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# Impact of Homotopy Theory

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***Abstract:*** In this paper, by framework for the Homotopy Analysis Method (HAM), the blueprints was of some nonlinear. Arithmetical topology is a standout amongst the most vital manifestations in science which utilizes mathematical instruments to examine topological spaces. The HAM is a proficient and a clear systematic apparatus for taking care of nonlinear issues and does not require little parameters in the overseeing conditions and limit/beginning conditions.

The consequence of this examination introduces the utility and adequacy of HAM strategy.The essential objective is to discover arithmetical invariants that group topological spaces up to homeomorphism (however for the most part group up to homotopy comparability). The most imperative of these invariants are homotopy, homology, and cohomology gatherings. This subject is an exchange between topology and polynomial math and concentrates mathematical invariants gave by homotopy and homology hypotheses. Thetwentieth century saw its most noteworthy improvement.

In this article, another homotopy structure is exhibited for the numerical examination of finding the game-plan of a first-sort out in Partial Differential Equation (PDE)ux(x,y)+a(x,y)uy(x,y)+b(x,y) g(u) = f(x,y).

The Homotopy Analysis Method (HPM) and the isolating of a source most out of reach demonstrate are used together develop this new framework. The homotopy made in this structure relies upon the ruin of a source work. Specific debilitating of source limits incite to various homotopys. Using the way that the breaking down of a source work impacts the union of an answer drives us to change of another framework for the deteriorating of a source ability to resuscitate the meeting of an answer. The edification behind this examination is to show that building the best homotopy by separating f(x,y) verifiably picks the course of action with less computational work than using the present approach while giving quantitatively strong outcomes. Besides, this method can be summed up to all inhomogeneous PDE issues.

***Keywords*** *Homotopy Analysis Method (HAM), Partial differential equation, Parabolic-Hyperbolic.*

**INTRODUCTION**

The homotopy perturbation method was proposed by Ji-Huan He in 1999. In this strategy, the arrangement is considered as a summation of an unending arrangement, which generally meets quickly to the correct arrangement. Numerous scientists utilized this technique say, utilized this strategy to process Laplace change, comprehended utilitarian indispensable conditions by utilizing the homotopy bother technique, committed the homotopy bother strategy for illuminating exceptional sorts of direct fractional differential conditions with variable coefficients, utilized this technique to settle unique sorts of nonlinear integral equations.

Homotopy Theory, which is the principle part of logarithmic topology, considers topological
questions up to homotopy comparability. Homotopy identicalness is a weaker connection than topological identicalness, i.e., homotopy classes of spaces are bigger than homeomorphism classes.

Scientific demonstrating of genuine issues as a rule brings about practical conditions, for example, standard or incomplete differential conditions, basic and fundamental differential conditions and so on. The hypothesis of indispensable condition is one of the significant themes of connected science. In this paper another Homotopy Perturbation Method (HPM) is acquainted with get correct arrangements of the frameworks of essential conditions differential and is given cases to the precision of this strategy. This paper displays a prologue to new Mathematical demonstrating of genuine issues for the most part brings about utilitarian conditions, for example, standard or halfway differential conditions, fundamental and basic differential conditions and so forth. The hypothesis of basic condition is one of the significant points of connected arithmetic.

In this paper another Homotopy Perturbation Method (HPM) is acquainted with acquire correct arrangements of the frameworks of basic conditions differential and is given cases to the exactness of this strategy. This paper shows a prologue to new strategy for HPM, at that point presents the arrangement of essential - differential straight conditions and furthermore presents applications and writing. In second segment we will present arrangements of averaging basic - differential and a few techniques to understand this sort of accomplishment.

