Transition Analysis of Spherical Shell under Uniform Pressure

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Abstract

This paper presents non-linear deformation behaviour of compressible spherical shell examined under pressure by using Seth's transition theory. The purpose of the paper is to present the study of elastic - plastic stress analysis in a spherical shell under the combined effect of compressibility and temperature. The solution of the problem has been obtained by using the concept of generalized strain measures and Seth's transition theory. The radial and hoop stresses are calculated under the effect of pressure and compressibility factor. Results have been discussed numerically & discussed graphically.

Keywords: Pressure, Elastic-Plastic, Stress, Strain, Compressible

1. INTRODUCTION

The problem of flow of an absolutely rigid sphere by the viscous incompressible fluid has been solved by Stokes in 1851. He used the approximate method of neglecting inertial terms and exterior forces in the basic equations of motion. In the books by Kochin *et al.* (1965) and Landau *et al.* (1987) containing the classical solution of such problem, the surface strains are not considered, and the fluid motion is carried out at low Reynolds numbers. Analysis & design of shells are very significant for various engineering applications. Due to continuously interest of the engineering community in the shells, the elastic - plastic behaviour of shells has been very closely studied by various authors. The problem of elastic - plastic transition in transversely isotropic shells under uniform pressure has been solved by Hill[5], Johnson [7] et al in the theory of plasticity. The elastic solution of the problem has been obtained in the elastic range and then derived the stresses at fully plastic state using Tresca' yield condition. In 1963, Seth[8] has discussed the concept of elastic - plastic transition in shells without using adhoc assumptions and explained that the transition takes place is non-linear in nature , where as classical model does not explained non -linearity of transition state. B. R. Seth worked on problem of elastic-plastic transition in shells and tubes under Pressure. Seth obtained elastic-plastic transition in paper with the help of a semi-empirical yield condition like that of Tresca or Von Mises. The stresses are calculated from the elastic solution and then used in the yield condition to get the transition surface. The possibility of treating it as a transition or turning point phenomenon in finite deformation has not been explored. When the plastic state tends to set in, the stress strain relations undergo a change. Seth has obtained the results for fully plastic state by using the theory of finite deformation. S. Hulsurkar (1966) discussed on creep transition theory of spherical shell. Seth's transition theory of creep neither uses the creep laws nor the yield criteria and the associated flow rules. It has been used to find the creep stresses in shells under uniform internal pressure by defining various transition functions when the steady state of creep has been achieved. Hulsurkar found that the transition points may be multiple points and these can be used to find either plastic or creep stresses depending upon the transition functions. This theory can be also applicable to the primary and tertiary stages of creep.

Here in this current literature, the elastic plastic transition problem for spherical shells under internal pressure is solved using the concept of generalized measures. The results obtained agree with the Seth [8] for n=2. Therefore, the concept of generalized measures has been successfully applied to large number of problems [1-5]. Seth [9] has defined the concept of generalized strain measures as

$$e_{ii} = \int_0^{e_{ii}^A} [1 - 2e_{ii}^A]^{\frac{n-2}{2}} de_{ii}^A = \frac{1}{n}[1 - (1 - 2e_{ii}^A)^{\frac{n}{2}}, \ i=1,2,3 \qquad (1)$$

where n is the measure & e_{ii}^A is the almansi finite strain components.

In this paper, the analysis of the elastic - plastic transition in shell is done by concept of generalized measures and Seth' transition theory. Results have been discussed numerically & discussed graphically.

2. GOVERNING EQAUTIONS

We consider a spherical shell of constant thickness under internal pressure. Due to the symmetry in the elastic properties, the displacement is purely radial. Therefore, we take the displacements in spherical coordinates (r, θ, ϕ) as

u = r(1−β) , v = 0 , w = 0 where β is function of $r = \sqrt{x^2 + y^2 + z^2}$ (2)

The strain components for finite deformation are given as

$$e_{rr}^A = \frac{1}{2}[1-(r\,\beta' + \beta)^2] \quad , e_{\theta\theta}^A = e_{\varphi\varphi}^A = \frac{1}{2}(1-\beta^2)$$

$$e_{r\theta}^A = e_{\theta\varphi}^A = e_{r\varphi}^A = 0 \quad (3)$$

where $\beta' = \frac{d\beta}{dr}$

Substituting equation (3) in equation (1) , the generalized components of the strain are

$$e_{rr} = \frac{1}{2}[1-(r\,\beta' + \beta)^n] \quad , e_{\theta\theta}^A = e_{\varphi\varphi}^A = \frac{1}{2}(1-\beta^n) \quad (4)$$

$$e_{r\theta}^A = e_{\theta\varphi}^A = e_{r\varphi}^A = 0$$

The stress - strain relations for isotropic material are given by[11]

$$T_{ij} = \lambda\delta_{ij}I_1 + 2\mu e_{ij} \ , \ (i,j = 1,2,3) \quad (5)$$

where λ and μ are lame's constants and $I_1 = e_{kk}$ is called first strain invariant.

