $x_i^*(k)$ and $\Delta_i(k) = |x_i^0(k) - x_i^*(k)|$. Δ_{\max} and Δ_{\min} are global maximum and global minimum values in different data series, respectively. The distinguishing coefficient (ζ) lays between 0 and 1, which is to expand or to compress the range of GRC, generally, $\zeta = 0.5$ is taken.

3.1.3. Grey Relational Grade (GRG)

In third step, the grey relational grade is computed by finding the average of the grey relational coefficient corresponding to each performance characteristics. This grade is being estimated with the following equation (4):

$$y_i = \frac{1}{n} \sum_{k=1}^n (\xi_i(k)) \quad (4)$$

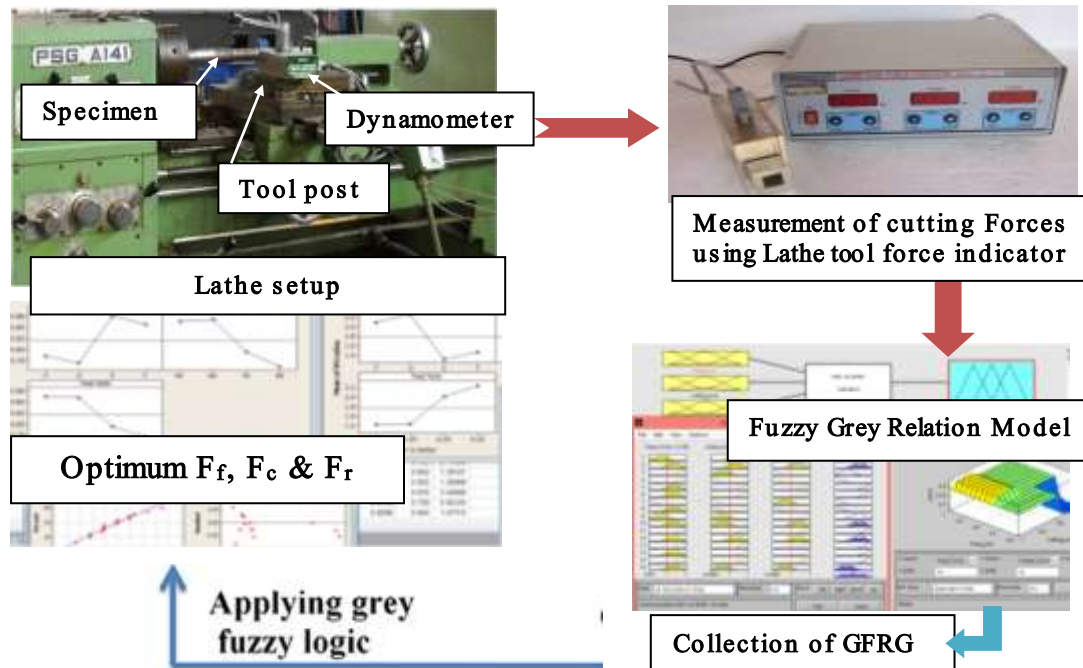


Figure. 2 Experimental setup.

Table 2. Experimental response data.

Exp. No.	Input Parameters			Cutting forces		
	α	k	f	F_f	F_c	F_r
1	-7	45	0.16	157	245	39
2	-7	60	0.2	127	235	88
3	-7	75	0.25	88	265	186
4	-7	90	0.32	108	333	255
5	-3	45	0.2	137	216	29
6	-3	60	0.16	127	216	39
7	-3	75	0.32	226	382	108
8	-3	90	0.25	118	333	216
9	3	45	0.25	157	275	39
10	3	60	0.32	137	265	59
11	3	75	0.16	108	186	39
12	3	90	0.2	137	225	69
13	7	45	0.32	98	196	49
14	7	60	0.25	118	196	39
15	7	75	0.2	108	206	49
16	7	90	0.16	137	284	98

where, y_i the grey relational grade and n is the number of process response. The optimal value of the GRG can be predicted by using Eq. (5)

$$y_i = y_m + \sum_{i=1}^q (\bar{y}_i - y_m) \quad (5)$$

where, y_m is total mean of the GRG value, q is number of input process parameters, and y_i is mean GRG value at the optimal level for the i^{th} parameter.

