

A Novel for PAPR Reduction in Band Limited OFDM Using Peak Cancellation

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Abstract

In the era of wireless communication, many research works have been done to reduce the peak average power ratio (PAPR). But due to some undefined reasons, many researchers have been failed to reach the perfect destination. One of the main reasons is that for different signals the PAPR will work differently. Thousands of techniques have been proposed to reduce PAPR one of the technique is proposed to reduce PAPR in this paper is Peak cancellation (PC). Peak cancellation is selected due to ease of hardware implementation. But the main drawback in peak cancellation is it gives rise to in-band distortion and out-of-band radiation. To restrict the distortions made by the signal to an acceptance level, it is little bit complicated to design the cancelling pulses as well as envelope threshold. This paper is used to focus on theoretical analysis of signal and its validity and limitations. Based on the level-crossing rate approximation of the peak distribution, we derive a closed-form expression for the achievable signal-to-distortion power ratio (SDR). We also analyze the adjacent channel leakage ratio (ACLR) as well as error vector magnitude (EVM), with which the symbol error rate (SER) over an additive white Gaussian noise (AWGN) channel is obtained. All the theoretical results developed in this work are compared with those based on the corresponding computer simulations to justify our analytical approach. Index Terms—Adjacent channel leakage ratio (ACLR), error vector magnitude (EVM), orthogonal frequency division multiplexing (OFDM), peak cancellation (PC), peak-to-average power ratio (PAPR), signal-to-distortion power ratio (SDR), symbol error rate (SER).

1. Introduction

Orthogonal frequency division multiplexing (OFDM) [1] is a robust transmission technique to combat the influence of wireless fading channels. However,

OFDM suffers from high peak-to-average power ratio (PAPR) that significantly reduces the efficiency of the high power amplifier (HPA). To alleviate this, many approaches have been proposed to reduce the PAPR [2,3], among which the pre-distortion approach such as clipping [4], is an efficient one.

Clipping cancels the signal peak by adding a scaled pulse function, at the cost of out-of-band radiation due to the infinite frequency response of the pulse function. Therefore, filtering should be combined at the cost of peak re-growth [5]. To control the PAPR and out-of-band radiation simultaneously, repeated clipping and filtering (RCF) is proposed in [6] with a high complexity.

To simultaneously make a better tradeoff among PAPR, out-of-band radiation and computational complexity, peak cancellation [7] is proposed as a candidate of clipping-based techniques. It introduces designed windowing function, with finite response in both time and frequency domains, to replace the original pulse function in clipping. In this case, peak cancellation will not cause severe out-of-band radiation, thus saving the complexity for RCF [8,9].

Peak cancellation will introduce additional distortion to the transmitted signals, thus degrading the system performance. For clipping-based techniques, the influence on the system performance is analyzed in [5]. However, for peak-cancellation combined OFDM system, an overall estimation of the nonlinear distortion is still missing. In this paper, the interference of peak cancellation is modeled and analyzed, aiming to give an evaluation of the system performance. Through theoretical analysis, a closed form of the BER performance is derived, with popular modulation schemes such as QPSK and 16QAM taken into consideration. Finally, the analyzed results are verified by numerical simulations.

2. Literature Survey

[8] **Alain Y. Kibangou and Gérard Favier:** In this letter, we first present explicit relations between block-oriented nonlinear representations and Volterra models. For an identification purpose, we show that the estimation of the diagonal coefficients of the Volterra kernels associated with the considered block-oriented nonlinear structures is sufficient to recover the overall model. An alternating least squares-type algorithm is provided to carry out this model identification.

[9] **S. O. Rice:** THIS paper deals with the mathematical analysis of noise obtained by passing random noise through physical devices. The random noise considered is that which arises from shot effect in vacuum tubes or from thermal agitation of electrons in resistors. Our main interest is in the statistical properties of such noise and we leave to one side many physical results of which Nyquist's law may be given as an example.

[10] **Ahmad R. S. Bahai, Andrea J. Goldsmith, Burton R. Saltzberg:** Multicarrier signals are known to suffer from a high peak-to-average power ratio, caused by the addition of a large number of independently modulated subcarriers in parallel at the transmitter. When subjected to a peak-limiting channel, such as a nonlinear power amplifier, these signals may undergo significant spectral distortion, leading to both in-band and out-of-band interference, and an associated degradation in system performance. This paper characterizes the distortion caused by the clipping of multicarrier signals in a peak-limiting (nonlinear) channel. Rather than modeling the effects of distortion as additive noise, as is widespread in the literature, we identify clipping as a rare event and focus on evaluating system performance based on the conditional probability of bit error given the occurrence of such an event. Our analysis is based on the asymptotic properties of the large excursions of a stationary Gaussian process, and offers important insights into both the true nature of clipping distortion, as well as the consequent design of schemes to alleviate this problem.