Homotopy type theory is another branch of arithmetic that joins parts of a few extraordinary fields in a very surprising way. Mainly it depends on an as of late found association between homotopy hypothesis what's more, sort hypothesis. Homotopy hypothesis is an outgrowth of mathematical topology and homological variable based math, with connections to higher class hypothesis; while sort hypothesis is a branch of numerical rationale and hypothetical software engineering. In spite of the fact that the associations between the two are right now the concentration of extreme examination, it is progressively evident that they are quite recently the start of a subject that will take additional time and all the more diligent work to completely get it. It addresses subjects as apparently removed as the homotopy gatherings of circles, the calculations for sort checking, what's more, the meaning of feeble ∞-groupoids.

Despite the fact that a definitive objective of topology is to group different classes of topological spaces up to a homeomorphism, in logarithmic topology, homotopy identicalness assumes a more essential part than homeomorphism, basically on the grounds that the fundamental devices of mathematical topology (homology and homotopy gatherings) are invariant concerning homotopy proportionality, and don't recognize topologically nonequivalent, however homotopic objects.

Non-linear fragmentary differential conditions are known to depict a wide assembling of wonders not simply in material science, where applications extend over magneto uid-dynamnics, water surface gravity waves, electromagnetic radiation reactions, and particle acoustic waves in plasma, just to give two or three blueprints, in addition in science and science, and a few distinct fields.

Not a lot of issues in material science, or doubtlessly in any branch of common science, can be enlightened direct hence, one regularly first investigation a flawless model, which is reflected however much as could sensibly be anticipated from the characteristic of the bonafide legitimate structure, as a fitting appraisal and after that handle diverse effects through some intense perturbative or conceivably non perturbative systems.

Perturbance Theory (PT) is comprehensively used to get some information about physical frameworks that can be authoritatively grasped yet contain little aggravations parameters while applying PT to such a structure, change around the inconvenience parameter is combined, and approximants are passed on as power game-plan of these parameters. On the other hand, non perturbative strategies have been delivered to explore physical issues which don't to some degree physical parameter to be used as the exacerbation parameter.

The non-parameter increment approach, the oupgraded trouble hypothesis (OPT) the Variational Disturbance Theory(VPT) and the straight δ-progression method (LDE), are regular nonperturbative frameworks, and have been made as fit contraptions in quantum field theory and in move physical settings in the midst of the past three decades.

These methods do reject disturbance strategy in forces of physical parameters, and the union of accumulated is controlled by some made parameters which don't exists in the essential issues. The reproduced parameters are settled toward the finish of figurings as showed by some perspective, for example, guideline of negligible affectability (PMS), which requires the approximants have immaterial reliance on these parameters over aggravation techniques. A champion among the most without a doubt comprehended non perturbative frameworks is homotopy analysis method (HAM), at first proposed a suitable legitimate method for seeing straight and nonlinear differential and basic conditions.

The HAM was adequately associated with deal with various nonlinear issues, for instance, nonlinear Riccati differential condition with partial demand, nonlinear Vakhnenko condition, the Glauert-stream issue, fragmentary KdV-Burgers-Kuramoto Equation, a summed up Hirota-Satsuma coupled KdV condition, nonlinear warmth trade, to shot development with the quadratic law, to confine layer stream of nano fluid past an expanding sheet, to the Poisson-Boltzmann state of semiconductor devices, solitary course of action of discrete MKdV condition, to the game plan of Fractional differential conditions, to the Oldroyd 6-reliable fluid with appealing field, MHD-stream of an Oldroyd 8-enduring fluid, to the nonlinear streams with slip restrain condition and so forth.

Nonlinear marvels play urgent govern in connected science. Express answers for the nonlinear conditions are of key significance. Different strategies for getting express answers for nonlinear conditions have been proposed. In this work, the homotopy irritation strategy is utilized for illuminating the underlying worth issues of extraordinary sorts of nonlinear first request Fredholm integro-differential conditions with a few illustrative cases. The last consequences of these cases acquired by methods for the homotopy irritation technique where contrasted and the outcomes got from the correct arrangements demonstrate that this technique gave successful outcomes.

