

An Approach to implement viscous dissipative Couette flow between vertical permeable plates with Thermal radiation effect on Magneto Hydro Dynamic

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Abstract:

The objective of this paper to find the numerical solution of unsteady magneto hydro dynamic free convective Couette flow of viscous incompressible fluid confined between two vertical permeable parallel plates in the presence of thermal radiation is performed. A uniform magnetic field which acts in a direction orthogonal to the permeable plates, and uniform suction and injection through the plates are applied. The magnetic field lines are assumed to be fixed relative to the moving plate. The momentum equation considers buoyancy forces while the energy equation incorporates the effects of thermal radiation. The fluid is considered to be a gray absorbing – emitting but non – scattering medium in the optically thick limit. The Roseland and approximation is used to describe the radioactive heat flux in the energy equation. The two plates are kept at two constant but different temperatures and the viscous and Joule dissipations are considered in the energy equation. The non – linear coupled pair of partial differential equations are solved by an efficient Crank Nicholson method. With the help of graphs, the effects of the various important flow parameters entering into the problem on the velocity, temperature and concentration fields within the boundary layer are discussed. Also the effects of these flow parameters on skin friction coefficient and rates of heat and mass transfer in terms of the Nusselt and Sherwood numbers are presented numerically in tabular form.

Keywords: Free convection, MHD, Viscous dissipative fluid, Thermal radiation, Coquette flow, Finite difference method.

Introduction:

The study of free convection flow is useful for energy – related engineering applications such as geophysics and problems involving the spread of pollution. Soriano *et al.* [1] studied free convection

flow along an isothermal vertical finite plate using perturbation analysis, and the velocity and temperature fields were obtained for small Gash of number. This problem was extended by Soriano and Wang – Tzu [2] to include moderate Gash of numbers by using a numerical finite difference scheme. The effect of natural convection on unsteady Coquette flow was studied by Singh [3]. The Laplace transform technique was used to obtain the velocity and temperature fields, the skin friction and rate of heat transfer. It was observed that an increase in the Gash of number results in an increase in the flow velocity. Jha [4] extended the work of Singh [3] by discussing the combined effects of natural convection and a uniform transverse magnetic field when the magnetic field is fixed relative to the plate or fluid. Using the Laplace transform technique, exact solutions were obtained for the velocity and temperature fields.

The trends observed with respect to the magnetic field strength were consistent with those observed in [5]. Singh *et al.* [6] compared the unsteady free convection Coquette flow at large values of time with the corresponding steady-state problem and found that they are in good agreement. It was also observed that the flow velocity decreases with increasing Prandtl number. Jha and Apere [7] extended the work of Jha [4] by considering the unsteady MHD free convection Coquette flow between two vertical parallel porous plates with uniform suction and injection. The cases where the magnetic field is considered fixed relative to the fluid and fixed relative to the moving plate were considered. The velocity and temperature distributions were obtained using the Laplace transform technique. The results revealed that both temperature and velocity decrease with increasing Prandtl number and with increasing suction/injection parameter. The effect of magnetic field strength on the velocity is consistent with the results obtained in

[3] and [5]. The velocity has also been found to increase with increasing Gash of number.

Couette flows find widespread applications in geophysics, planetary sciences and also many areas of industrial engineering. For many decades engineers have studied such flows with and without rotation and also for both the steady case and unsteady case. Both Newtonian and non – Newtonian flows with for example magnetic field effects and heat transfer have also been examined. Such studies have entailed many configurations including the flow between rotating plates, rotating concentric cylinders, etc. In rotating Couette flows a viscous layer at the boundary is instantaneously set into motion. An early study of unsteady Couette flow was reported by Vidyanidhi and Nigam [8] who studied the viscous flow between rotating parallel plates under constant pressure gradient. Verma and Sehgal [9] used the micro polar flow model to obtain analytical solutions for the Couette flow of fluids which can support couple stresses and distributed body couples. Liu and Chen [10] investigated computationally the transient rotating Couette flow problem. Jana and Datta [11] studied the steady Couette flow of a viscous incompressible fluid between two infinite parallel plates, one stationary and the other moving with uniform velocity, in a rotating frame of reference. Heat transfer rates were shown to decrease with an increase in rotation parameter. Seth *et al.* [12] studied the transient magneto hydro dynamic Couette flow between parallel rotating plates with one plate moving with a time – dependent velocity in its own plane. Both impulsive start and accelerated startup of the moving plate were considered.