[11] **P. Banelli and S. Cacopardi:** L Orthogonal frequency-division multiplexing (OFDM) baseband signals may be modeled by complex Gaussian processes with Rayleigh envelope distribution and uniform phase distribution, if the number of carriers is sufficiently large. The output correlation function

of instantaneous nonlinear amplifiers and the signal-to-distortion ratio can be derived and expressed in an easy way. As a consequence, the output spectrum and the bit-error rate (BER) performance of OFDM systems in nonlinear additive white Gaussian noise channels are predictable both for uncompensated amplitude modulation/amplitude modulation (AM/AM) and amplitude modulation/pulse modulation (AM/PM) distortions and for ideal pre-distortion. The aim of this work is to obtain the analytical expressions for the output correlation function of a nonlinear device and for the BER performance. The results in closed-form solutions are derived for AM/AM and AM/PM curves approximated by Bessel series expansion and for the ideal pre-distortion case.

3. Proposed Method

3.1 Peak cancellation:

The principle of PC is to generate cancelling pulses at the time instances where the peaks higher than the predetermined threshold γ are found and to subtract them from the original signal. An example block diagram suitable for practical implementation [5] is depicted in Fig. 1. In what follows, we denote the polar expression of the OFDM signal by $s(t) = r(t)e^{j\theta(t)}$, where $r(t)$ and $\theta(t)$ represent the envelope and phase of the original signal $s(t)$, respectively. Suppose that there are $N_p(\gamma)$ peaks that are higher than the threshold γ in the envelope process $r(t)$ during one OFDM symbol interval, and let t_i denote the time instant at which the i th peak is observed, where $i \in \{1, 2, \dots, N_p(\gamma)\}$. The signal after peak cancellation can then be written as

$$s_c(t) = s(t) - \sum_{i=1}^{N_p(\gamma)} p_i(t - t_i) \quad (1)$$

Where $p_i(t)$ denotes the cancelling pulse corresponding to the i th peak with an appropriate time shift such that $p_i(t)$ has a peak at $t = 0$, and $p(t)$ is the sum of all the cancelling pulses generated within one OFDM symbol. In this work, we express the i th cancelling pulse in the following form [5]

$$p_i(t) = (\rho_i - \gamma)e^{j\theta_i}g(t) \quad (2)$$

Where $p_i = r(t_i)$, $\theta_i = \theta(t_i)$, and $g(t)$ is a dedicated impulse response function referred to as a cancelling pulse kernel in what follows.

3.2 Signal-To-Distortion Power Ratio Analysis of Peak Cancellation for OFDM Signals:

Assuming that the OFDM signal is approximated by a complex Gaussian random process, the standard approach for statistical characterization of the signal after nonlinear processing is the use of Busgang's theorem [10]. We model the OFDM signal after PC as a linear transform of the input signal and additive distortion given by

$$s_c(t) = \alpha_\gamma s(t) + d(t) \quad (3)$$

$$\alpha_\gamma = \frac{E\{s^*(t)s_c(t)\}}{E\{s^*(t)s(t)\}} = E\{s^*(t)s_c(t)\} \quad (4)$$

due to the fact that $E\{|s(t)|^2\} = 1$. Note that when $s_c(t)$ is modeled as a non-stationary process, which is the case for the OFDM signal with PC, the time average (over the period of T_s) should be applied to (9) as it is still a function of t [21]. The SDR can be then defined as [22], [23]

$$SDR = \frac{E\{|\alpha_\gamma s(t)|^2\}}{E\{|d(t)|^2\}} = \frac{|\alpha_\gamma|^2}{P_{av,d}} \quad (5)$$

