

Application of Higher Order Image Co-Segmentation in Medical Images

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Abstract:

A novel interactive image co-segmentation algorithm using likelihood estimation and higher order energy optimization is proposed for extracting common foreground objects from a group of related images. Our approach introduces the higher order clique's, energy into the co-segmentation optimization process successfully. A region-based likelihood estimation procedure is first performed to provide the prior knowledge for our higher order energy function. Then, a new co-segmentation energy function using higher order cliques is developed, which can efficiently co-segment the foreground objects with large appearance variations from a group of images in complex scenes. Both the quantitative and qualitative experimental results on representative datasets demonstrate that the accuracy of our co-segmentation results is much higher than the state-of-the-art co-segmentation methods.

The delineation of tumor boundaries in medical images is an essential task for the early detection, diagnosis and follow-up of cancer. However accurate segmentation remains challenging due to presence of noise, inhomogeneity and high appearance variability of malignant tissue. In this paper, we propose an automatic segmentation approach using fully-connected higher-order co-segmentation (HOC) where potentials are computed within a discriminant Grassmannian manifold. First, the framework learns within-class and between-class similarity distributions from a training set of images to discover the optimal manifold discrimination between normal and pathological tissues. Second, the conditional optimization scheme computes non-local pairwise as well as pattern-based higher-order potentials from the manifold subspace to recognize regions with similar labeling's and incorporate global consistency in the inference process. Our HOC framework is applied in the context of metastatic brain tumor segmentation in CT images.

Compared to state of the art methods, our method achieves better performance on a group of 30 intracerebral hemorrhage images and can deal with highly pathological cases.

Keywords

Image Co-Segmentation, Likelihood Estimation, Higher Order Energy Optimization, Higher-Order Co-Segmentation (HOC), Medical Images, Tumor, Cancer, Detection and Diagnosis of Cancer.

1. Introduction

IMAGE co-segmentation is commonly referred as jointly partitioning multiple images into foreground and background components. The idea of co-segmentation is first introduced by Rother et al. where they simultaneously segment common foreground objects from a pair of images. The co-segmentation problem has attracted much attention in the last decade, most of the co-segmentation approaches are motivated by traditional Markov Random Field (MRF) based energy functions, which are generally solved by the optimization techniques such as linear programming dual decomposition and network flow model. The main reason may be that the graph-cuts and MRF methods work well for image segmentation and are also widely used to solve the combinatorial optimization problems in multimedia processing. Similar rationale is also adopted by some co-saliency methods.

The existing image co-segmentation methods can be roughly classified into two main categories, including unsupervised co-segmentation techniques and interactive co-segmentation approaches. The common idea of the unsupervised techniques formulates image co-segmentation as an energy minimization and binary labeling problem. These approaches usually define the energy function using standard MRF terms and histogram matching term. The former encourages the consistent segmentations in every single image while the later penalizes the differences between the foreground histograms of



multiple images. Inspired by interactive single-image segmentation methods several interactive co-segmentation approaches using user scribbles have been proposed in recent years. The user usually indicates scribbles of foreground or background as additional constraint information to improve the co-segmentation performance. These interactive co-segmentation approaches can handle a group of related images and improve the co-segmentation results by user scribbles.

Batra et al. proposed an interactive image co-segmentation approach to segment foreground objects with user interactions. They learned foreground/background appearance models using user scribbles. Recently, Collins et al. formulated the interactive image co-segmentation problem as the random walk model and added the consistency constraint between the extracted objects from a set of input images. Their method utilized the normalized graph Laplacian matrix and solved the random walk optimization scheme by exploiting its quasi-convexity of foreground objects.