3.2. Fuzzy-Grey Relational Analysis

Fuzzy Grey relational analysis is an approach in which the fuzzy system developed based on grey relational coefficients and grey relational grade. The key elements of fuzzy grey relation system are explained below.

3.2.1. Fuzzy logic unit: A fuzzy logic unit consists of a fuzzifier for the input of data (grey relational coefficient) into an inference engine with the help of fuzzy subsets, membership functions, and a fuzzy

Table 3. Grey relation coefficients and grey relational grade.

S.No	Normalization			Grey Relation Coefficient			GR Grade
	Ff	Fc	Fr	$\xi_1(Ff)$	$\xi_2(Fc)$	$\xi_3(Fr)$	γ
1	0.5000	0.6990	0.9558	0.7421	0.8638	0.9393	0.8484
2	0.7174	0.7500	0.7389	0.8358	0.8842	0.7241	0.8147
3	1.0000	0.5969	0.3053	1.0000	0.8256	0.4965	0.7740
4	0.8551	0.2500	0.0000	0.9085	0.7179	0.4065	0.6776
5	0.6449	0.8469	1.0000	0.8021	0.9258	1.0000	0.9093
6	0.7174	0.8469	0.9558	0.8358	0.9258	0.9393	0.9003
7	0.0000	0.0000	0.6504	0.5900	0.6562	0.6621	0.6361
8	0.7826	0.2500	0.1726	0.8687	0.7179	0.4529	0.6799
9	0.5000	0.5459	0.9558	0.7421	0.8078	0.9393	0.8298
10	0.6449	0.5969	0.8673	0.8021	0.8256	0.8377	0.8218
11	0.8551	1.0000	0.9558	0.9085	1.0000	0.9393	0.9493
12	0.6449	0.8010	0.8230	0.8021	0.9056	0.7947	0.8341
13	0.9275	0.9490	0.9115	0.9521	0.9740	0.8856	0.9372
14	0.7826	0.9490	0.9558	0.8687	0.9740	0.9393	0.9273
15	0.8551	0.8980	0.9115	0.9085	0.9493	0.8856	0.9144
16	0.6449	0.5000	0.6947	0.8021	0.7924	0.6917	0.7621

rule base. The inference engine performs a fuzzy reasoning on fuzzy rules to generate a fuzzy value. At last, the defuzzifier converts the fuzzy value into a Grey-Fuzzy grade [7]. The structure built for this study is a three input- one-output fuzzy logic unit as shown in Figure 3.

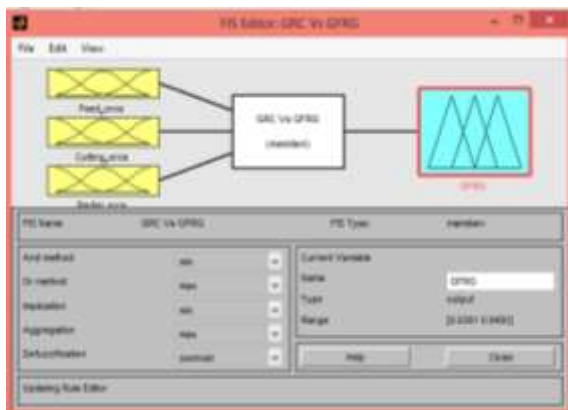


Fig. 3. Fuzzy logic unit.

3.2.2. Fuzzy Sets and Membership functions: The fuzzifier uses triangle form of membership function to fuzzify inputs ($\xi_1 = \text{GRC for } F_f, \xi_2 = \text{GRC for } F_c$ and $\xi_3 = \text{GRC for } F_r$). They are changed into linguistic fuzzy subsets using membership functions of a triangle form [9], and are uniformly assigned into four fuzzy subsets—(1).Low, (2).Below–Middle, (3).Above–Middle, (4).High grades [10]. The output variable is the Grey-Fuzzy Relational Grade (γ_o) also converted into similar linguistic fuzzy subsets using membership functions of gauss form, as shown in Figure.4.