**MOTIVIC HOMOTOPY THEORY:**

Logarithmic geometry and topology share a long history of association, crossfertilization furthermore, rivalry. The most recent stage includes the recently made field of motivic homotopy hypothesis. This can be thought of as a development of homotopy hypothesis to a setting that straightforwardly includes mathematical geometry, and has empowered the presentation of procedures of mathematical topology to issues in variable based math, number hypothesis and mathematical geometry.

We will talk about the sources of this improvement together with a gander at the current applications of the hypothesis. One imperative setting for homotopy hypothesis is the stable homotopy portrayal; this is the sensible universe in which the flow homotopy-scientist works. The key bounce forward instigating to the making of the subject of motivic homotopy hypothesis was the change by Morel-Voevodsky of new sorts of this portrayal, which brought the "standard" varieties from homotopy hypothesis together with responsibilities from arithmetical geometry.

Voevodsky's headway of the motivic stable homotopy portrayal empowered one shockingly to work with the imperative material of logical geometry, plans of polynomial conditions, with the adaptability and power authoritatively just accessible in homotopy theory.

The motivic hypothesis has been a recognizable achievement. The Fields Medalist Voevodsky utilized these homotopical systems as a bit of his confirmation of the watched Milnor figure, and motivic homotopy theory had essentially more focal impact in his devotion to the check of the Bloch-Kato figure, the beneficial pinnacle of thirty years of elevated research.

Other than these totally staggering applications, the way that one could now utilize the insights and procedures for homotopy hypothesis to manage issues in arithmetical geometry has pulled in mathematicians from the two fields and has actuated to a wealth of new enhancements and applications, for example, game-plan works out as expected for logarithmic vector groups.

Motivic homotopy hypothesis has been beneficial for the regular homotopy scientists also.

The late work of Hill, Hopkins and Ravenel on the Kervaire invariant one figure, settling one of the real open issues in stable homotopy hypothesis, utilized as a piece of an earnest way the "cut filtration" in equivariant stable homotopy hypothesis, which in this manner was moved by Voevodsky's cut filtration in motivic stable homotopy hypothesis[3].

## HOMOTOPY ANALYSIS METHOD

The homotopy analysis method is produced in 1992 by Liao. It is a diagnostic way to deal with get the arrangement arrangement of direct and nonline arpartial differential equations. The distinction with the other annoyance methods is that this method is free of little/substantial physical parameters. It additionally gives a straightforward approach to guarantee the union of arrangement. This method has been effectively connected to understand numerous direct and non straight partial differential equations in different fields of science and building by many creators. The homotopy analysis method is valuable and effective for acquiring both logical and numerical approximations of direct or nonlinear differential equations.

With a particular extraordinary objective to show the key considered HAM, consider the running with differential condition

$$N\,[u(x)] = 0;$$

Where is a nonlinear executive, x and t prescribes the self-ruling factors and u is an abscure limit dull farthest point. For straightforwardness, we disregard all most unprecedented or starting conditions, which can be overseen in the vague way? By system for the HAM, we initially shape the gathered zeroth demand turning condition Where $q \in [0; 1]$ is the introducing parameter, is a partner parameter, L is a collaborator coordinate executive, H(x; q) is a dark limit, u0(x; t) is a basic supposition of and H(x; t) implies a nonzero aide work. Plainly when the introducing parameter q = 0 and q = 1, condition gets the chance to be exclusively. Thusly as q increases from 0 to 1, the course of action vacillates from the fundamental supposition u0(x; t) to the game plan u(x; t). Developing (x; t; q) in Taylor course of action concerning q, one has.

It ought to be featured that um(x; t) for $m \geq 1$ is overseen by the straight condition with coordinate explanation behind limitation conditions that comes layout the essential issue, which can be understood by the major figuring programming, for example, Mathematical or Maple. For the meeting of the above method we suggest the peruser to Liao.