This study formulates the interactive image co-segmentation problem in terms of the higher-order energy optimization, which complements the existing MRF segmentation framework and improves the accuracy of co-segmenting the challenging images with foreground objects that have variations in color and texture only by a few of user seeds. Higher-order energy optimization has been widely used in many fields of computer vision like image denoising [14] and single-image segmentation. We construct higher-order clique as a composed group of three parts: the foreground region, the background region and the over-segmentation region, which considers the correspondence between the over-segmentation region and the labeled region. This strategy makes our framework effective enough in realistic scenarios, instead of a simple foreground/background appearance histogram model. Additionally, our higher-order energy efficiently utilizes the statistical information on a group of pixels by estimating the segmentation quality on higher-order cliques.

Compared to existing image co-segmentation methods, the proposed approach offers the following contributions.

1) We formulate the interactive image co-segmentation via likelihood estimation and high-order energy optimization, which utilizes the region likelihoods of multiple images and considers the quality of segmentation to achieve promising co-segmentation performance.

2) A novel higher-order clique construction method is proposed using the estimated

foreground/background regions and the regions of original images.

3) A new region likelihood estimation method is presented, which provides enough prior information for higher-order energy item for generating final co-segmentation results.

2. Literature Review

1. In “Efficient Co-Segmentation of Image Using Higher Order Clique” A new interactive image co segmentation algorithm using possibility estimation and higher order energy is proposed for extracting general foreground objects from a group of interrelated images. Our approach introduces the higher order cliques, energy into the co segmentation optimization process successfully. A region based likelihood estimation procedure is first performed to provide the primary knowledge for our higher order energy function. A new co segmentation energy function using higher order clique is developed, which can capably co segmentation energy function using higher order clique is developed, which can efficiently co segment the foreground objects with huge manifestation variations from a group of images in complex scenes. Both the quantitative and qualitative experimental results on representative datasets reveal that the accuracy of our co segmentation results is much higher than the state-of-the-art co segmentation methods IMAGE co-segmentation is commonly referred as jointly partitioning multiple images into foreground and background components.

2. In “Extracting Primary Objects by Video Co-Segmentation” Video object segmentation is a challenging problem. Without human annotation or other prior information, it is hard to select a meaningful primary object from a single video, so extracting the primary object across videos is a more promising approach. However, existing algorithms consider the problem as foreground/background segmentation. Therefore, we propose an algorithm that learns the model of the primary object by representing the frames/videos as a graphical model. The probabilistic graphical model is built across a set of videos based on an object proposal algorithm. Our approach considers appearance, spatial, and temporal consistency of the primary objects. A new dataset is created to evaluate the proposed method and to compare it to the state-of-the-art on video object co-segmentation.

The experiments show that our method obtains state-of-the-art results, outperforming other algorithms by 1.5% (pixel accuracy) on the MOVICS dataset and 9.6% (pixel accuracy) on the new dataset T HE amount of video data is explosively increasing.

3. In “Co-segmentation of Image Pairs by Histogram Matching Incorporating a Global Constraint into MRFs” We introduce the term co-segmentation which denotes the task of segmenting simultaneously the common parts of an image pair. A generative model for co-segmentation is presented. Inference in the model leads to minimizing an energy with an MRF term encoding spatial coherency and a global constraint which attempts to match the appearance histograms of the common parts. This energy has not been proposed previously and its optimization is challenging and NP-hard. For this problem a novel optimization scheme which we call trust region graph cuts is presented. We demonstrate that this framework has the potential to improve a wide range of research: Object driven image retrieval, video tracking and segmentation, and interactive image editing. The power of the framework lies in its generality, the common part can be a rigid/non-rigid object (or scene), observed from different viewpoints or even similar objects of the same class. We introduce the term co-segmentation which denotes the task of segmenting simultaneously the common parts of an image pair. A generative model for co-segmentation is presented.

3. Overview and Design Approach

Our co-segmentation procedure includes two main steps. The first step is a fast but effective likelihood estimation process, which calculates the probabilities of pixels belonging to foreground/background over entire dataset according to user scribbles. The estimated likelihood offers a rough estimation for foreground /background and is fed into next step as prior knowledge. This process is described. In the second stage, a higher-order energy based co-segmentation function is proposed to obtain final accurate co-segmentation results on a group of images, which is based on higher order cliques. Our higher-order cliques are constructed from a set of foreground and background regions by user scribbles, where all the regions in each image are matched to produce better co-segmentation performance. Additionally, our approach considers the quality of segmentation in higher-order energy to obtain more accurate estimations of foreground or background.