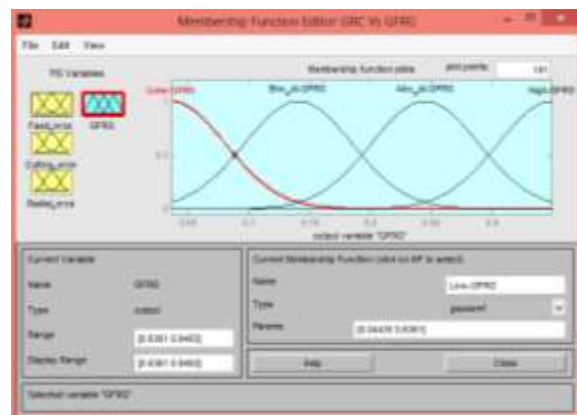


Fig.4. Fuzzy membership functions.

3.2.3. Fuzzy rules: The fuzzy rule base consists of ‘if-then’ control rules to express the inference relationship between input and output. A typical linguistic fuzzy rule called Mamdani is described as:

Rule 1: If (ξ_1 is F_2) and (ξ_2 is C_3) and (ξ_3 is R_4) then (γ_o is GF_3) else,

Rule 2: If (ξ_1 is F_3) and (ξ_2 is C_3) and (ξ_3 is R_3) then (γ_o is GF_3) else,

...

Rule 16: If (ξ_1 is F_3) and (ξ_2 is C_2) and (ξ_3 is R_2) then (γ_o is GF_2).

where F_i (Feed force), C_i (Cutting force), R_i (Radial force) and GF_i (Grey Fuzzy Grade) are the fuzzy subsets by the corresponding membership functions, i.e., $\mu_{F_i}(\xi_1)$, $\mu_{C_i}(\xi_2)$, $\mu_{R_i}(\xi_3)$ and $\mu_{GF_i}(\gamma_o)$. The inference engine then performs fuzzy reasoning

on fuzzy rules by taking max–min inference (Eq. (6)) for generating a fuzzy value $\mu_{D0}(y)$.

$$\mu_{D0}(y) = (\mu_{F1}(\xi_1) \wedge \mu_{C1}(\xi_2) \wedge \mu_{R1}(\xi_3) \wedge \mu_{GF1}(y_o)) \vee (\mu_{F2}(\xi_1) \wedge \mu_{C2}(\xi_2) \wedge \mu_{R2}(\xi_3) \wedge \mu_{GF2}(y_o)) \dots \vee (\mu_{Fi}(\xi_1) \wedge \mu_{Ci}(\xi_2) \wedge \mu_{Ri}(\xi_3) \wedge \mu_{GF_i}(y_o)) \quad (6)$$

where, \wedge is minimum operation, and \vee is maximum operation respectively.

Finally the defuzzifier converts the fuzzy value into crisp output using the centroid defuzzification method (Eq. 7); i.e. grey fuzzy reasoning grade (y) is calculated from the fuzzy multi-response output $\mu_{D0}(y_o)$ using the following equation:

$$y = \frac{\sum y_o \mu_{D0}(y_o)}{\sum \mu_{D0}(y_o)} \quad (7)$$

The non-fuzzy value y_o gives Fuzzy Grey Relational Grade. Invariably, a larger Fuzzy grey relational grade is opted, which gives an improved performance characteristic. Table.4 shows the results of fuzzy-grey relational grade for different experiments.

Table 4. Fuzzy Grey Relational Grade.

S.No	Fuzzy values(GFRG)	Rank
1	0.844	7
2	0.842	11
3	0.742	13
4	0.725	14
5	0.852	6
6	0.855	4
7	0.674	16
8	0.72	15
9	0.844	8
10	0.843	10
11	0.911	1
12	0.844	9
13	0.864	3
14	0.875	2
15	0.855	5
16	0.788	12

4. Analysis of Experimental Results Using Fuzzy-GRG

In this work the experiments are performed based on Taguchi and fuzzy grey relational analysis, by which it is possible to identify the significant effect of each machining parameter on the Fuzzy-GRG (GFRG) at different levels. The mean Fuzzy-GRG at each level for the different machining parameters is presented in Table 5, which is referred to as a response table. The influence of each machining

parameter can be more clearly presented by means of the Fuzzy-GRG response graph. The Fuzzy-GRG graph shows the change in the response when a given factor goes from level 1 to level 4. The response graph for the machining parameters of turning process is presented in Figure.5.