An asymptotic solution was presented for both large and small times and shear stresses at the moving plate due to primary and secondary flows computed. An increase in rotation parameter was shown to elevate the secondary flow shear stress component but depress values of the primary flow component. Mandal and Mandal [13] obtained analytical solutions for the effects of magnetic field and Hall currents on rotating parallel plate Couette flow. They also studied the case where the plates have arbitrary conductivity and thickness. The transient dusty suspension Couette flow problem was studied by Kythe and Puri [14]. Singh *et al.* [15] obtained closed form solutions for velocity and skin friction for rotating hydro magnetic Couette flow, showing that the Ekman number decreases primary velocities but boosts the secondary velocity values.

The converse effect was reported for the magnetic parameter (Hartmann number).

Main purpose of this present paper is to find the numerical solution of unsteady magneto hydrodynamic free convective Couette flow of viscous incompressible fluid confined between two vertical permeable parallel plates in the presence of thermal radiation is performed. A uniform magnetic field which acts in a direction orthogonal to the permeable plates, and uniform suction and injection through the plates are applied. The magnetic field lines are assumed to be fixed relative to the moving plate. The momentum equation considers buoyancy forces while the energy equation incorporates the effects of thermal radiation. The fluid is considered to be a gray absorbing – emitting but non – scattering medium in the optically thick limit. The Roseland approximation is used to describe the radioactive heat flux in the energy equation.

The non – linear coupled pair of partial differential equations are solved by an efficient Crank Nicholson method which is more economical from computational point of view. The resulting system of equations are solved to obtain the velocity and temperature distributions. These solutions are useful to gain a deeper knowledge of the underlying physical processes and it provides the possibility to get a benchmark for numerical solvers with reference to basic flow configurations. The mathematical analysis and the method of solution of this paper have been presented in Section 2 and 3 respectively and the results are discussed in Section 4 and the conclusions are set out in Section 5.

Mathematical formulation:

The incompressible Newtonian fluid flows between two parallel vertical non – conducting permeable plates. These plates are located on planes $y' = 0$ and $y' = h$, and are infinite in the x' and z' – directions. The plate at $y' = h$ is stationary and the other plate moves with time – dependent velocity $U_o t'^n$ in the positive x' – direction (where U_o is constant and n is a non – negative integer). The temperature of the moving and stationary plates are fixed at T'_w and T'_h respectively, with $T'_w > T'_h$. Uniform suction through the moving plate and uniform injection through the stationary plate are

applied through the plates at $t' = 0$ in the negative y' – direction.