In the event that condition yields one of a kind game-plan, at that point this framework will pass on the astonishing course of action. On the off chance that condition does not aggregate an emerge strategy, the HAM will give an answer among different other conceivable courses of action.

**DIFFERENTIAL HOMOTOPY THEORY AND ARAKELOV THEORY**

Differential homotopy speculation relies upon refining the impediment to manifolds of conventional homotopy invariants of spaces by solidifying additional structures, for instance, differential structures or affiliations. In one course, this approach appears in arithmetical geometry through Arakelov theory, while document speculation outlines another basic bearing. Starting late this speculation has gone up against a "motivic" character, in that things are produced as presheaves on various groupings of smooth manifolds, for the most part as the motivic theory relies upon presheaves on the class of smooth designs.

Arakelov presented a segment at vastness in number-crunching contemplations, subsequently offering ascend to worldwide hypotheses like those of the hypothesis of surfaces, yet in a math setting over the ring of whole numbers of a number field.

Arakelov geometry is a method for examining diophantine issues from a geometrical point of view. To put it plainly, given a diophantine issue, one considers a number juggling plan related with that issue, and includes the unpredictable purposes of that plan by method for "compactification". Next, one supplies every single number-crunching group on the plan with an extra structure over the complex numbers, which means one blesses them with certain hermitian measurements.

To begin a prologue to Arakelov geometry is to consider the least difficult sort of number-crunching scheme conceivable, in particular the spectrum of a ring of whole numbers in a number field, for example Spec(Z). In the nineteenth century, a few creators, as Kummer, Kronecker, Dedekind and Weber, attracted regard for the noteworthy analogy that one has between the properties of rings of whole numbers in a number field, from one viewpoint, and the properties of organize rings of relative non-particular bends on the other. Specifically, they began the parallel development of a hypothesis of "spots" or "prime divisors" on the two sides of the analogy.

Most vital, ethically, was however that the achievement of this hypothesis enabled mathematicians to see that number hypothesis from one perspective, and geometry on the other, are bound together by a greater picture. Thusly of intuition kept on being worried in the twentieth century, most remarkably by Weil, and any reasonable person would agree that the later development of the idea of a scheme by Grothendieck is specifically identified with these early thoughts.

It is outstanding from conventional topology or geometry that compactifying a space frequently presents a helpful structure to it, which makes an investigation of it less demanding by and large. Similar holds for our situation: by presenting an extra Arakelov structure to a given number juggling circumstance one winds up with a helpful set-up to detail, think about and even demonstrate diophantine properties of the first circumstance. For example one could consider questions managing the span of the answers for a given diophantine issue.

Fermat's method of plunge can maybe be seen as a model of Arakelov geometry on number arthemetics schemes.

From the two viewpoints, the exchanging of contemplations and methods between motivic homotopy speculation and differential homotopy is both standard and appealing [4].

Traditional differential cohomology has been displayed by Cheeger and Simons as a refinement of ordinary cohomology with important coefficients, and fills in as a goal for refined trademark classes and trademark outlines. This approach relies upon an express delineation of the essential social occasions by cycles and relations. Using a more homotopy theoretic approach, Hopkins and Singer exhibit how one can refine a summed up cohomology speculation, for instance, K-theory, to a differential one.

Relationship with material science are discussed and a uniqueness theory for differential cohomology theories is proficient. In a general setup using boundlessness classes is created to describe differential extensions as a heap of spectra on the arrangement of smooth manifolds and is associated with the advancement of differential arithmetical K-speculation of number rings. Theories to standard designs over the entire numbers are given.

Work of Holmstrom-Scholbach uses parts of motivic homotopy theory to create Arakelov motivic cohomology and Arakelov K-speculation; a relative approach is used by Hopkins-Quick as a piece of their improvement of "Deligne"- cobordism[5].