Table 5. Response table for FGRG

Level	Rake Angle	Approach Angle	Feed Rate
1	0.7883	0.851	0.8495
2	0.7752	0.8538	0.8482
3	0.8605	0.7955	0.7953
4	0.8455	0.7692	0.7765

4.1. Analysis of Variance (ANOVA)

ANOVA is used to analyze which machining parameters significantly affect the performance characteristics [11]. This is accomplished by separating the total variability of the Fuzzy-GRG, which is measured by the sum of the squared deviations from the total mean of the Fuzzy-GRG, into contributions by each machining parameter and the error.

Table.6 ANOVA table for GFRG.

Source	DF	Sum of Squares	Mean of Squares	F	% Contribution
α	3	0.035248	0.011749	2.23	24.30
k	3	0.04949	0.016497	3.13	34.12
f	3	0.028721	0.009574	1.82	19.80
Error	6	0.031599	0.005266		
Total	15	0.145058			

From the ANOVA Table.6 it was found that approach angle has the most significant effect on cutting forces.

4.2. Confirmation Test

Finally a confirmation test was conducted to verify the improvement in the cutting forces for the estimated Fuzzy-GRG, using the optimal level of the machining parameters [12].

Table 7 shows the comparisons of predicted and actual machining responses using their optimal machining parameters.

Table 7. Predicted vs Actual machining responses.

Responses	Initial m/c parameter ($\alpha=3, k=75, f=0.16$)	Optimum m/c parameters ($\alpha=3, k=60, f=0.16$)	
		Predicted	Experimental
F_r	108	127	108
F_c	136	216	206
F_t	39	42	39

Based on the confirmation experiments, for the final optimal combination of parameters (rake angle= 3°, Approach angle=60° and feed rate=0.16) the cutting force were reduced. Hence there is a

significant improvement in responses after optimization.

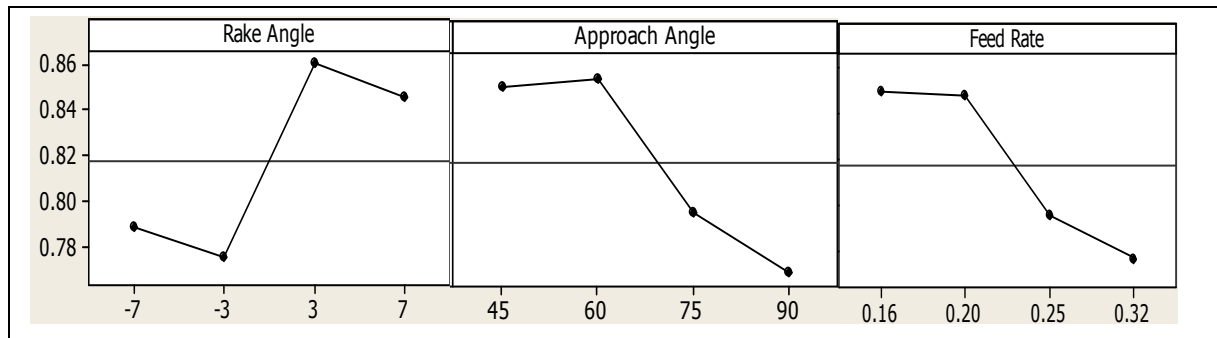


Figure.5 Response graph for means of Fuzzy-GRG.

5. Conclusions

In this paper the Taguchi, Fuzzy Grey Relational Analysis are used for the determination optimum levels of feed rate, rake angle and approach angle; which influences the cutting forces in turning. ANOVA is also used to find out the significantly most effective parameter on responses. From the analysis the following conclusions are drawn:

1. The optimal combination of input parameters is identified to be 'rake angle=3°', 'Approach angle=60°' and 'feed rate=0.16'.
2. From the ANOVA it is found that the most effective parameter is 'Approach angle'.
3. The experimental responses for the above optimal combination are reveals that there is a considerable reduction in the cutting forces which means they are optimized.

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