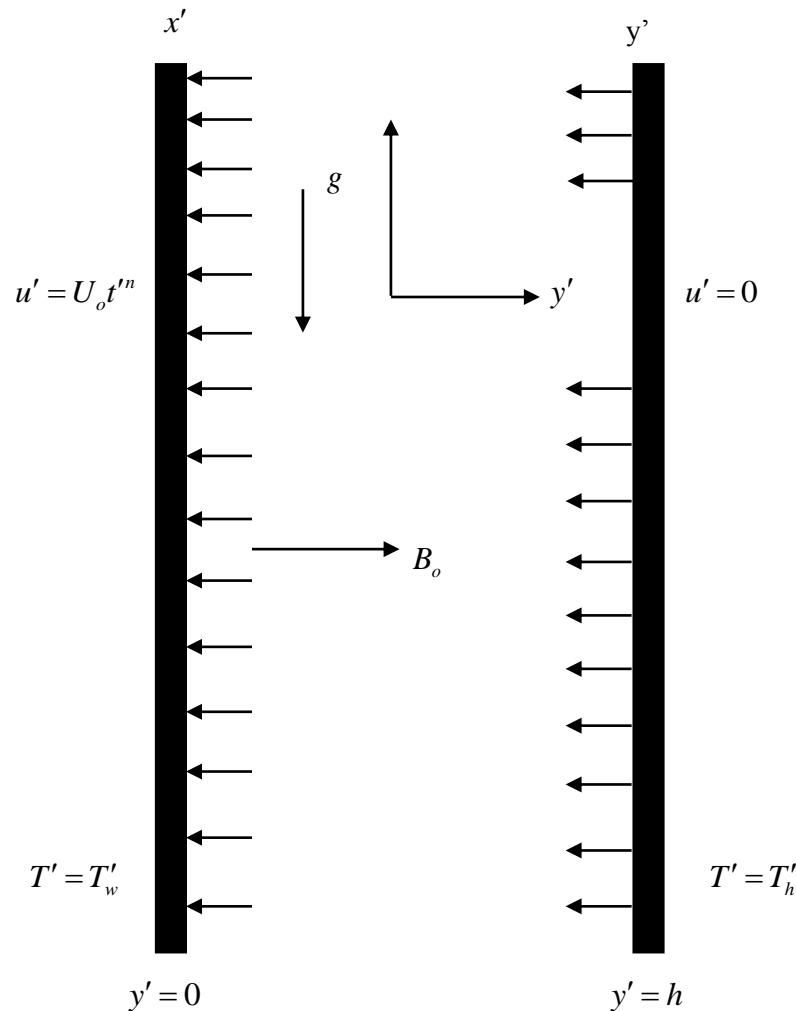


Figure 1. Physical model of the problem

We make the following simplifying assumptions.

1. A uniform magnetic field B_o is applied in the positive y' – direction and is assumed undisturbed as the induced magnetic field is neglected by assuming a very small magnetic Reynolds number.
2. It is assumed that the external electric field is zero and the electric field due to the polarization of charges is negligible.
3. The homogeneous chemical reaction of first order with rate constant between the diffusing species and the fluid is neglected.
4. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present and hence Soret and Duffer effects are negligible.
5. The fluid has constant kinematic viscosity and constant thermal conductivity and the Bossiness's approximation have been adopted for the flow.

6. The fluid is a gray and optically thick absorbing emitting but non – scattering medium.
7. The fluid has a refractive index of unity.

Under the above assumptions, the governing equations are:

Continuity Equation:

$$\frac{\partial u'}{\partial y'} = 0 \quad (1)$$

Momentum Equation:

$$\rho \left(\frac{\partial u'}{\partial t'} - v_o \frac{\partial u'}{\partial y'} \right) = \mu \frac{\partial^2 u'}{\partial y'^2} + \rho g \beta (T' - T'_h) - \gamma B_o^2 (u' - U_o t'^n) \quad (2)$$

Energy Equation:

$$\rho C_p \left(\frac{\partial T'}{\partial t'} - v_o \frac{\partial T'}{\partial y'} \right) = \kappa \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 + \gamma B_o^2 u'^2 - \frac{\partial q_r}{\partial y'} \quad (3)$$

Where u' is the flow velocity in x' – direction, ρ is the fluid density, g is the acceleration due to gravity, μ is the coefficient of viscosity, β is the coefficient of thermal expansion, T' is the fluid temperature, γ is the electrical conductivity, C_p is the specific heat capacity at constant pressure, κ is the thermal conductivity and q_r is the radioactive heat flux.

The corresponding initial and boundary conditions are

$$t' \leq 0 : \left. \begin{array}{l} u' = 0, \quad T' = T'_h \text{ for } 0 \leq y' \leq h \\ t' > 0 : \left\{ \begin{array}{l} u' = U_o t'^n, \quad T' = T'_w \text{ at } y' = 0 \\ u' = 0, \quad T' = T'_h \text{ at } y' = h \end{array} \right. \end{array} \right\} \quad (4)$$

The radioactive heat flux term is simplified by making use of the Roseland approximation [36] as

$$q_r = - \frac{4\bar{\sigma}}{3k^*} \frac{\partial T'^4}{\partial y'} \quad (5)$$

Here $\bar{\sigma}$ is Stefan – Boltzmann constant and k^* is the mean absorption coefficient. It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expressed as a linear function of T' after using Taylor's series to expand T'^4 about the free stream temperature T'_h and neglecting higher – order terms.

This results in the following approximation:

$$T'^4 \cong 4T_h'^3 T' - 3T_h'^4 \quad (6)$$

Using equations (5) and (6) in the last term of equation (3), we obtain:

$$q_r = - \frac{16\bar{\sigma}T_h'^3}{3k^*} \frac{\partial^2 T'}{\partial y'^2} \quad (7)$$

Introducing (7) in the equation (3), the energy equation becomes:

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_h'^3}{3k^*} \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 + \gamma B_o^2 u'^2 \quad (8)$$

To find the solutions of equations (2), (3) and (8) subject to the initial and boundary conditions (4), two cases are considered:

1. Impulsive movement of the plate at $y' = 0$ (i.e. $n = 0$) and
2. Uniformly accelerated movement of the plate at $y' = 0$ (i.e. $n = 1$).