Using equation (4) in (5), the corresponding stresses are given as

$$T_{rr} = \frac{\lambda+2\mu}{n}[1-(r\,\beta' + \beta)^n] + \frac{2\lambda}{n}(1-\beta^n)$$

$$T_{\theta\theta} = T_{\varphi\varphi} = \frac{\lambda}{n}[1-(r\,\beta' + \beta)^n] + \frac{2\lambda+2\mu}{n}(1-\beta^n) \quad (6)$$

$T_{r\theta} = T_{\theta\varphi} = T_{r\varphi} = 0$

The equations of equilibrium are all satisfied except

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{2(\tau_{rr} - \tau_{\theta\theta})}{r} = 0 \tag{7}$$

Using equation (6) in (7) , we get the non - linear differential equation in β as

$$\frac{\lambda+2\mu}{n}[1-(r\,\beta' + \beta)^n] + \frac{2\lambda}{n}(1-\beta^n) + \frac{4\mu}{n}\int \frac{\beta^n - (r\,\beta' + \beta)^n}{r}\,dr = \mathrm{k} \tag{8}$$

where k is constant

Put $c = 2\mu/\lambda + 2\mu = (1-2\sigma)/1-\sigma$, r $\beta' = \beta$P

and differentiating (8) with respect to r , we get equation as

$$\frac{d\beta}{dP}\left[(1+P)^n + 2(1-c) - \frac{2c(1-(1+P)^n)}{nP}\right] + \beta\,(1+P)^{n-1} = 0 \tag{9}$$

Turning points of β from equation (9) are $P \to 1$ *and* $P \to \pm\infty$.

: The boundary conditions of the problem are given as

$T_{rr} = -p$ at r = a and $T_{rr} = 0$ at r = b

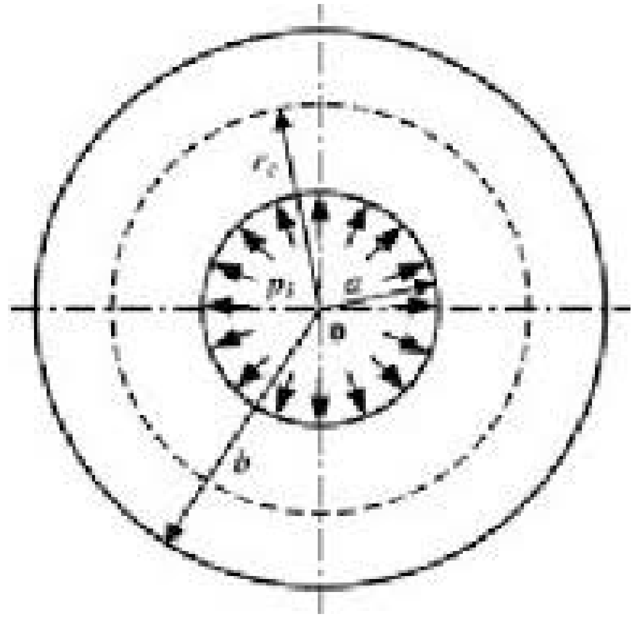


Figure1: Geometry of spherical shell under internal pressure

3. DETERMINATION OF PLASTIC STRESESS THROUGH THE PRINCIPAL STRESS

In these types of problems, the solution of the problem depends upon the nature of the problem. If the problem is based on elastic-plastic transition then we define the transition function through principal stress. If problem deals with the creep behaviour of solid, then we define transition function through principal stress difference. In order to find the plastic stress, the transition function is defined using the principal stress as taken by Seth [8], Gupta et al. [1-5] at the transition point $P \to \pm\infty$. Define the transition function as

$$R = 1 - \frac{nT_{rr}}{3\lambda + 2\mu} = \frac{\beta^n}{3 - 2c}\left[(1+P)^n + 2(1-c)\right] \tag{10}$$

Taking the logarithmic differentiation of equation (10) with respect to r and using equation (9), we get

$$\frac{dlogR}{dr} = \frac{2c\left[1-(1+P)^n\right]}{r\left[(1+P)^n + 2(1-c)\right]} \tag{11}$$