3.3 Level-Crossing Rate Approximation of Peak Distribution

The generation of cancelling pulses depends on the event that the peaks higher than a given threshold occurs. Therefore, the knowledge of the peak distribution of OFDM signals is necessary. However, as discussed in [2], even if we assume the baseband OFDM signal as a band-limited complex Gaussian process, the exact form of peak distribution is complicated and may not be expressed in a closed form. Nevertheless, the level crossing rate of a Gaussian process can be represented in a closed form following the work of Rice [25]. For a strictly band limited OFDM signal, the average number of positive crossings of a given level r per OFDM symbol is expressed as

$$v_c^+(r) = \sqrt{\frac{\pi}{3}} \frac{N}{T_s} r e^{-r^2} \quad (6)$$

Furthermore, since the number of the level-crossings and that of the peaks tend to agree as the threshold r increases, the conditional probability that a peak ρ exceeds r given that it exceeds a reference level γ is expressed by

$$\begin{aligned} Pr\{\rho > r | \rho > \gamma\} &\approx \frac{v_c^+(r)}{v_c^+(\gamma)} = \frac{r e^{-r^2}}{\gamma e^{-\gamma^2}}, e > \gamma \\ &> \frac{1}{\sqrt{2}} \quad (7) \end{aligned}$$

The conditional pdf of the peaks that exceed γ is then approximated as

$$\begin{aligned} f_\rho(r|\gamma) &= \frac{d}{dr} Pr\{\rho > r | \rho > \gamma\} \\ &\approx \frac{(2r^2 - 1)}{\gamma} e^{\gamma^2 - r^2}, r > \gamma \\ &> \frac{1}{\sqrt{2}} \quad (18) \end{aligned}$$

Recall that for a given threshold γ , denotes the number of the peaks above γ during one OFDM symbol period. Conditioned that we observe $N_\rho(\gamma)$ peaks and by referring to (5), the cross-correlation term α_γ in can be calculated as

$$\begin{aligned} \alpha_\gamma &= E\{s^*(t)s_c(t) | N_\rho(\gamma)\} \\ &= E\{s^*(t)s(t) - p(t) | N_\rho(\gamma)\} \\ &= \frac{1}{N_\rho(\gamma)} \\ &\quad - \sum_{i=1}^{N_\rho(\gamma)} E\{s^*(t)p_i(t) - t_i | \rho_i > \gamma\} \quad (9) \end{aligned}$$

We may express (9) as

$$\alpha_\gamma = 1 - \sum_{i=1}^{N_\rho(\gamma)} D(\tau; \gamma) |g(\tau)|^2 \quad (10)$$

This is because we are evaluating the distortion conditioned that the peak exceeds a certain threshold γ , and this amount cannot be zero. Since the probability of having such a high peak itself approaches zero, this behavior can be justified.

Closed-Form Expression of Attenuation Factor
Considering the asymptote of $N \rightarrow \infty$ and taking the time average, together with resorting to the strong law of large numbers, (10) can be simplified as

3.4 Closed-Form Expression of Attenuation Factor:

Considering the asymptote of $N \rightarrow \infty$ and taking the time average, together with resorting to the strong law of large numbers, (10) can be simplified as

$$\alpha_\gamma \approx 1 - E\{N_\rho(\gamma)\}C(\gamma)\beta \quad (11)$$

and it follows that $Si(2\pi) = 1.41815 \dots$. As a result, an asymptote of α_γ in the case of $N \rightarrow \infty$ can be expressed in the following closed-form expression:

$$\alpha_\gamma \approx 1 - \frac{Si(2\pi)}{\sqrt{3\pi}} \left\{ \gamma e^{-\gamma^2} + \sqrt{\pi} \operatorname{erfc}(\gamma) \right\} \quad (12)$$

3.5 Distortion Power Analysis based on Level-Crossing Rate Approximation

The average power of the distortion term $P_{av,d}$ caused by PC can be written as

$$P_{av,d} = E\{|s_c(t)|^2 | N_\rho(\gamma)\} - |\alpha_\gamma|^2 \quad (13)$$

Collecting the above results and noticing that α_γ has a real value, we obtain

$$P_{av,d} = \left(2\alpha_\gamma - 1 - |\alpha_\gamma|^2 \right) + E\{N_\rho(\gamma)\}B(\gamma)\eta \quad (14)$$

And substituting α_γ of (12) into (14) leads to the following closed-form expression:

$$P_{av,d} = \frac{\operatorname{erfc}(\gamma) \left(Si(2n\pi) + \frac{(\cos(2n\pi) - 1)}{2n\pi} \right)}{3\pi} - \frac{e^{-2\gamma^2} \left(\gamma + e^{\gamma^2} \sqrt{\pi} \operatorname{erfc}(\gamma) \right)^2 Si(2\pi)^2}{3\pi} \quad (15)$$

Which indicates that the distortion term is dependent on the length of the window function n .