Case – I: Impulsive movement of the plate $y' = 0$:

Taking $n = 0$ in equation (2) and introducing the following non – dimensional quantities in equations (2), (3) and (8):

$$\left. \begin{aligned} y = \frac{y'}{h}, \quad u = \frac{u'}{h}, \quad t = \frac{t'\mu}{h^2}, \quad \theta = \frac{T' - T_h'}{T_w' - T_h'}, \quad Gr = \frac{\rho g \beta h^2 (T_w' - T_h')}{\mu U_o}, \\ M^2 = \frac{\gamma B_o^2 h^2}{\mu}, \quad a = \frac{\rho h^2}{\mu}, \quad R = \frac{\kappa k^*}{4\sigma T_h'^3}, \quad Pr = \frac{\mu C_p}{\kappa}, \quad Ec = \frac{U_o^2}{C_p (T_w' - T_h')} \end{aligned} \right\} \quad (9)$$

The equations (2) and (3), using non – dimensional quantities (9) reduce to the following non – dimensional form of equations:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - M^2(u - 1) \quad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \left(\frac{3R + 4}{3R} \right) \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y} \right)^2 + M^2(Ec)u^2 \quad (11)$$

Where Gr is the thermal Gash of number, M is the Magnetic field (Hartmann number), a is the Accelerating parameter, Pr is the Prandtl number, R is the thermal radiation parameter and Ec is the Eckert number.

The initial and boundary conditions (4) reduce to:

$$\left. \begin{aligned} t \leq 0: \quad u = 0, \quad \theta = 0 \quad \text{for} \quad 0 \leq y \leq 1 \\ t > 0: \quad \left\{ \begin{aligned} u = 1, \quad \theta = 1 \quad \text{at} \quad y = 0 \\ u = 0, \quad \theta = 0 \quad \text{at} \quad y = 1 \end{aligned} \right. \end{aligned} \right\} \quad (12)$$

Case – II: Uniformly accelerated movement of the plate (at $y' = 0$)

Taking $n = 1$ in equation (2) and using equation (9), the equation (2) reduces to the following non – dimensional form of equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - M^2(u - at) \quad (13)$$

The initial and boundary conditions (4) reduce to:

$$\left. \begin{array}{l} t \leq 0: u = 0 \text{ for } 0 \leq y \leq 1 \\ t > 0: \left\{ \begin{array}{l} u = at \text{ at } y = 0 \\ u = 0 \text{ at } y = 1 \end{array} \right\} \end{array} \right\} \quad (14)$$

For practical engineering applications and the design of chemical engineering systems, quantities of interest include the following Skin – friction, Nussle number and Sherwood number are useful to compute.

Skin – friction: The skin friction or the shear stress at the moving plate of the channel in non – dimensional form is given by:

$$\tau_1 = - \left[\frac{\partial u}{\partial y} \right]_{y=0} \quad (15)$$

Nussle Number: The rate of heat transfer at the moving hot plate of the channel in non – dimensional form is given by:

$$Nu_0 = - \left[\frac{\partial \theta}{\partial y} \right]_{y=0} \quad (16)$$

Further the rate of heat transfer on the stationary plate is given by:

$$Nu_1 = - \left[\frac{\partial \theta}{\partial y} \right]_{y=1} \quad (17)$$

The mathematical formulation of the problem is now completed. Equations (10), (11)& (13) present a coupled nonlinear system of partial differential equations and are to be solved by using initial and boundary conditions (12) & (14). However, exact solutions are difficult, whenever possible. Hence, these equations are solved by the Crank Nicholson method.