A. Likelihood Estimation

Given a group of images $\{I_1, \dots, I_n\}$ and the user scribbles that indicate foreground or background objects, we first compute pixel likelihood x_i^k for foreground/background in image I_i . The likelihood of pixel x_i^k is denoted by π_i^k, l where l is a label indicating foreground (1) or background (0) and k is the index value of x_i^k . We compute the likelihoods of regions instead of pixels for computational

efficiency. Each input image I_i of the group is divided into regions $ris \in Ri$ using the over-segmentation methods such as mean shift [1] or efficient graph [6] method. For each region ris , the region likelihoods of foreground and background are defined as z_i^s, l , which is further formulated in a quadratic energy function as follows:

$$F_i l = F_1 + F_2 = \lambda_i N(R_i) \sum_{s=1}^2 (z_i^s, l - \varepsilon_i^s, l)^2 + N(R_i) \sum_{s,s'=1}^2 w_i^s, s' (z_i^s, l - z_i^{s'}, l)^2 \quad (1)$$

Where the first term F_1 defines an unary constraint that each region tends to have the initial likelihood ε_i^s, l estimated through the appearance similarity to foreground/background. The second term F_2 gives the interactive constraint that all regions of the whole image should have same likelihood when their representative colors are similar. The parameter λ is a positive coefficient for balancing the relative influence between F_1 and F_2 . $w_i^s, s' = \exp(-c_i^s - c_i^{s'})$ is a weighting function that gives a similarity measure for regions ris and ris' in color space, and c_i^s is the mean color of region ris . $N(R_i)$ is the number of regions of R_i and the parameter z_i^s, l indicates the likelihood of region ris . ε_i^s, l defines the initial likelihood for region ris . Given the user scribbles, we can get the background region set $uj \in U(0)$ and foreground region set $uj \in U(1)$. We use the shortest Euclidean distance between region ris and the background/foreground region set (U_0 / U_1) in color space to compute the initial likelihood ε_i^s, l for region ris .

$$\varepsilon_i^s, l = \begin{cases} \frac{\min_{uj \in U^0} (\|\bar{c}_s^i - \bar{c}_j\|)}{\min_{uj \in U^0} (\|\bar{c}_s^i - \bar{c}_j\|) + \min_{uj' \in U^1} (\|\bar{c}_s^i - \bar{c}_{j'}\|)} & \text{if } l = 1 \\ \frac{\min_{uj' \in U^1} (\|\bar{c}_s^i - \bar{c}_{j'}\|)}{\min_{uj \in U^0} (\|\bar{c}_s^i - \bar{c}_j\|) + \min_{uj' \in U^1} (\|\bar{c}_s^i - \bar{c}_{j'}\|)} & \text{if } l = 0 \end{cases}$$

Where c_j (c_j') is the mean color of background region uj (foreground region uj'). Based on the region likelihoods $z_i l = [z_i^s, l] N(R_i) \times 1$ and their initial region likelihoods $\varepsilon_i l = [\varepsilon_i^s, l] N(R_i) \times 1$, the quadratic energy function $F_i l$ is formulated as the following matrix forms: $F_i l = (z_i l - \varepsilon_i l)^T \Lambda_i (z_i l - \varepsilon_i l) + z_i^T T l (D_i - W_i) z_i l$ (3) where $W_i = [w_i^s, s'] N(R_i) \times N(R_i)$ and $D_i = \text{diag}([d_i^1, \dots, d_i^{N(R_i)}])$. The diagonal elements of the metric D_i are the degrees of the weight matrix W_i : d_i