**NEW HOMOTOPY PERTURBATION METHOD**

To delineate the fundamental musings of the homotopy analysis method, we consider the running with nonlinear differential condition:

$$A(u)-f(r)=0, r\in\Omega$$

with the boundary conditions

$$B(u,) = 0,\ r \in \Gamma,$$

Where A = General different Operator

B= Limit Operator

F(r)= Analytical function, and Γ is also known as the boundary of domain Ω . [6]

Th Operator A in this condition (1) can be fixed up when all is said in done of L and N, where L and N are prompt and nonlinear parts of A, freely, as takes after:

$$L(u)+N(u)-f(r) = 0.$$

By this equation of the homotopy method, we build the homotopy

$$H(v, p)=(1-p)(L(v)-L(u_0))+p(A(v)-f(r))=0,(3)$$

which is equivalent to

$$H(v, p)=L(v)-L(u_0)+pL(u_0)+p(N(v)-f(r))=0,(4)$$

Where p is an embeddings parameter, u0 is a focal estimation of (1), which satisfies the most remote point conditions. As p changes from zero to solidarity, v(r,p) changes from u0 to u(r). In this procedure, the union of an answer depends on upon the choice of u0, that is, we can have unmistakable evaluated answers for different u0.

Give us a chance to disintegrate the source work as $f(r) = f_1(r)+f_2(r)$. In the event that we take $L(u_0)=f_1(r)$in (3), we get the accompanying homotopy:

$$H(v,p)=(1-p)(L(v)-f_1(r))+p(A(v)-f(r))=0,(5)$$

which is equivalent to

$$H(v, p)=L(v)-f_1(r)+p(N(v)-f_2(r))=0 \qquad (6)$$

Obviously, from (6) we have

$$H(v, 0) = L(v)-f_1(r) = 0,$$

$$H(v, 1) = A(v)-f(r) = 0,$$

As the embedding parameter $p$ changes from zero to unity, $v(r, p)$ changes from $L^{-1}(f_1(r))$ to $u(r)$.

As showed by He's homotopy annoyance method, we would first be able to use the displaying presenting parameter p as a little parameter and perceive that the strategy of (6) can be made as a power approach in p as takes after:

$$V = v_0+pv_1+p^2v_2+p^3v_3+\cdots.$$

Setting p=1, we get the approximate arrangement of (1)

$$U==v_0+v_1+v_2+v_3+\cdots.$$

In the running with part in the segment, it is shown that the arrangement of the right-hand side additionally called a point of confinement basically impacts the measure of figuring and the speed of meeting of the game-plan [8].

**Improvement of another homotopy in light of the decay of a source work**

Enable us to consider the running with limit respect issue with the going with in homogeneous PDE:

$$U_x(x,y) + a(x,y)u_y(x,y) + b(x,y)g(u) = f(x,y)(8)$$

$$u(0, y) = h(y), \qquad (9)$$

where a, b, g and f are unfaltering limits in some district of the plane and g(0)=0.

By managing this most distant point respect issue by the homotopy disturbance method, we get a, we get an off base or right course of action u(x,y). Before continuing with further, let us basic administrator S described in the running with shape:

At that point the subsidiary of operator S as for y is characterized asCreating f(x,y) as a total of two points of confinement $f(x,y)=f_1(x,y)+f_2(x,y)$ and after that building a homotopy in condition, we have

$$H(v,p)=(1-p)(v_x-f_1)+p(v_x+a(x,y)vy+b(x,y)g(v)-f),$$

which is equivalent to

$$H(v,p)=v_x(x,y)-f_1(x,y)+p(a(x,y)vy(x,y)+b(x,y)g(v(x,y))-f_2(x,y))=0. \qquad (12)$$

Substituting (7) into (12), and looking coefficient of the terms by a relative power in p, we have