Numerical Solution by Crank Nicholson Method:

Equations (10), (11) & (13) represent coupled system of non – linear partial differential equations which are solved numerically under the initial and boundary conditions (12) & (14) using the finite difference approximations. A linearization technique is first applied to replace the non – linear terms at a linear stage, with the corrections incorporated in subsequent iterative steps until convergence is reached. Then the Crank Nicolson implicit method is used at two successive time levels [17]. An iterative scheme is used to solve the linear zed system of difference equations. The solution at a certain time step is chosen as an initial guess for next time step and the iterations are continued till convergence, within a prescribed accuracy. Finally, the resulting block tridiagonal system is solved using the generalized Thomas – algorithm [17]. Finite difference equations relating the variables are obtained by writing the equations at the midpoint of the computational cell and then replacing the different terms by their second order central difference approximations in the y – direction. The diffusion terms are replaced by the average of the central differences at two successive time – levels. The computational domain is divided into meshes of dimension Δt and Δy in time and space respectively as shown in figure 2. We define the variables $B = u_y$ and $L = \theta_y$ to reduce the second order differential equations (10) & (11) to first order differential equations. The finite difference representations for the resulting first order differential equations (10), (11) & (13) take the following forms:

$$\left(\frac{u_{i+1,j+1} - u_{i,j+1} + u_{i+1,j} - u_{i,j}}{2(\Delta t)}\right) = \left(\frac{(B_{i+1,j+1} + B_{i,j+1}) - (B_{i+1,j} + B_{i,j})}{2(\Delta y)}\right) + Gr \left(\frac{\theta_{i+1,j+1} + \theta_{i,j+1} + \theta_{i+1,j} + \theta_{i,j}}{4}\right) - M^2 \left(\frac{u_{i+1,j+1} + u_{i,j+1} + u_{i+1,j} + u_{i,j}}{4}\right) + M^2 \quad (18)$$

$$\left(\frac{\theta_{i+1,j+1} - \theta_{i,j+1} + \theta_{i+1,j} - \theta_{i,j}}{2(\Delta t)}\right) = \frac{1}{Pr} \left(\frac{3R+4}{3R}\right) \left(\frac{(L_{i+1,j+1} + L_{i,j+1}) - (L_{i+1,j} + L_{i,j})}{4}\right) + M^2 (Ec) \left(\frac{u_{i+1,j+1} + u_{i,j+1} + u_{i+1,j} + u_{i,j}}{4}\right)^2 + QZFO \quad (19)$$

$$\left(\frac{u_{i+1,j+1} - u_{i,j+1} + u_{i+1,j} - u_{i,j}}{2(\Delta t)}\right) = \left(\frac{(B_{i+1,j+1} + B_{i,j+1}) - (B_{i+1,j} + B_{i,j})}{2(\Delta y)}\right) + Gr \left(\frac{\theta_{i+1,j+1} + \theta_{i,j+1} + \theta_{i+1,j} + \theta_{i,j}}{4}\right) + Gc \left(\frac{\phi_{i+1,j+1} + \phi_{i,j+1} + \phi_{i+1,j} + \phi_{i,j}}{4}\right) - M^2 \left(\frac{u_{i+1,j+1} + u_{i,j+1} + u_{i+1,j} + u_{i,j}}{4}\right) + M^2 at \quad (20)$$

Where *QZFO* represents the viscous dissipation term which are known from the solution of the momentum equations and can be evaluated at the midpoint $\left(i + \frac{1}{2}, j + \frac{1}{2}\right)$ of the computational cell. Computations have been made for $Gr = 2.0$, $Pr = 0.71$, $R = 2.0$, $M = 2.0$, $F = 0.3$, $a = 2.0$ and $t = 0.1$. Grid – independence studies show that the computational domain $0 < t < \infty$ and $-1 < y < 1$ can be divided into intervals with step sizes $\Delta t = 0.0001$ and $\Delta y = 0.005$ for time and space respectively. The truncation error of the central difference schemes of the governing equations is $O(\Delta t^2, \Delta y^2)$. Stability and rate of convergence are functions of the flow and heat parameters. Smaller step sizes do not show any significant change in the results. Convergence of the scheme is assumed when all of the unknowns u and θ for the last two approximations differ from unity by less than 10^{-6} for all values of y in $-1 < y < 1$ at every time step. Less than 7 approximations are required to satisfy this convergence criteria for all ranges of the parameters studied here.