$d_i = \sum_{s=1}^2 N(R_i) w_i^s, s'$. The diagonal elements of the metric Λ_i are $\text{diag}([\lambda_i, \dots, \lambda_i]) N(R_i) \times N(R_i)$. (3) is then solved by the following convex optimization:

$$\frac{\partial F_i l}{\partial z_i l} = \Lambda_i (z_i l - \varepsilon_i l) + (D_i - W_i) z_i l = 0. \quad (4)$$

After solving (4), we finally obtain the region likelihoods $z_{I,l}$ as follows:

$$z_{I,l} = \lambda_i \varepsilon_{I,l} / (\lambda_i \varepsilon_{I,l} + D_i - W_i). \quad (5)$$

Considering the definition of $\varepsilon_{I,l}$ in (2), we have $\varepsilon_{I,0} + \varepsilon_{I,1} = 1$.

$\varepsilon_{I,1} = 1$. According to $\varepsilon_{I,0} + \varepsilon_{I,1} = 1$ and (5), we have

$$z_{I,0} + z_{I,1} = 1. \quad (6)$$

We only need to calculate either $z_{I,0}$ or $z_{I,1}$ using (5). (5) is easily computed by least-square and the optimization only takes 0.02 s for 500 over-segmentation regions per image in our tests. After the region likelihood $z_{I,l}$ is obtained, the pixel likelihood $\pi_{I,k,l}$ is set to the same value as the likelihood of the region that this pixel belongs to $\pi_{I,k,l} = z_{I,sk,l}$ where sk indicates the region $r_{I,sk}$ that pixel x_{ik} belongs to.

B. Higher-Order Energy Co-Segmentation

Via our likelihood estimation, we have a fast and rough estimate for foreground/background in each image. For generating more accurate co-segmentation results, we further propose a higher-order energy based co-segmentation function. In order to simultaneously segment a group of input images $\{I_1, \dots, I_n\}$ with the labeled images T , we first build a global term $E_{\text{global}}(I_1, \dots, I_n, T)$ to match all the images with the labeled images T . The proposed energy of our co-segmentation algorithm is expressed as follows:

$$F = \sum_{i=1}^n E_i^{\text{unary}} + \sum_{i=1}^n E_i^{\text{pairwise}} + E_{\text{global}}(I_1, \dots, I_n, T) \quad (7)$$

where E_i^{unary} and E_i^{pairwise} denote unary term and pairwise term respectively and the global term E_{global} is proposed to match all the input images $\{I_1, \dots, I_n\}$ with labeled images T . The scalars λ weight various terms. The unary term E_i^{unary} and the pairwise term E_i^{pairwise} for image I_i are defined as follows:

$$E_i^{\text{unary}} = \sum_k -\log(\pi_{I,k,1}) \cdot \varphi(x_{ik}) - \log(\pi_{I,k,0}) \cdot (1 - \varphi(x_{ik}))$$

$$E_i^{\text{pairwise}} = \sum_{k,k' \in \mathcal{N}_{\text{cik}}} -\text{cik}_{kk'} \cdot |\varphi(x_{ik}) - \varphi(x_{ik'})| \quad (8)$$

where cik denotes the color value of pixel x_{ik} and $\pi_{I,k,l}$ is obtained in our likelihood estimation step. The set \mathcal{N}_{cik} contains all the four-neighbors within one image. $\varphi(x_{ik})$ is a binary function indicating the assignment of pixel x_{ik} to the background (0) or foreground (1). The unary term E_i^{unary} is based on the likelihood estimation results and penalizes assignments of pixels with lower likelihood to

foreground. The pairwise term E_i^{pairwise} imposes intra-image label smoothness by constraining the segmentation labels to be consistent, which tends to assign the same label to neighboring pixels that have similar color. The co-segmentation model in (7) is intuitive. Next we discuss how to design the global energy item in the following paragraphs. Previous co-segmentation approaches performed co-segmentation on image pairs and made simple assumption that two input images shared a same/similar foreground object. In contrast, we try to extract common foreground objects that have large variations in color, texture and shape from a group of images with complex background. Rather than building a simple foreground or background appearance model, we collect a region set of foreground/background according to user interaction.

