Discrete Mathematical Structures

Gaurav Verma[1], Harmandeep Singh[2]

[1]Department of Mathematics, Gobindgarh Public College, Alour, Punjab, India.

[2]Department of Math, Stat & Physics, Punjab Agricultural University, Ludhiana, Punjab, India.

Email id: gkdon85@gmail.com, harmansingh@pau.edu

Abstract

The present paper discusses the concept of mathematical discrete structures. The paper describes various types of discrete mathematical structures briefly. Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. Although the main objects of study in discrete mathematics are discrete objects, analytic methods from continuous mathematics are often employed as well.

Keywords: discrete, mathematics, structures.

1. INTRODUCTION

The historical backdrop of discrete science has included various testing issues which have centered consideration inside territories of the field. In graph theory, much research was inspired by endeavors to demonstrate the four color theorem, first expressed in 1852, yet not demonstrated until 1976 (by Kenneth Appel and Wolfgang Haken, utilizing considerable PC help). In logic, the second issue on David Hilbert's rundown of open issues displayed in 1900 was to demonstrate that the axioms of arithmetic are consistent. Gödel's second inadequacy hypothesis, demonstrated in 1931, demonstrated this was unrealistic – in any event not inside math itself. Hilbert's tenth issue was to decide if a given polynomial Diophantine condition with number coefficients has a whole number arrangement. In 1970, Yuri Matiyasevich demonstrated this wasn't possible. The need to break German codes in World War II prompted propels in cryptography and hypothetical software engineering, with the primary programmable advanced electronic PC being created at England's Bletchley Park with the direction of Alan Turing and his original work on Computable Numbers. In the meantime, military necessities roused propels in operations inquire about. The Cold War implied that cryptography stayed essential, with basic advances, for example, open key cryptography being created in the next decades. Operations research remained important as a tool in business and project management, with the critical path method being developed in the 1950s. The telecommunication industry has also motivated advances in discrete mathematics, particularly in graph theory and information theory. Formal verification of statements in logic has been necessary for software development of safety-critical systems, and advances in automated theorem proving have been driven by this need. Computational geometry has been an important part of the computer graphics incorporated into modern video games and computer-aided design tools. Several fields of discrete mathematics, particularly theoretical computer science, graph theory, and combinatorics, are important in addressing the challenging bioinformatics problems associated with understanding the tree of life.

Discrete mathematics is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics – such as integers, graphs, and statements in logic – do not vary smoothly in this way, but have distinct, separated values[1,2]. Discrete mathematics therefore excludes topics in "continuous mathematics" such as calculus and analysis. Discrete objects can often be enumerated by integers. More formally, discrete mathematics has been characterized as the branch of mathematics dealing with countable sets (sets that have the same cardinality as subsets of the natural numbers, including rational numbers but not real numbers). However, there is no exact definition of the term "discrete mathematics". Indeed, discrete mathematics is described less by what is included than by what is excluded: continuously varying quantities and related notions. The set of objects studied in discrete mathematics can be finite or infinite. The term finite mathematics is sometimes applied to parts of the field of discrete mathematics that deals with finite sets, particularly those areas relevant to business. Research in discrete mathematics increased in the latter half of the twentieth century partly due to the development of digital computers which operate in discrete steps and store data in discrete bits. Concepts and notations from discrete mathematics are useful in studying and describing objects and problems in branches of computer science, such as computer algorithms, programming languages, cryptography, automated theorem proving, and software development. Conversely, computer implementations are significant in applying ideas from discrete mathematics to real-world problems, such as in operations research. Although the main objects of study in discrete mathematics are discrete objects, analytic methods from continuous mathematics are often employed as well[3-5]. In university curricula, "Discrete Mathematics" appeared in the 1980s, initially as a computer science support course; its contents were somewhat haphazard at the time. The curriculum has thereafter developed in conjunction with efforts by ACM and MAA into a course that is basically intended to develop mathematical maturity in freshmen; therefore it is nowadays a prerequisite for mathematics majors in some universities as well. Some high-school-level discrete mathematics textbooks have appeared as well. At this level, discrete mathematics is sometimes seen as a preparatory course, not unlike precalculus in this respect.