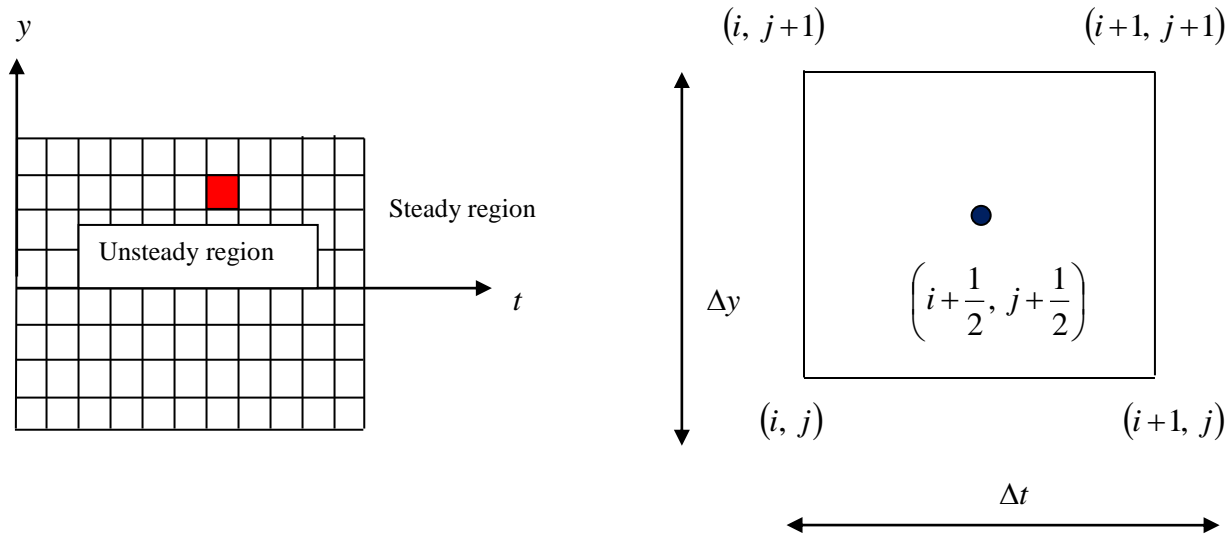


Figure 2. Mesh Layout

Results and Discussions:

We solve the similarity equations (18), (19) and (20) numerically subject to the boundary conditions given by (12) & (14). Graphical representations of the numerical results are illustrated in figure (3) through figure (15) to show the influences of different numbers on the boundary layer flow. In this study, we investigate the influence of the effects of material parameters such as Grashof number (Gr), Prandtl number (Pr), Hartmann number (M), Thermal radiation parameter (R) and Eckert number (Ec) separately in order to clearly observe their respective effects on the velocity and temperature profiles of the flow have been observed through graphically in the cases of impulsive movement of the plate (i.e. $n = 0$) and uniformly accelerated movement of the plate (i.e. $n = 1$). And also skin friction coefficient (τ_o), Rate of heat and mass transfer coefficients (Nu_o & Nu_1) respectively have been observed through tabular forms. During the course of numerical calculations of the velocity (u) and temperature (θ), the values of the Prandtl number are chosen for Mercury ($Pr = 0.025$), Air at $25^\circ C$ and one atmospheric pressure ($Pr = 0.71$), Water ($Pr = 7.00$) and Water at $4^\circ C$ ($Pr = 11.40$). For the physical significance, the numerical discussions in the problem $a = 2.0$ and $t = 0.1$ stable values for velocity and temperature and fields are obtained. Figures (3) and (4) illustrate the effects of Gr (thermal Grashof number) on the velocity field. We observe that the velocity of fluid increases with increasing thermal Grashof number in both the cases. It is due to reason that increases in give rise to buoyancy effects resulting in more induced flows. Figures (5) and (6) display the effects of Hartmann number M (magnetic parameter) on the velocity field. It is observed that the velocity of fluid increases with increasing magnetic parameter in case of impulsive movement of the plate (i.e. $n = 0$), but decreases with increasing magnetic parameter in case of uniformly accelerated movement of the plate (i.e. $n = 1$). Figures (4) and (5) represent the velocity profiles for different values of Pr (Prandtl number). It is observed that the velocity of fluid decreases as the value of Prandtl number increases in both the cases. This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing.