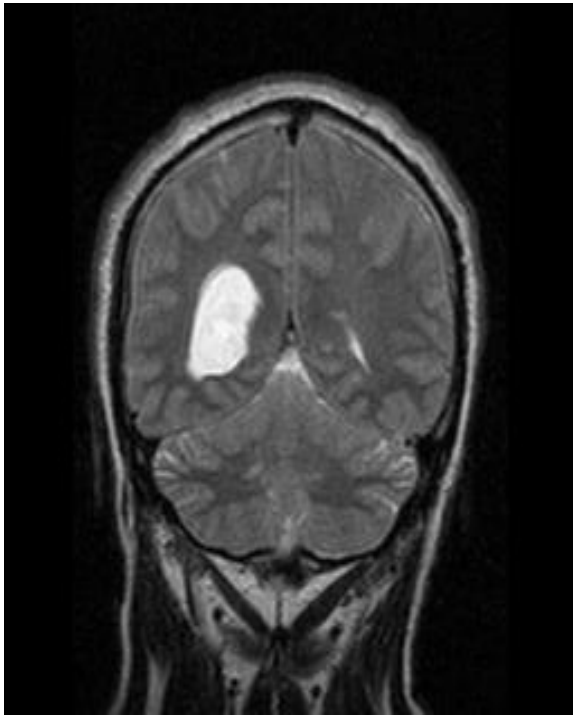
4. Experimental Results

In this section, we first discuss our experiments for evaluating the performance between our algorithm and previous well-known co-segmentation approaches. Then, we give qualitative and quantitative results obtained by the proposed method with and without the higher-order energy. The experimental evaluations are designed to assess the running time statistics of these algorithms. Then, we give qualitative and quantitative results obtained by the proposed method with and without the higher-order energy. The experimental evaluations are designed to assess the running time statistics of these algorithms. Three parameters λ , λ_1 and λ_2 are used in our two energy functions (1) and (7). We empirically set $\lambda = 10$, $\lambda_1 = 1$ and $\lambda_2 = 30$ for all the test image sets in our experiments.

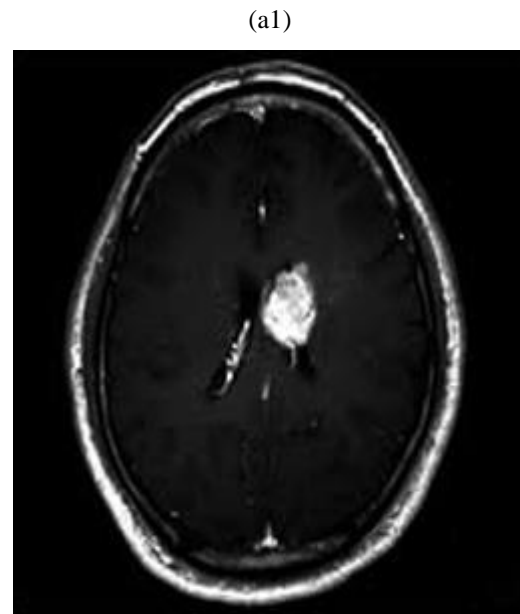
A. Co-segmentation Results

Our method is first compared with the state-of-the-art interactive co-segmentation methods: intelligent scribble guided co-segmentation (ICOSEG) and RWCS on previous benchmark datasets. To achieve a relatively fair comparison, both the proposed method and other interactive co-segmentation methods we have taken Medical intracerebral hemorrhage images as input in all experiments. In the experiments, we collect a variety of image groups from well-known image databases such as Brain Cerebral image database. These two datasets are very popular for image co-segmentation experiments where the ground-truth segmentation masks are also provided. Each group of image collections has a common theme or common foreground object, which makes it challenging to co-segment them with user scribble seeds. We then quantitatively compare the co-segmentation performance of our algorithm with other eight unsupervised approaches: discriminative clustering