Finally, substituting (12) and (15) in the (5) to form a closed-form expression of SDR as a function of n and γ , i.e.,

$$SDR(n, \gamma) = \frac{(\sqrt{3\pi} - Si(2\pi) \{ \gamma e^{-\gamma^2} + \sqrt{\pi} \operatorname{erfc}(\gamma) \})^2}{\sqrt{3\pi} \operatorname{erfc}(\gamma) Si(2n\pi) - e^{-2\gamma^2} \left(\gamma + e^{\gamma^2} \sqrt{\pi} \operatorname{erfc}(\gamma) \right)^2 Si(2\pi)^2} \quad (16)$$

For $n=1, 2, \dots$

3.6 Steps to evaluate the performance of PAPR

1. Let us assume a random signal $s(t)$ with specific bits

2. Based on some threshold limit signal-to-distortion power ratio (SDR) is evaluated to describe the performance of the signal.

3. SDR should be as high as possible. Because if distortion of the signal is low then the SDR will be high.

4. The adjacent channel leakage ratio (ACLR) as well as error vector magnitude (EVM) is evaluated with which the symbol error rate (SER) over an additive white Gaussian noise (AWGN) channel is obtained.

5. Cancelling pulse generator (CPG) is used to detect the peak signal beyond the threshold limit so as to reduce PAPR as well as improve the performance of the signal.

6. The detected peak signal is named as $p(t)$.

7. Now the detected peak signal $p(t)$ is subtracted from the original signal $s(t)$ to obtain a reduced peak signal with improved system performance.

3.7 IN-BAND AND OUT-OF-BAND DISTORTION ANALYSIS

In this section, we theoretically analyze the effect of the cancelling pulse on the ACLR. This also used to derive an EVM expression that takes place only in the in-band distortion.

A. Length of Cancelling Pulse Kernel

power spectrum density (PSD) can be calculated as

$$S_g(f) = \frac{T}{2N\pi^3} \left[Si(n\pi(1 - 2Tf)) + Si(n\pi(1 + 2Tf)) \right]^2 \quad (17)$$

Hence, the corresponding ACLR of $g(t)$ can be expressed as

$$\zeta_g(n) = \frac{\int_{D_{out}} S_g(f) df}{\int_{D_{in}} S_g(f) df} \quad (18)$$

B. ACLR Analysis

ACLR is usually evaluated by the average (long term) PSD of the peak cancellation. The

autocorrelation is evaluated for every sample and is defined as

$$R_p(\tau) = \sum_{i=1}^{N_p(\gamma)} R_{pi}(\tau) \quad (19)$$

C. EVM Analysis

In OFDM, EVM is used to measure the in-band distortion of a signal with respect to different effective SDR value as

$$EVM = \sqrt{\frac{1}{SDR_{eff}}} \times 100\% \quad (20)$$

the EVM of the PC output can be written as

$$EVM = \frac{1}{\alpha_\gamma} \sqrt{\int_{D_{in}} P_p(f) df - (1 - \alpha_\gamma)^2} \times 100\% \quad (21)$$

Based on the fact that $\int_{D_{in}} P_s(f) df = 1$

D. SER Over an AWGN Channel

The average SER $P_{e,total}$ is expressed as

$$P_{e,total} = T_c(\gamma) \cdot P_e(SNDR) + (1 - T_c(\gamma)) \cdot P_e(SNR) \quad (22)$$

The effective (in-band) SNDR which assimilate both channel noise and distortion to analyse the performance. It can be written as

$$SNDR = \frac{\alpha_\gamma^2 E\{|s(t)|^2\}}{P_{av,n} + \int_{D_{in}} P_d(f) df} \quad (23)$$

