

Disciplines of Particles

Paper on mathematical methods of particles

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Louis de Broglie wrote a paper in 1924 for his doctoral thesis in which he hypothesized the concept of wave nature of matter. His 1924 PhD thesis evolved the concept of de Broglie wavelength associated with particles like photons and electrons. His central idea was that, the matter particles are associated with dualistic nature wave and particle. Albert Einstein explained the mechanism of photoelectric effect by applying the Planck's quantum theory in which light behaves as a particle. Physicists faced a problem in early 20th century to explain the observable phenomena like interference, diffraction and polarization etc, then physicists concluded that matter and radiation behaves both as a particle and a wave, this simply explains that matter and radiation holds both characteristics a wave form and a particle form. This new concept and its mathematical disciplines are collectively named as quantum physics that studies the mathematical concept of motion of particles, energy and their interaction. In 1927 major advance came in quantum mechanics when Werner Karl Heisenberg published a paper in which he postulated his principle which today we know as a principle of uncertainty which describes the limitation of how particles behave in a subatomic level. The uncertainty principle can simply be stated as follows, it is impossible to determine the position and momentum (or velocity) of the particle (say electron) at the same instant or simultaneously. This simply means that if one measures the position of an electron with extremely high degree accuracy then it is impossible to determine the momentum or

velocity of an electron at that instant and the vice-vice. For his outstanding contribution in quantum physics Werner K Heisenberg was been awarded by Nobel Prize in physics in 1932. However Einstein raised questions about this newly postulated uncertainty principle. De Broglie's work led to invention of electron microscope. Broglie's 1924 PhD papers predictions and the correctness of its mathematical equations was confirmed experimentally in 1927 by Davisson and Germer working at Bell Telephone laboratory. Their electron diffraction experiment shows the wavelength of accelerated electron beam is almost equivalent to what estimated by Prince Louis de Broglie equation. De Broglie was awarded by Nobel Prize in physics in 1929, two years after the experimental confirmation of wave nature of electrons. Broglie's wave equation was hypothesized by assuming two criteria of photons.

1. Assuming photon as an electromagnetic radiation or wave of frequency f , the energy associated with it is given by Planck's equation.

$$E = hf$$

2. Secondly assuming photon as a particle of mass m then the energy associated with it is given by Einstein equation.

$$E = mc^2$$

Broglie's genius equivalent relation from both above equations,

$$hf = mc^2$$

From Planck's work we know $f = c/\lambda$, substituting we get

$$\frac{hc}{\lambda} = mc^2$$

From this relation Broglie's equation can be derived

$$\lambda = \frac{h}{mc}$$

This is the wave equation derived by de Broglie in 1924 is for calculating the wavelength of a photon. Broglie then realized that this equation must be true for fundamental particles like electrons, protons and neutrons. He then concluded by extending his own equations that a free particle of mass m moving with a velocity v must be associated with a wavelength (the wave nature) given by

$$\lambda = \frac{h}{mv}$$

This paper discusses the new method to calculate the wavelength of an electron, signifying the answer of the problem that was found during my careful analyzing and studying the wave nature of electrons (and other particles) and also studying the Davisson and Germer experiment in 1927. The problem so called proposition 1 is given here,

Proposition 1. How to calculate the wavelength of an electron (any particles) in terms of speed of light c and the mathematical constant π . In other words insert variable of c (speed of light) and π to calculate the wavelength of electrons.

The solution of this first proposition led me to develop the new mathematical concept which I call as 'latx particle scale' or L.P.S. this is the mathematical parameter found to derive the equation to calculate the wavelength of the electron as per mentioned in first proposition. The mathematical notation of latx scale is L_X . This scale is explained as follows, if a particle (say electron) of mass m moving with a

velocity v then the L.P.S. associated with it is given by

$$L_X = \int_0^h \frac{4e^3mv}{c^3\pi^3} de$$

In this expression e is the charge of the electron and h is the Planck's constant.

The latx scale equation is applied to derive the equation to calculate the wavelength of an electron.

So therefore the wavelength of the electron associated with the latx particle scale must be equivalent with de Broglie's wave and is given by

$$\lambda = \frac{h^5}{L_X c^3 \pi^3} = \frac{h}{mv}$$

So we concluded that the equation of the wavelength of the electron from latx particle scale is

$$\lambda = \frac{h^5}{L_X c^3 \pi^3}$$

This equation is the solution of proposition 1 it includes the variable c and π .

The study of the wave nature of electrons rise the second problem, this problem is termed as proposition 2. It is explained as follows

Proposition 2. Determine the method to calculate the wavelength of an electron in terms of electrons charge (e) and wavelength of an arbitrary photon λ' .