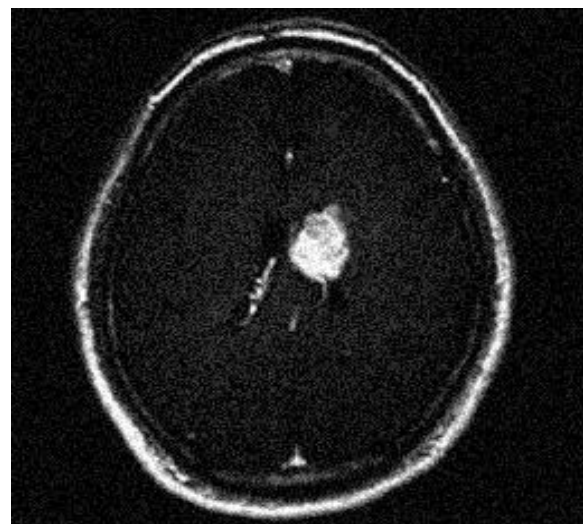
co-segmentation (DCCS) multi-class co-segmentation (MCCS) distributed co-segmentation (DCS) region matching based co-segmentation (RMCS) consistent functional maps based co-segmentation (CFCS) joint object discovery and segmentation (JODS) multi-class joint segmentation (MJS) and multiple random walkers based co-segmentation (MRCS) [40]. The experimental results by DCCS, MCCS,



(a)

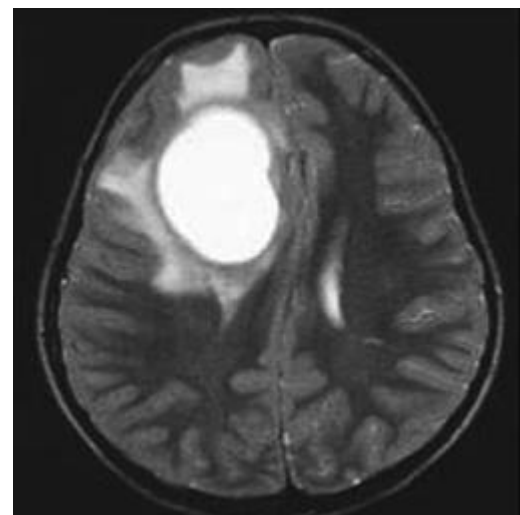


(a1)



(b)

(b1)