In the same way SNR is also defined as

$$SNR = \frac{E\{|s(t)|^2\}}{P_{av,n}} \quad (24)$$

4. Results

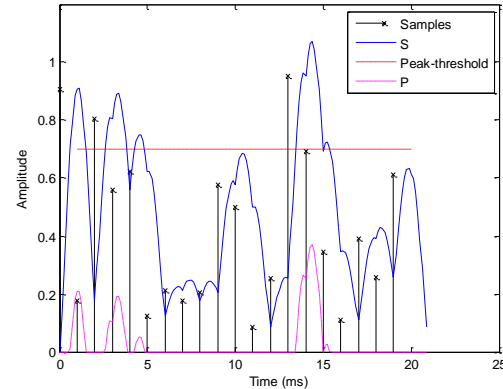


Fig.1. Waveforms associated with Peak Cancellation

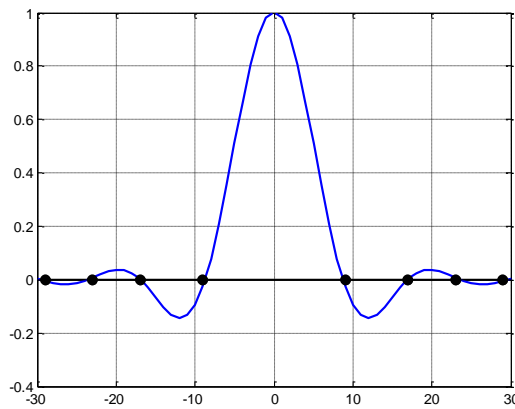


Fig.2. cancelling pulse kernel with the oversampling rate J=4. Here black dots indicates when n is taken as integer value.

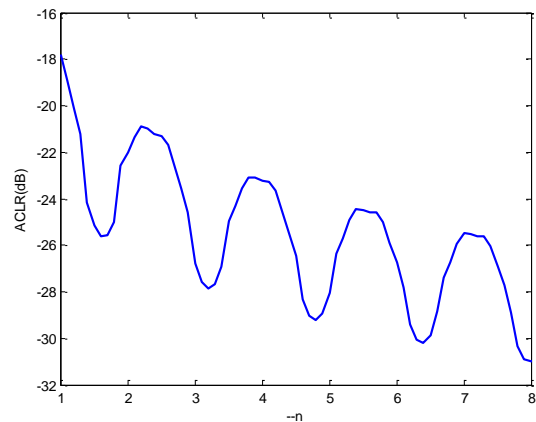


Fig.3.the comparison of both ACLR and the length n of a cancelling pulse kernel g(t) is evaluated with a sinc function.

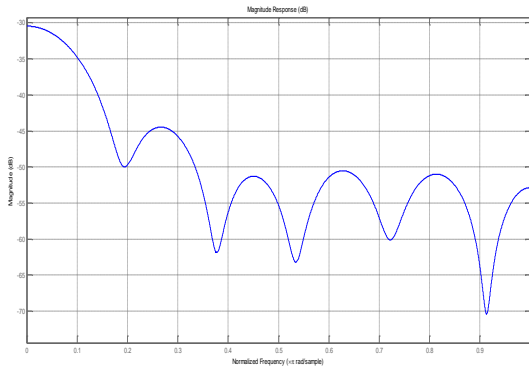


Fig.4. Averaged PSD of the cancelling pulses with $n = 5$, $\gamma = 4\text{dB}$, and $J = 8$, The bandwidth is normalized by the inverse of the Nyquist interval, i.e., $1/T$, which is equivalent to the bandwidth of the in-band OFDM signal.

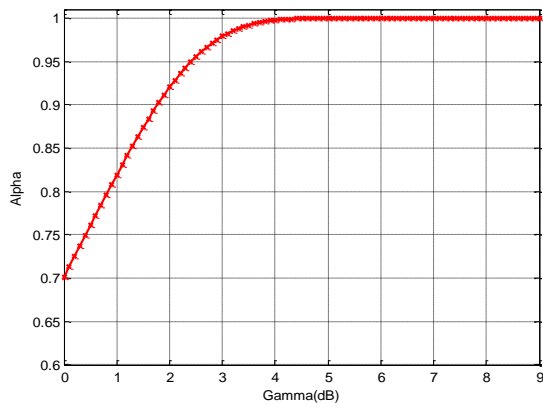


Fig.5. Performance analysis of attenuation factor with multiple PAPR values

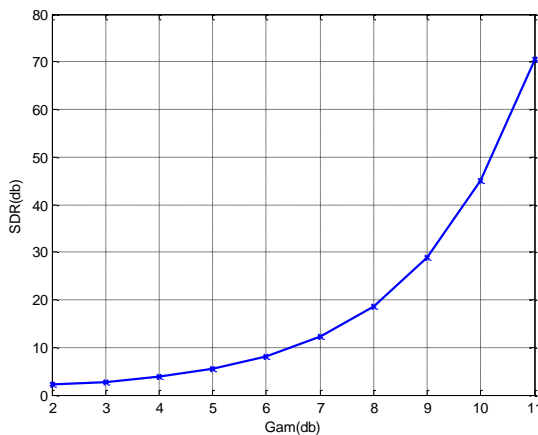


Fig.6. Performance analysis of SDRs with different target PAPR γ .

