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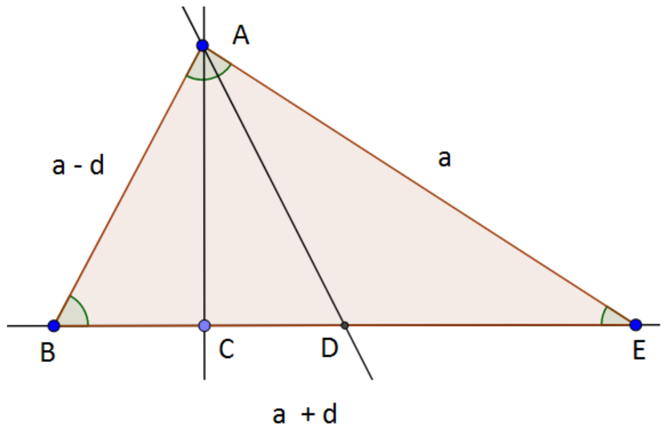
Piyush Theorem #2

Statement: *In an AP Series △ Triangle, if distance of median point to perpendicular point and sum of other two sides is divisible by 10, then triangle is a Right Angle Triangle.*



What we have: *Sides are in AP Series (a-d, a, a+d)*

Let's prove it:

Distance CD = (AB+AE)/10

10.CD = (a-d) +a

5.CD + 5.CD = 2a-d

5(BD-BC) + 5(CE-DE) = 2a-d

5BD – 5BC + 5CE – 5DE = 2a-d

5CE – 5BC = 2a-d—– (BD=DE)

5(CE-BC)= 2a-d -- eqn.1

Now,

CE^2 = AE^2 – AC^2

BC^2 = AB^2 – AC^2

CE^2- BC^2 = AE^2 – AB^2

(CE+BC)(CE-BC) = (AE +AB) (AE – AB)

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(a +d)(CE – BC) = (a +a –d) (a-a +d)

(a +d)(CE-BC) = (2a-d) d

CE – BC =(2a-d)d/((a +d))

put value from eqn.1

5(2a –d) d/ (a +d) = (2a-d)

5d/ (a +d) = 1

5d = a +d

5d – d = a

a = 4d

If it is a Right Angle Triangle,

(a +d)^2 = a^2 + (a-d) ^2

(a +d)^2 – (a-d) ^2 = a^2

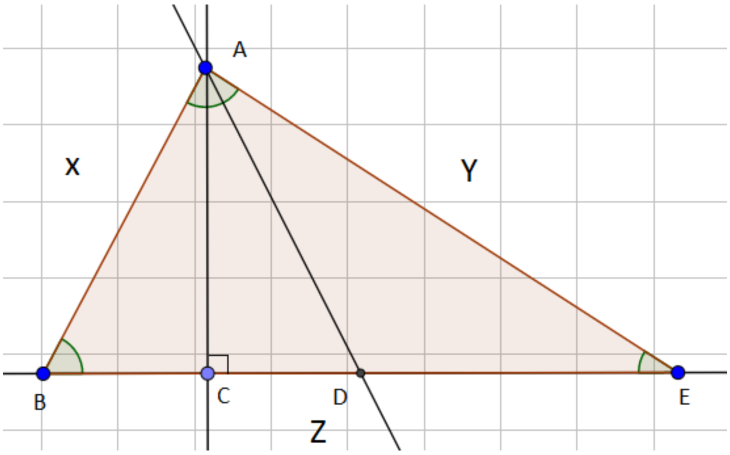
(a + d + a – d)(a + d –a +d) = a^2

(2a)(2d) = a^2

a = 4d QED.

Piyush Theorem #3

Statement: *In a Right-angled Triangle, if distance between middle point and perpendicular point and sum of other two sides is divisible by 10, then sides will be in A.P. Series.*



Let's prove it:

$$Z^2=X^2+Y^2$$

$$X^2= AC^2+ BC^2 \qquad X^2= AC^2+ (Z/2- X+Y/10)$$

$$Y^2= AC^2+ CE^2 \qquad Y^2= AC^2+ (Z/2+ X+Y/10)$$

$$X^2- Y^2=AC^2-AC^2+(Z/2-X+Y/10+Z/2- X+Y/10)(Z/2- X+Y/10-Z/2-X+Y/10)$$

$$(X+Y)(X - Y)= (Z)(- X+Y/5)$$

$$(X-Y)= (-Z/5)$$

$$Z = 5Y-5X \text{ ...Put this value into } Z^2= X^2+ Y^2$$

$$(5Y -5X)^2= X^2+ Y^2$$

$$25Y^2+ 25 X^2- 50XY = X^2+ y^2$$

$$24X^2- 50XY + 24Y^2= 0$$

$$24X^2- 32XY - 18XY + 24Y^2= 0$$

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$$8X(3X - 4Y) - 6Y(3X - 4Y) = 0$$

$$(3X - 4y)(8X - 6Y) = 0$$

$$2(3X - 4Y)(4X - 3Y) = 0$$

$$(3X - 4Y)(4X - 3Y) = 0$$

$3X - 4Y = 0 \quad 3X = 4Y \quad 3/4 = Y/X$ or $X = 4Y/3$ or $Y = 3X/4$

$4X - 3Y = 0 \quad 4X = 3Y \quad 4/3 = Y/X$ or $X = 3Y/4$ OR $Y = 4X/3$

Put value of Y in eqn. $5Y - 5X = Z$

$$5*4X/3 - 5X = Z$$

$$20X - 15X = 3Z$$

$$5X = 3Z$$

$$X/Z = 3/5$$

$$Y/X = 4/3$$

So $X = 3$; $Y = 4$; $Z = 5$ that is an A P Series.