If at all there would be a relationship which exists then

$$a(x,y)[S_y(f_1(x,y))+h_y(y)]+b(x,y)g(S(f_1(x,y))+h(y))=f_2(x,y) \qquad (14)$$

Between $f_1(x,y)$and $f_2(x,y)$, then we have from (13)

$$(v_1)x=0,v_1(0,y)=0\Rightarrow v_1=0$$

and

At that point we can consider that the estimation or likewise called as right plan of issue may be (8)- (9) which can be named as$u(x,y)=v_0(x,y)$, the main source capacity which is obtained from the previous equation in the same structure.

$$f(x,y)=f_1+a(x,y)[S_y(f_1)+h_y(y)]+b(x,y)g(S(f_1)+h(y)) \quad (15)$$

Thusly condition is a principal condition to breath life into the combining. In the event that condition has a game plan, at that point we pick up the evaluated or right strategy of issue in two stages [9].

Regardless, it isn't all around conceivable to separate the source work f(x,y) in a way that the limits f1(x,y) and f2(x,y) have the relationship.

On the off chance that we have such a case, at that point we are chasing down a self-convincing P(x,y) with the true blue concentrate on that the reasons for suppression f1(x,y) and f2(x,y) have the running with relationship:

$$a(x,y)[S_y(f_1)+h_y(y)]+b(x,y)g(S(f_1)+h(y))=f_2+P(x,y) \quad (16)$$

For this condition, we get the unforgiving or right approach of the issue in more than two stages. What's more, we can get the diagram as system.

**A Detailed Examination of the new homotopy perturbation method for linear problems**

In this area we address the hypothesis of the new homotopy inconvenience system given for some wonderful f(x,y)in the running with straight issue:

$U_x(x, y) + a\, u_y(x, y) + bu(x, y) = f(x, y)$ $u(0, y) = h(y)$, where $a$ and $b$ are constants.

**Case 1: the source work f(x,y) is a polynomial**

We consider that h(y) = 0 and f(x,y) is a n th-organize polynomial in issue (17)- (18). From this time forward f(x,y) can be framed in the running with structure:

Where Aαβ are constants and α, β are standard numbers. In case the polynomial f(x,y) is decayed as

$$f(x,y)=f_1(x,y)+aS_y(f_1(x,y))+bS(f_1(x,y))$$

from (15), we can get the arrangement of the issue in two stages. We take f1(x,y) as a n th-coordinate polynomial given as [10]

*where* $c_{\alpha\beta}$ *are constants. Then* $S(f_1(x,y))$ *becomes*and $S_y(f_1(x,y))$ becomes

Keeping in mind the end goal to decide the obscure coefficients cαβin terms of Aαβ, we substitute:

$f(x,y)=f_1(x,y)+aS_y(f_1(x,y)$
$+bS(f_1(x,y))$

With this homotopy which is constructed by the result:

$H(v,p)=(1-p)(v_x-f_1(x,y))+p(v_x+av_y+b_v-f(x,y))=0$

leads us to reaching the solution in two steps [11].

### 1.1.1. Problem 1:

Consider the inhomogeneous straight boundary value problem (BVP) with steady coefficients

$$u_x-u_y+u=e_y+e_x,u(0,y)=e_y \qquad (32)$$

The homotopy given below is constructed:

$$H(v,p)=(1-p)(v_x-r_1(x)-t_1(y))+p(v_x-v_y+v-e_x-e_y)=0, \qquad (33)$$

which is equivalent to

$$H(v,p) = v_x-r_1(x)-t_1(y)+p(-v_y+v+r_1(x)+t_1(y)-e_x-e_y)=0, \qquad (34)$$

we get the capacities $r_1(x)$, $t_1(y)$ as it is cleared up. Plainly $t_1(y)=e_y$ satisfies condition. Also, from condition, $r_1(x)$ can be found as takes after: then we get the given below equation

Hence, the solution $u(x,y)$

$$u(x,y)=v_0\frac{e^x+e^{-x}}{2}+e^y x+e^y$$

The answer we get out of the problem which will be with an minimum amount of calculation.