2. TYPE OF VARIOUS MATHEMATICAL DISCRETE STRUCTURES

(A) Theoretical Computer Science: Hypothetical computer engineering incorporates regions of discrete arithmetic applicable to computing. It draws intensely on graph theory and mathematical logic. Included inside theoretical computer engineering is the investigation of calculations for scientific mathematical outcomes. Computability thinks about what can be processed on a basic level, and has close binds to logic, while complexity concentrates the time taken by calculations. Automata theory and formal theory are firmly identified with computability. Petri nets and process algebras are utilized to model computer frameworks, and strategies from discrete arithmetic are utilized as a part of examining VLSI electronic circuits. Computational geometry applies calculations to geometrical issues, while computer image investigation applies them to portrayals of pictures. Theoretical computer science also includes the study of various continuous computational topics.

(B) Information theory: Information theory involves the quantification of information. Closely related is coding theory which is used to design efficient and reliable data transmission and storage methods. Information theory also includes continuous topics such as: analog signals, analog coding, analog encryption. A key measure in information theory is "entropy". Entropy quantifies the amount of uncertainty involved in the value of a random variable or the outcome of a random process. For example, identifying the outcome of a fair coin flip (with two equally likely outcomes) provides less information (lower entropy) than specifying the outcome from a roll of a die (with six equally likely outcomes). Some other important measures in information theory are mutual information, channel capacity, error exponents, and relative entropy.

(C) Mathematical Logic: Mathematical logic is a subfield of mathematics exploring the applications of formal logic to mathematics. Mathematical logic is often divided into the fields of set theory, model theory, recursion theory, and proof theory. These areas share basic results on logic, particularly first-order logic, and definability. Since its inception, mathematical logic has both contributed to, and has been motivated by, the study of foundations of mathematics. This study began in the late 19th century with the development of axiomatic frameworks for geometry, arithmetic, and analysis. In the early 20th century it was shaped by David Hilbert's program to prove the consistency of foundational theories.

(D) Set theory: Set theory is the branch of mathematics that studies sets, which are collections of objects, such as {blue, white, red} or the (infinite) set of all prime numbers. Partially ordered sets and sets with other relations have applications in several areas. In discrete mathematics, countable sets (including finite sets) are the main focus. The beginning of set theory as a branch of mathematics is usually marked by Georg Cantor's work distinguishing between different kinds of infinite set, motivated by the study of trigonometric series, and further development of the theory of infinite sets is outside the scope of discrete mathematics. Indeed, contemporary work in descriptive set theory makes extensive use of traditional continuous mathematics.

(E) Combinatorics: Combinatorics studies the way in which discrete structures can be combined or arranged. Enumerative combinatorics concentrates on counting the number of certain combinatorial objects - e.g. the twelvefold way provides a unified framework for counting permutations, combinations and partitions. Analytic combinatorics concerns the enumeration (i.e., determining the number) of combinatorial structures using tools from complex analysis and probability theory. In contrast with enumerative combinatorics which uses explicit combinatorial formulae and generating functions to describe the results, analytic combinatorics aims at obtaining asymptotic formulae. Design theory is a study of combinatorial designs, which are collections of subsets with certain intersection properties. Partition theory studies various enumeration and asymptotic problems related to integer partitions, and is closely related to q-series, special functions and orthogonal polynomials. Originally a part of number theory and analysis, partition theory is now considered a part of combinatorics or an independent field. Order theory is the study of partially ordered sets, both finite and infinite[6].

(F) Graph theory: Graph theory, the study of graphs and networks, is often considered part of combinatorics, but has grown large enough and distinct enough, with its own kind of problems, to be regarded as a subject in its own right. Graphs are one of the prime objects of study in discrete mathematics. They are among the most ubiquitous models of both natural and human-made structures. They can model many types of relations and process dynamics in physical, biological and social systems. In computer science, they can represent networks of communication, data organization, computational devices, the flow of computation, etc. In mathematics, they are useful in geometry and certain parts of topology, e.g. knot theory. Algebraic graph theory has close links with group theory. There are also continuous graphs, however for the most part research in graph theory falls within the domain of discrete mathematics[7-9]. Graph theory has close links to group theory. This truncated tetrahedron graph is related to the alternating group $A4$.