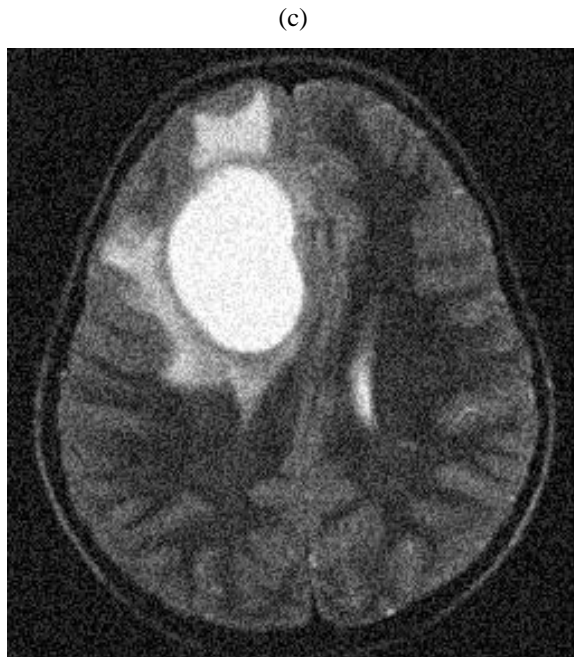
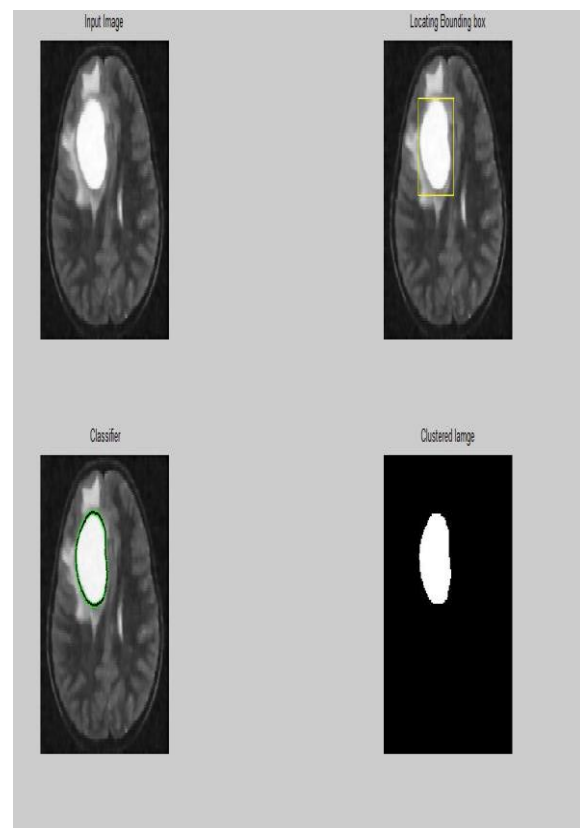
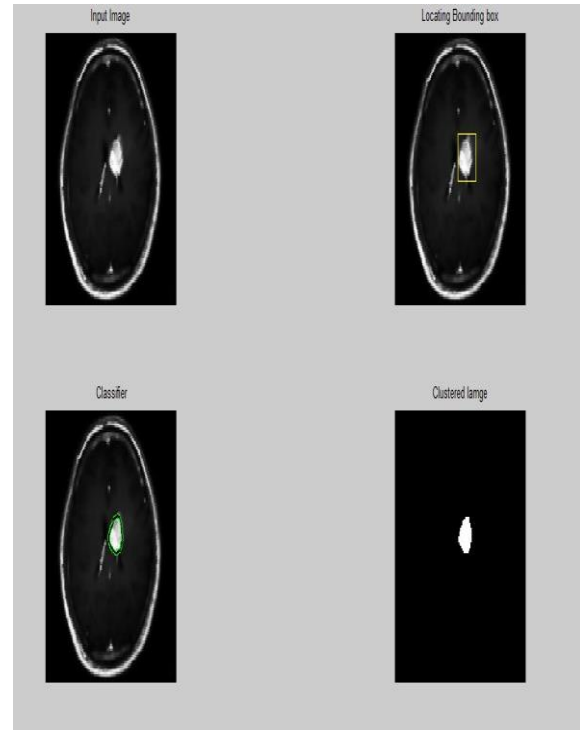


Fig. 1. Comparison results. (a)(a1), (b)(b1), (c)(c1) are three different positions of Brain cerebral internal views taken as input.

Our algorithm achieves better co-segmentation results than both the ICOSEG [21] and RWCS [28] algorithms. And DCS are produced by directly running the implementation codes from their websites. And the co-segmentation results of ICOSEG are generated by the implementation code from the authors [21]. The experimental results of JODS are downloaded from their websites. The results by RMCS, CFCS, MJS and MRCS are mainly borrowed from original works, therefore only parts of these results are reported. Fig. 4 gives a comparison between our algorithm and two well-known interactive co-segmentation approaches: ICOSEG and RWCS for a group of challenging images. The group of Brown bear images is a relatively difficult group in iCoseg dataset. From the co-segmentation results by ICOSEG, we can see that most of regions of the common objects are generally segmented.

However, there are still many background regions of which color is similar to foreground are classified falsely. The reason is that the established common appearance model does not work well when the color distributions of foreground and background pixels across the entire dataset have too many overlaps. Another important interactive co-segmentation method is the RWCS using random walker algorithm as its optimization framework. Based on their appearance model with foreground objects, their algorithm achieves better co-segmentation results shown in the output figures than the ICOSEG

method. However, the similarity between foreground and background color histograms still influences the performance of RWCS, which may lead to the incorrect segmentation of some foreground regions.