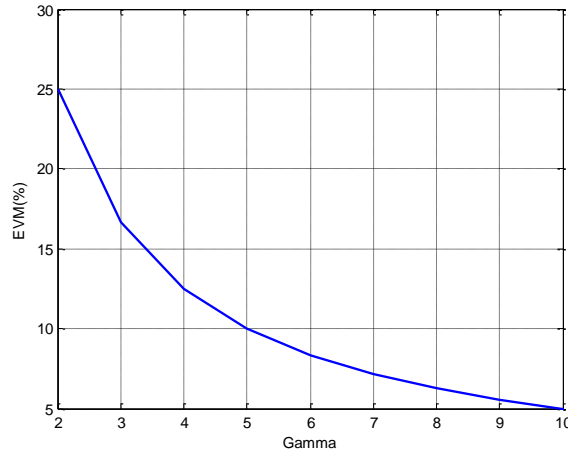


Fig.7. Performance analysis of Simulated EVM with respect to different target PAPR γ

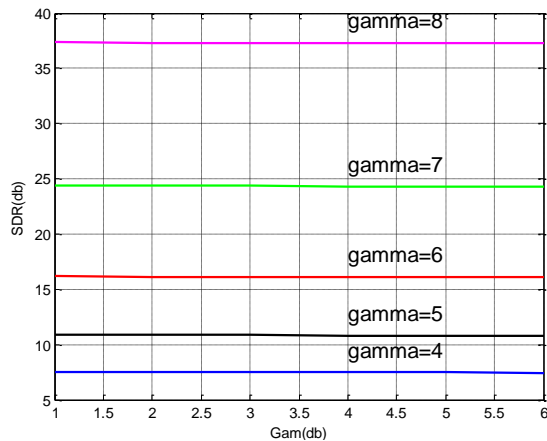


Fig.8. performance of SDR with respect to the length of cancelling pulse n under different PAPR γ .

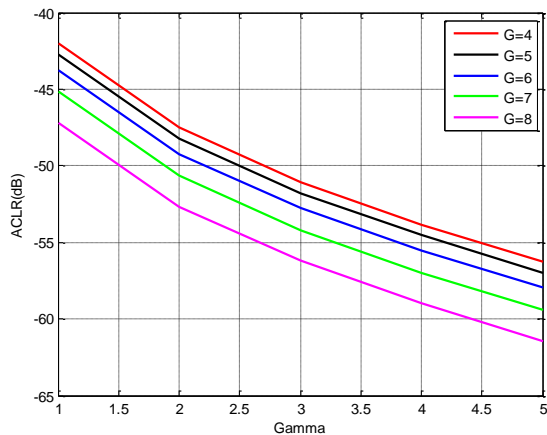


Fig.9. Graphical analysis of ACLRs with respect to multiple pulse length n under multiple target PAPR γ .

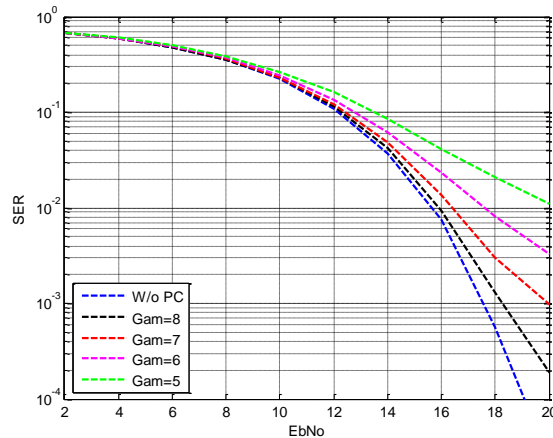


Fig.10. SER over an AWGN channel with respect to different target PAPR γ of 16-QAM OFDM signals in the presence of PC.

4. Conclusion

In this paper, theoretical evaluation of both in-band and out-of-band distortion usual overall performance of top cancellation is performed for OFDM systems. The closed-form expression of SDR due to PC is first acquired, with which the impact of PC in terms of EVM and ACLR related to the height decreased sign is analyzed from the frequency area mindset. In addition, the following SER degradation because of PC while transmitted over an AWGN channel is mathematically formulated. The effectiveness of the analytical approach given on this paper is properly confirmed with the useful resource of simulation effects. The analytical gear evolved on this art work serve as a beneficial guideline for designing cancelling pulses of PC-based totally OFDM systems. They may be considerably applied for estimating its effectiveness in addition to boundaries in view of device requirements.

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