Taking the asymptotic value of equation (11) at $P \to \pm\infty$ and integrating, we get

$$R = A_0 r^{-2c} \tag{12}$$

where A_0 is a constant of integration , which can be determined by boundary condition. From equation (10) , we have the transition value of T_{rr} is

$$T_{rr} = \frac{2\mu\,(3-2c)}{nc}\left[1 - A_0 r^{-2c}\right] \tag{13}$$

The value of E in the transition range is given by Seth [10]

$$Y = \frac{E}{n} = \frac{2\mu(1+\sigma)}{n} = \frac{2\mu\,(3-2c)}{n(2-c)} \tag{14}$$

Using equation (14) in (13), we have

$$T_{rr} = \frac{Y\,(2-c)}{c}\left[1 - A_0 r^{-2c}\right] \tag{15}$$

Applying boundary condition, $T_{rr} = 0$ at r = b and using equation (7)

$$T_{rr} = \frac{Y\,(2-c)}{c}\left[1 - \left(\frac{b}{r}\right)^{2c}\right] \quad , \quad T_{\theta\theta} - T_{rr} = Y(2-c)(b/r)^{2c} \tag{16}$$

For initial yielding, $|\,T_{\theta\theta} - T_{rr}\,|$ is maximum at r = a. Therefore yielding will start at the internal surface of the sphere.

$$|\,T_{\theta\theta} - T_{rr}\,|_{r=a} = Y(2-c)(b/a)^{2c}$$

By boundary condition $T_{rr} = -p$ at r = a in equation (16). Therefore pressure required to start initial yielding.

$$p_i = \frac{Y(2-c)}{c}\left[\left(\frac{b}{a}\right)^{2c} - 1\right] \tag{17}$$

The results obtained are same as given by Seth [8].

Fully plastic state:-

In order to find fully plastic state, we make c→0 at the external surface r = b and equation (16) and (17) become.

$$T_{rr} = 4Y \log \frac{r}{b}, \quad |T_{\theta\theta} - T_{rr}| = 2Y \tag{18}$$

Pressure required for attaining the fully plastic state is given as

$$p_f = 4Y \log \frac{b}{a} \tag{19}$$

which is twice the result given by Hill [5].

Further we introduce the non-dimensional components as

R = r/b, R_o = a/b, $\sigma_r = T_{rr}$/Y, $\sigma_r = T_{\theta\theta}$/Y, $p^* = p_i$/Y, $p^* = p_f$/Y

Therefore, Elastic-plastic stresses and pressure from equation (16) and (17) is given as

$$\sigma_r = \frac{2-c}{c}\left[1 - (\frac{b}{r})^{2c}\right] \quad , \quad \sigma_\theta - \sigma_r = (2-c)(b/r)^{2c}$$

Pressure required for initial yielding is given $p_i^* = \frac{(2-c)}{c}\left[\left(\frac{b}{a}\right)^{2c} - 1\right]$ (20)

And for fully plastic state, stresses and pressure is given as

$$\sigma_r = 4 \log \frac{r}{b}, \quad \sigma_\theta - \sigma_r = 2, \quad p_f^* = 4Y \log \frac{b}{a} \tag{21}$$

Calculation of pressure:

By using the above results, we can find the pressure required for initial yielding in shells for various materials having compressibility and incompressibility.

Table 1:

Pressure required for initial yielding with various compressibility factor

Compressibility	C=0	C=0.25	C=0.50	C=0.75
R_o	Pressure			

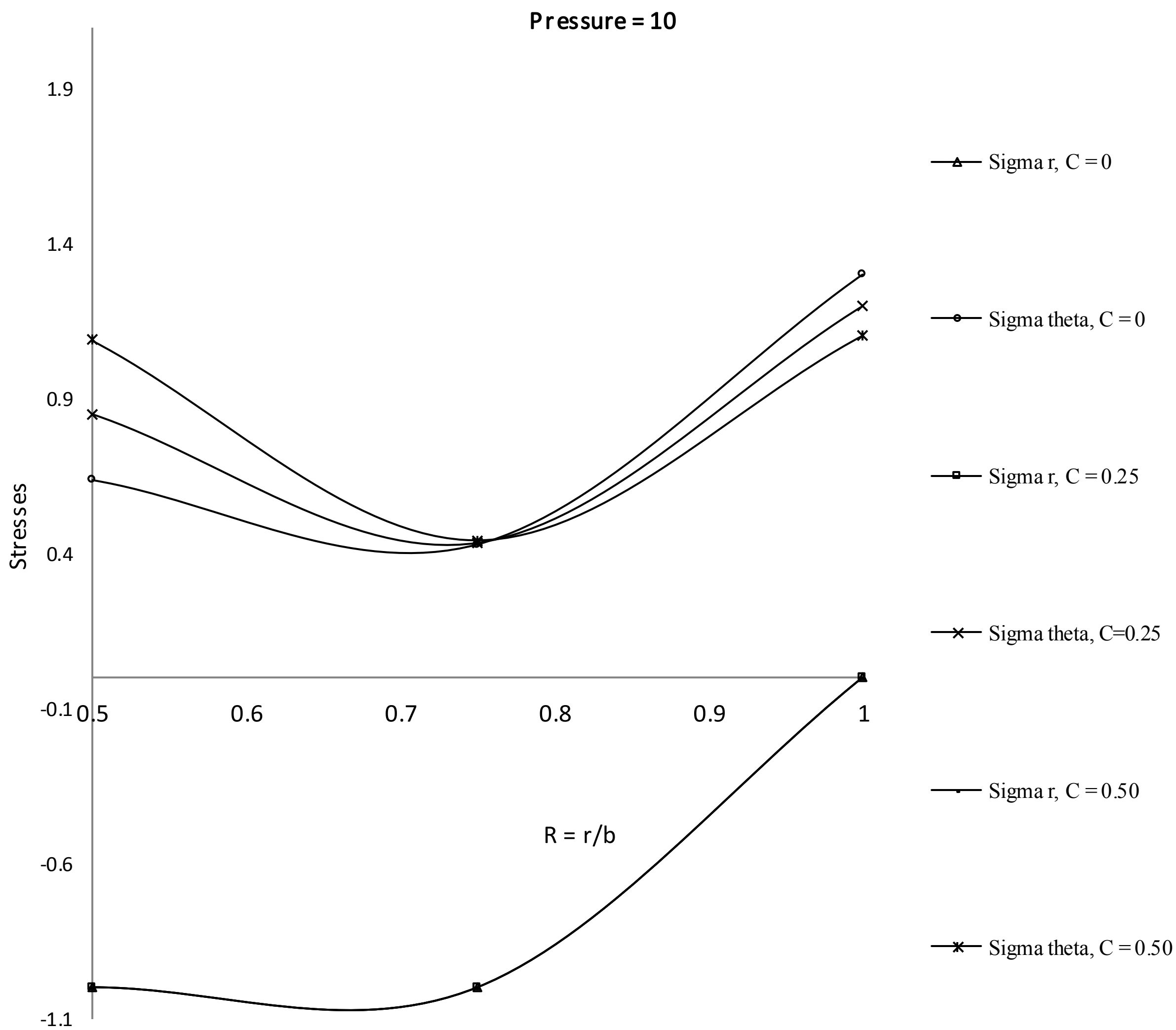
0.1	4.0000	15.1359	27.0000	51.14
0.2	2.7958	8.6524	12.0000	17.001
0.3	2.0897	5.7801	7.0000	8.478
0.4	1.5917	4.0679	4.5000	4.931
0.5	1.2041	2.8994	3.0000	3.0534