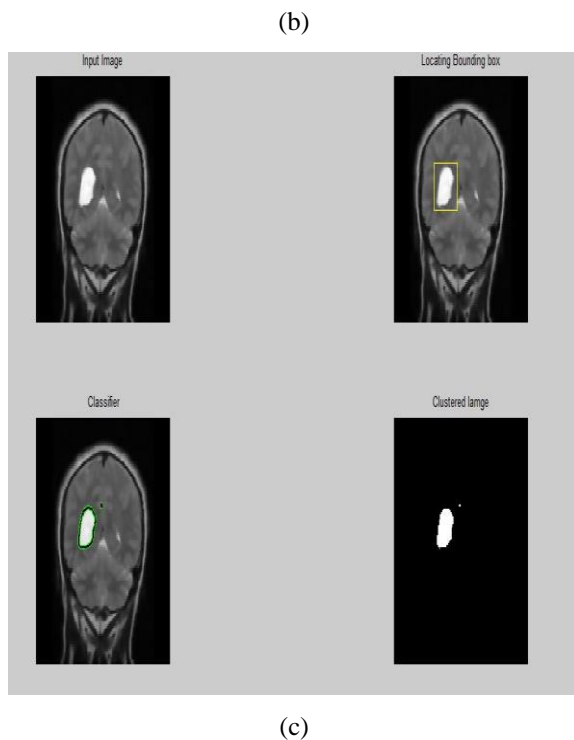


Fig. 2. (a),(b),(c), Comparison co-segmentation results between our method and ICOSEG [21], RWCS [28] approaches.

The first row shows the input images. The results in the second and third rows are obtained by ICOSEG and RWCS, respectively. The results in the fourth row are generated by our method. The ground truth masks are shown in the bottom row. Masks, there are still many regions of Medical intracerebral hemorrhage identifications cannot be correctly classified and segmented out by RWCS method. It is clear that our method produces high-quality co-segmentation results of foreground intracerebral hemorrhage, while the results by both ICOSEG and RWCS have more or less lost some important foreground regions. Our approach builds labeled region set instead of using foreground/background appearance model. That makes our method do not rely on the strong assumption that the foreground objects share a common appearance model. Therefore, our algorithm is more applicable and robust in realistic and complicated scenarios, which achieves more satisfying results using fewer scribbles.

Our approach is faster than the other two methods. We can observe that with an increase in the number of images in the group, the running time of our method only increases linearly. We further analyze the computational complexity of our algorithm.

The complexity of our likelihood estimation process is about $O(\sum_{i=1}^n [N(R_i)]^2)$, where $N(R_i)$

indicates the number of regions in R_i . Our higher-order energy function can be solved for each image individually and the higher-order cliques are optimized as a second-order function which can be solved by the conventional graph cut algorithm. Therefore, the complexity of higher-order co-segmentation step is about $O(\sum_{i=1}^n [N(I_i)]^2)$, where $N(I_i)$ is the number of pixels in I_i . The complexity of our full co-segmentation algorithm is about $O(\sum_{i=1}^n [N(I_i)]^2)$, since $N(I_i) \sim N(R_i)$. Therefore, the run-time of our method increases only linearly with additional images.

5. Conclusion

We have presented a novel interactive co-segmentation approach using the likelihood estimation and high-order energy optimization to extract the complicated foreground objects from a group of related images. A likelihood estimation method is developed to compute the prior knowledge for our higher-order co-segmentation energy function. Our higher-order cliques are built on a set of foreground and background regions obtained by likelihood estimation. Then our co-segmentation process from a group of images is performed at the region level through our higher-order cliques energy optimization. The energy function of our higher-order cliques can be further transformed into a second-order Boolean function and thus the traditional graph cuts method can be used to solve them exactly. The experimental results demonstrated both qualitatively and quantitatively that our method has achieved more accurate co-segmentation results than previous unsupervised and interactive co-segmentation methods, even though the foreground and background have many overlap regions in color distributions or in very complex scenes.

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