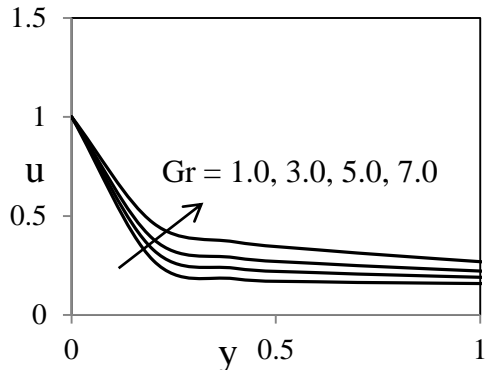


Figure 3. Velocity profiles for different values of Thermal Grashof number in case of impulsive movement of the plate

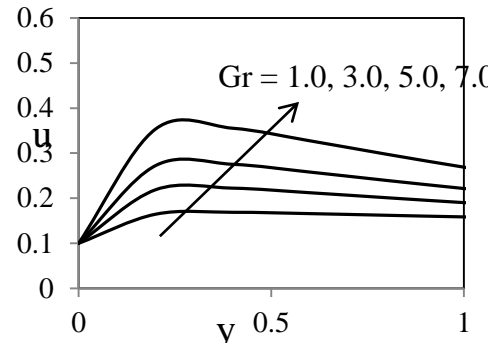


Figure 4. Velocity profiles for different values of Thermal Grashof number in case of uniformly accelerated movement of the plate

The effects of radiation parameter on the velocity profiles are presented in figures (3) and (4). From these figures, we observe that as the value of R (radiation parameter) increases, the velocity decreases in both the cases when the other physical parameters are fixed. The effect of viscous dissipation (Eckert number) on the velocity profiles is shown in the figures (4) and (5) in case of impulsive movement of the plate (i.e. $n = 0$) and accelerated movement of the plate (i.e. $n = 1$). The Eckert number (Ec) expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. From these figures the velocity profiles are increasing with increasing values of Eckert number in both the cases.

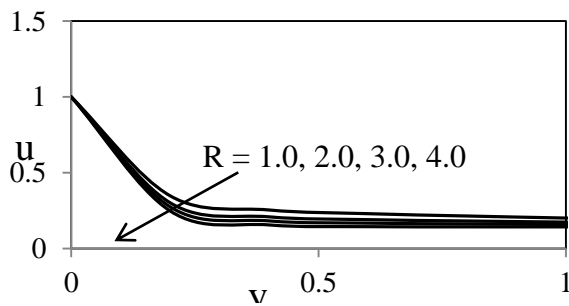


Figure 5. Velocity profiles for different values of Thermal radiation parameter in case of impulsive movement of the plate

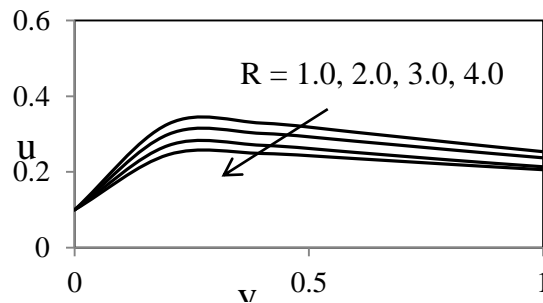


Figure 6. Velocity profiles for different values of Thermal radiation parameter in case of uniformly accelerated movement of the plate

The temperature profiles are illustrated in figures (5) to (6) for different values of Prandtl number, radiation parameter and Eckert number respectively. In figure (5), it can be seen that the temperature of the fluid is inversely proportional to the value of Pr. Thus increasing reduces the temperature in the system. This trend is generally due to the decrease of the thermal diffusivity at high value of Pr. From figure (3), it is clear that an increase in the radiation parameter results in decreasing the temperature within the boundary layer, as well as a decrease in the thermal boundary layer thickness. The influence of the viscous dissipation parameter i.e., the Eckert number (Ec) on the temperature profiles are shown in figure (7). From this figure (7) we observed that the temperature profiles are reducing with