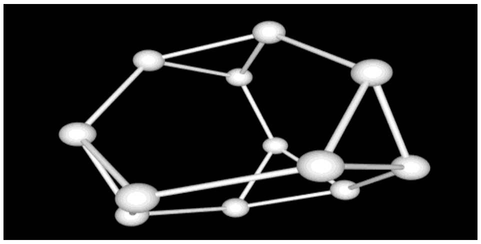

Fig 1: Graph of alternating group A_4

(G) Number theory: Number theory is a branch of pure mathematics devoted primarily to the study of the integers. It is sometimes called "The Queen of Mathematics" because of its foundational place in the discipline.[1] Number theorists study prime numbers as well as the properties of objects made out of integers (e.g., rational numbers) or defined as generalizations of the integers (e.g., algebraic integers).Number theory is concerned with the properties of numbers in general, particularly integers. It has applications to cryptography, cryptanalysis, and cryptology, particularly with regard to modular arithmetic, diophantine equations, linear and quadratic congruences, prime numbers and primality testing. Other discrete aspects of number theory include geometry of numbers. In analytic number theory, techniques from continuous mathematics are also used.

(H) Calculus of finite differences: A finite difference is a mathematical expression of the form $f(x + b) - f(x + a)$. If a finite difference is divided by $b - a$, one gets a difference quotient. The approximation of derivatives by finite differences plays a central role in finite difference methods for the numerical solution of differential equations, especially boundary value problems. A function defined on an interval of the integers is usually called a sequence. A sequence could be a finite sequence from a data source or an infinite sequence from a discrete dynamical system. Such a discrete function could be defined explicitly by a list (if its domain is finite), or by a formula for its general term, or it could be given implicitly by a recurrence relation or difference equation. Difference equations are similar to a differential equations, but replace differentiation by taking the difference between adjacent terms; they can be used to approximate differential equations or (more often) studied in their own right. Many questions and methods concerning differential equations have counterparts for difference equations. For instance, where there are integral transforms in harmonic analysis for studying continuous functions or analogue signals, there are discrete transforms for discrete functions or digital signals. As well as the discrete metric there are more general discrete or finite metric spaces and finite topological spaces.

(I) Discrete probability theory: Discrete probability theory deals with events that occur in countable sample spaces. For example, count observations such as the numbers of birds in flocks comprise only natural number values {0, 1, 2, ...}. On the other hand, continuous observations such as the weights of birds comprise real number values and would typically be modeled by a continuous probability distribution such as the normal. Discrete probability distributions can be used to approximate continuous ones and vice versa. For highly constrained situations such as throwing dice or experiments with decks of cards, calculating the probability of events is basically enumerative combinatorics. A discrete probability distribution is a probability distribution characterized by a probability mass function. Thus, the distribution of a random variable X is discrete, and X is called a discrete random variable.

(J) Algebra: Algebraic structures occur as both discrete examples and continuous examples. Discrete algebras include: boolean algebra used in logic gates and programming; relational algebra used in databases; discrete and finite versions of groups, rings and fields are important in algebraic coding theory; discrete semigroups and monoids appear in the theory of formal languages.

(K) Discrete geometry and Computational geometry: Discrete geometry and combinatorial geometry are about combinatorial properties of discrete collections of geometrical objects. A long-standing topic in discrete geometry is tiling of the plane. Computational geometry applies algorithms to geometrical problems. Discrete geometry has a large overlap with convex geometry and computational geometry, and is closely related to subjects such as finite geometry, combinatorial optimization, digital geometry, discrete differential geometry, geometric graph theory, toric geometry, and combinatorial topology[10].

3. APPLICATIONS OF DISCRETE STRUCTURES

The mathematics of modern computer science is built almost entirely on discrete math, in particular combinatorics and graph theory. This means that in order to learn the fundamental algorithms used by computer programmers, students will need a solid background in these subjects. Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science and data networking in this text, as well as applications to such diverse areas as chemistry, biology, linguistics, geography, business, and the Internet. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises.

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