Like the previous solution (proposition 1) requires the development of new mathematical scale so called mathematical parameter, this solution also requires the development of a mathematical scale. This new physical

parameter which is named as ‘electron wave scale’ or e.w.s. the mathematical notation of e.w.s is Δm^x . The electron wave scale is explained as follows, if the particle (say electron) moving with a velocity v and has mass m , and the wavelength of an arbitrary photon is λ' then the electron wave scale is given by the mathematical expression

$$\Delta m^x = \frac{\lambda' c^2 e}{h^2 m v}$$

This mathematical equation called electron wave scale is applied to determine the wavelength of an electron, so therefore the wavelength of an electron is given by the mathematical expression of electron wave scale

$$\lambda = \frac{h^3 \Delta m^x}{\lambda' c^2 e}$$

This is the solution of second proposition; it satisfies the variable of an arbitrary photon and the charge of an electron.

The above mentioned problems is to calculate the wavelength of the electron so from first and second problem we deduced the expressions as follows

$$\lambda = \frac{h^5}{L_x c^3 \pi^3}$$

$$\lambda = \frac{h^3 \Delta m^x}{\lambda' c^2 e}$$

Therefore since these are the wavelengths of electron based on their new mathematical scales so the mathematical calculations showed that both equations obey the de Broglie’s wavelength. In others words these newly deduced equations is indeed experimentally true.

So we finally concluded from both scales that

$$\lambda = \frac{h^5}{L_x c^3 \pi^3} = \frac{h^3 \Delta m^x}{\lambda' c^2 e}$$

This paper also shows the alternate results to calculate the Bohr’s velocity of an electron, angular momentum of an electron and energy of a photon.

The mathematical scale named as Electron Nuclear Scale is introduced, and upon numeral calculations this mathematical parameters derives equations to calculate the energy of a photon, angular momentum of an electron and Bohr’s velocity of a moving electron. This new mathematical parameter is defined as follows

An arbitrary photon of wavelength λ , then the electron nuclear scale or E.N.S is given by

$$N_x = \int_0^\lambda \frac{2KZ\pi}{h^3} dK$$

Where K is the constant of value 9×10^9 and Z is the atomic number of the element (Both variables appeared in the equations of Neil’s Bohr).

This new scale is applied to calculate the velocity of the electron which obeys the Bohr’s velocity of electron equation in terms of mathematical values. The equation is

$$v = \frac{2N_x h^2 K e^2}{\lambda^2 n}$$

Where λ is the wavelength of the assumed photon (assumed in this method)

The electron nuclear scale is also applied to calculate the energy of photon, if the photon

of wavelength λ is given then as per electron nuclear scale the energy of photon is

$$E = \sqrt{\frac{Z\pi c^2}{N_x h}}$$

Where Z is atomic number of the assumed element, which is assumed for this method

This electron nuclear scale is also explained to calculate the angular momentum of an electron on the basis of Bohr's $\frac{nh}{2\pi}$ relation, the equation to calculate the angular momentum of an electron is,

$$L = \sqrt[3]{\frac{\lambda^2 Z n^3}{8\pi^2 N_x}}$$

Where λ and Z are the wavelength of an assumed photon and atomic number of an assumed element (assumed for this method)

Method 3

Let the particle P be defined by the coordinates P(x, y) the idea is to calculate the wavelength of the particle, let us introduce the mathematical parameter called **particle wavelength parameter** and is given as follows,

If an electron be described by its coordinates (x, y) and has a mass m moving with a velocity v then the particle wavelength parameter is given by

$$V_x = \frac{xy\lambda'}{m^2 v^2} \int_0^h \frac{e^2}{\pi^3} de$$

Where λ' is the wavelength of an arbitrary photon or assumed photon (assumed in this method).

This particle wavelength parameter is applied to derive the equation to calculate the wavelength of an electron, so the wavelength of an electron is given by

$$\lambda = \sqrt{\frac{3\pi^3 V_x}{xy\lambda' h}}$$

Let the particle be defined by its three coordinates P(x, y, l), then I introduce the new mathematical parameter which is called as **particle coordinate scale** and it is explained as follows, if a particle of given coordinates moving with a velocity v and posses a mass m then by assuming the wavelength of assumed photon λ' the particle coordinate scale is given by

$$P_{px} = \frac{h^2 xy}{m^3 v^3} \int_0^e \frac{\lambda' \pi^3}{h^3 l} d\pi$$

Therefore the wavelength of an electron is given by

$$\lambda = \sqrt[3]{\frac{4h^4 l P_{px}}{xye^4 \lambda'}}$$

Conclusion: this paper introduces some new mathematical methods to calculate the wavelength of electrons, energy of a photon and velocity of an electron.