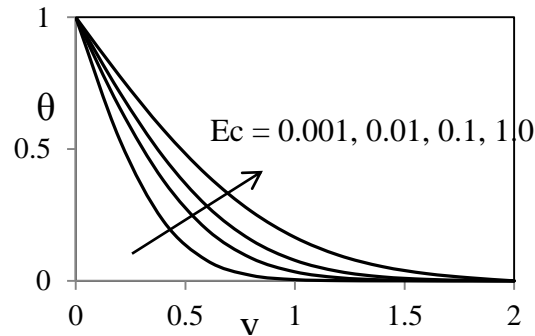


Figure 7: Temperature Profiles for different values of Eckert number

Table – 1: Skin friction in case of impulsive movement of the plate

Gr	Pr	M	R	Ec	τ_o
2.0	0.71	2.0	2.0	0.001	- 0.9973134
4.0	0.71	2.0	2.0	0.001	- 0.2873451
2.0	7.00	2.0	2.0	0.001	- 1.7731157
2.0	0.71	4.0	2.0	0.001	- 0.2014462
2.0	0.71	2.0	4.0	0.001	- 1.6500381
2.0	0.71	2.0	2.0	0.100	- 0.1179032

Table – 2: Skin friction in case of uniformly accelerated movement of the plate

Gr	Pr	M	R	Ec	τ_o
2.0	0.71	2.0	2.0	0.001	- 0.9273543
4.0	0.71	2.0	2.0	0.001	- 0.1827641
2.0	7.00	2.0	2.0	0.001	- 1.7126543
2.0	0.71	4.0	2.0	0.001	- 1.7993425
2.0	0.71	2.0	4.0	0.001	- 1.6722764
2.0	0.71	2.0	2.0	0.100	- 0.7466328

Tables (1) represent the skin friction in case of impulsive movement of the plate (i.e. $n = 0$) and in case of uniformly accelerated movement of the plate (i.e. $n = 1$) respectively. From the two tables, it is clear that in case of uniformly accelerated movement of the plate the skin friction increases as the value of Hartmann number (magnetic field) increases (keeping other parameters constant), but for impulsive movement of the plate, it decreases as the value of Hartmann number increases (keeping other parameters constant). Again, the skin friction decreases as the values of the Prandtl number and radiation parameter increase (keeping other parameters constant) in both the cases ($n = 0$ and $n = 1$). When the value of thermal Grashof number is increased (keeping other parameters constant) the value of skin friction also gets increased in both the cases ($n = 0$ and $n = 1$). Further, the skin friction increases as the values of Eckert number (keeping other parameters constant) are increased in both the cases ($n = 0$ and $n = 1$).

Table – 2 shows the effect of Thermal radiation parameter, Prandtl number and Eckert number on the Nusselt number (the rate of heat transfer) at the moving and stationary plates. We see that the Nusselt number at the moving plate Nu_o is increasing with increasing values of radiation parameter and Prandtl number but decreasing with increasing values of Eckert number. The Nusselt number at the stationary plate Nu_1 is decreasing with increasing values of radiation parameter and Prandtl number but increasing with increasing values of Eckert number. In order to ascertain the accuracy of the numerical results, the present results are compared with the previous results of Rajput and Sahu [12] in tables – 1 and 2 for both the cases I and II. They are found to be in an excellent agreement.

Conclusions:

This work investigated is to find the numerical solution of unsteady magneto hydro dynamic free convective Couette flow of viscous incompressible fluid confined between two vertical permeable parallel plates in the presence of thermal radiation is performed. The non – linear coupled pair of partial differential equations are solved by an efficient Crank Nicholson method. A parametric study illustrating the influence of different flow parameters on velocity, temperature and concentration fields are investigated in case of impulsive movement of the plate (i.e. $n = 0$) and

uniformly accelerated movement of the plate (i.e. $n = 1$). We conclude that the flow field and the quantities of physical interest are significantly influenced by these numbers.

1. A growing of Grashof number and Eckert number are to enhance the velocity of the flow field at all points in both the cases ($n = 0$ and $n = 1$).
2. A growing of Prandtl number and Thermal radiation parameter are to retard the velocity of the flow field at all points in both the cases ($n = 0$ and $n = 1$).
3. A growing of Hartmann number is to enhance the velocity of the flow in case of in case of impulsive movement of the plate (i.e. $n = 0$) and is to retard the velocity of the flow in case of uniformly accelerated movement of the plate (i.e. $n = 1$) at all points.
4. The temperature of the flow is increasing with increasing values of Eckert number while decreasing with increasing values of Prandtl number and Thermal radiation parameter at all the points.
5. Skin friction is increases with increasing of Hartmann number and Eckert number in case of impulsive movement of the plate (i.e. $n = 0$).
6. Skin friction is decreases with increasing of Hartmann number and increases with increasing of Eckert number in case of uniformly accelerated movement of the plate (i.e. $n = 1$).

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