4. Results and Discussion

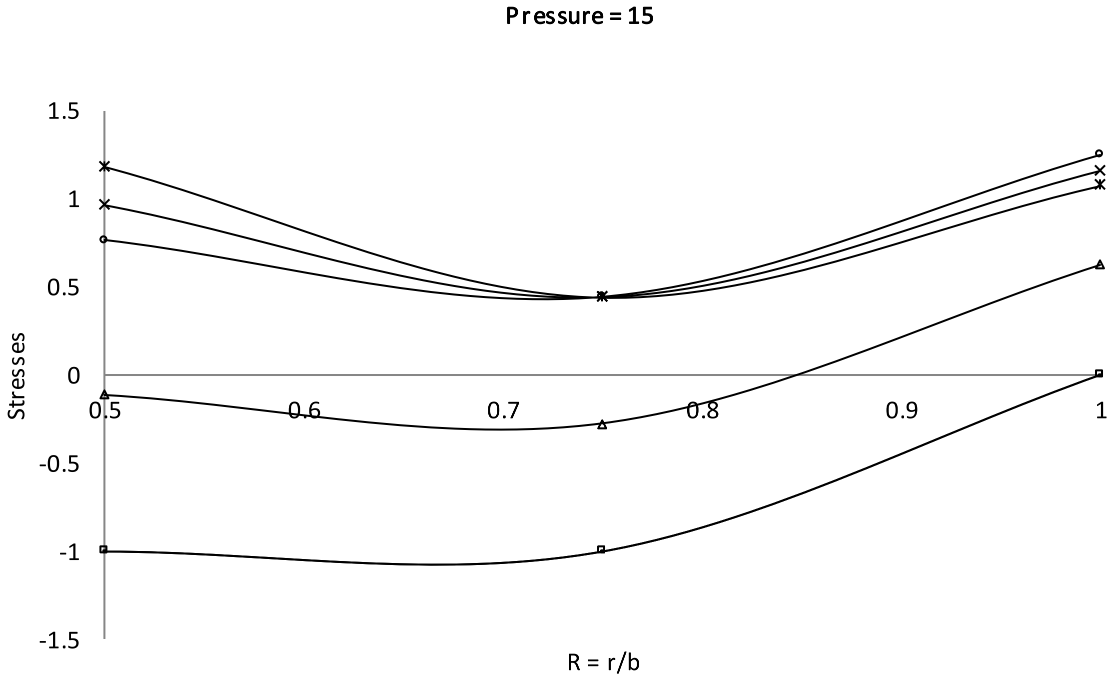
In Table 1, pressure is calculated along various radii ratios R_o = a/b for C=0, C=0.25 , C=0.5 , C=0.75. It has observed that pressure required for initial yielding at the internal surface is more than pressure at external surface. In figure 2, stresses are shown graphically along the radii ratio of spherical shell for compressible as well as in compressible materials. It is seen that circumferential stresses has maximum value at the internal surface as well as external surface of the spherical shell as compared to the radial stresses. It is also observed that the value of radial stresses lie between -1 to 0 due to boundary condition of the problem. The effect of pressure is seen on the stresses occurred in spherical shell made up of different materials. With increase in pressure, the stresses on the internal surface of the shell for compressible materials lead to damage of the spherical shell.

5. Conclusion

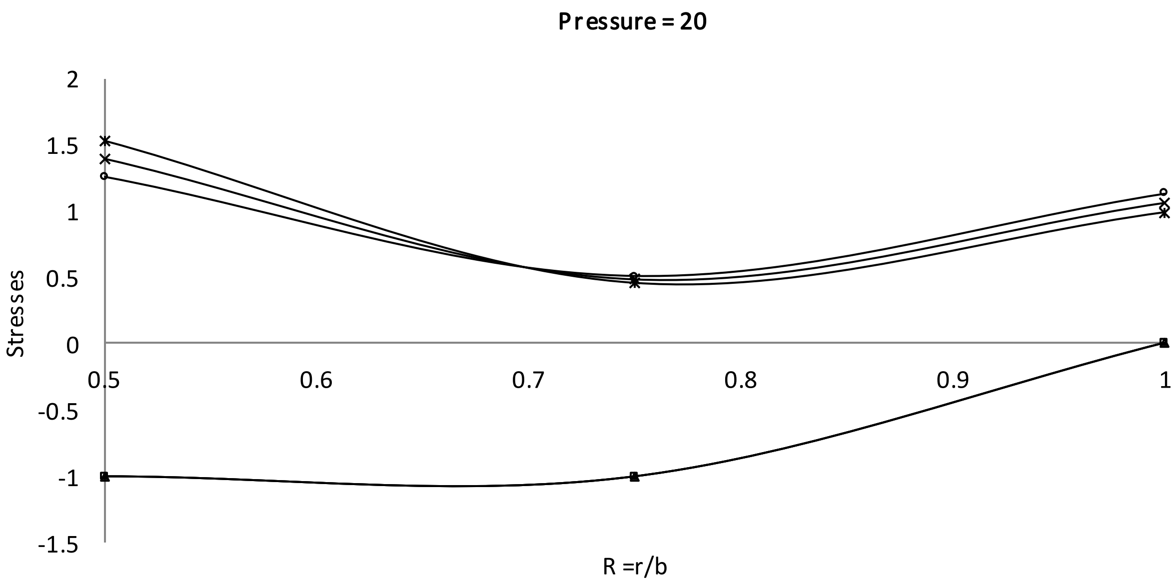
It can be concluded that spherical shell made up of incompressible material is on safer side of design as compared to spherical shell made up of compressible material under the effect of uniform internal pressure.


Pressure = 10
1.9
1.4
0.9
0.4
-0.1
-0.6
-1.1
Stresses
0.5
0.6
0.7
0.8
0.9
1
R = r/b
Sigma r, C = 0
Sigma theta, C = 0
Sigma r, C = 0.25
Sigma theta, C=0.25
Sigma r, C = 0.50
Sigma theta, C = 0.50

(a)



(b)



(c)

Figure 2. Stress distribution in spherical shell on dependence on compressibility and pressure

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