### Problem 2:

From the given below equation that is the inhomogenous linear BVP with coefficients of variables.

$$u(0, y) = 0;$$

In the given below equation which is on the right hand capacity f(x,y) the polynomial is a third arranged. It is given as

$$f(x,y)=f_1(x,y)+yS_y(f_1(x,y))-S(f_1(x,y))$$

From the main function $f(x,y)=2y^2+2xy^2$, if we take $f_1(x,y)$ as

$$f_1(x,y)=a_1x^3+a_2x^2y+a_3xy^2+a_4y^3+a_5x^2+a_6xy+a_7y^2+a_8x+a_9y+a_0 \qquad (40)$$

and substitute (40) into (39), we obtain

$$a_7a_1=2,a_2=a_3=a_4=a_5=a_6=a_8=a_9=a_0=0.$$

The outcome we get is given as $f_1(x,y)=2y^2$, The Homotopy is given in the format that is $f_2(x,y)$

$=2xy^2$
$H(v,p)=v_x-2y^2+p(yv_y-v-2xy^2)=0$,(41)

and the result is :

Finally we get the corrected equation in this format such as with the help of Calculation of short-length:

$(x,y) = v_0 = 2xy^2$ (42)

For showing ability and validity the proposed method we compare it with homotopy perturbation method to solve non-linear problem that used

**Application of (NHPM) and comparison with (HPM)**

$$\begin{cases} u'' + \omega^2 + 4q^2u^2u'' + 4q^2uu'^2 = 0, t \in \Omega \\ u(0) = A, u'(0) = 0 \end{cases}$$

(43)

Where $\omega$ and $q$ are known constants, also linear operator and non-linear operator chose as,

$$\begin{cases} L(u) = u'' + \omega^2 u, \\ N(u) = u^2u'' + uu'^2 \end{cases} \quad (44)$$

Where $v_0(t) = ac0s\alpha\omega t$ and $v_1$ problem has been given as follows:

$$\begin{cases} v_1'' + \omega^2 v_1 = (\alpha^2 + 2q^2A^2\alpha^2 - 1)\omega^2 A\cos\alpha\omega t + 2q^2\alpha^2\omega^2A^3\cos 3\alpha\omega t \\ v_1(0) = 0, v_1'(0) = 0 \end{cases}$$

(45)

In, problem (7) was solved by using variational iteration method and for obtaining $\alpha$ they chose $\alpha^2 + 2q^2A^2\alpha^2 - 1 = 0$ and $\alpha$ is given by,

$$\alpha = \frac{1}{\sqrt{1 + 2q^2A^2}} \quad (46)$$

But in the proposed method for finding $\alpha$ we use $v_1$ Galerkin method and to solve problem we employ analytic method because variational iteration method is an approximate method and analytic technique is given exact solution.

Also in the Galerkin method we obtain parameter $\alpha$ exactly, in generally NHPM convert non-linear problem to some easier linear problems in compare to HPM. In, by using Galerkin method for n=2 , is equal to (8), but in this article we use NHPM according to (4) and we have,

$$+\omega^2(1-2q^2A^2\alpha^2-6q^2\alpha^2A^2\cos 2\alpha\omega t)v_1$$

$$\left(1+2q^2A^2+2q^2A^2\cos 2\alpha\omega t\right)v_1^{"}-(4q^2A^2\alpha\omega\sin 2\alpha\omega t)v_1^{'}$$

$$=\left(\alpha^2-1+2q^2A^2\right)\omega^2A\cos\alpha\omega t+(2q^2\alpha^2\omega^2A^3)\cos 3\alpha\omega t$$

In the case of N=3 in Galerkin method we obtain the following form:

$$\alpha=\frac{1}{\sqrt{5-9q^2A^2-\sqrt{16-56q^2A^2+53q^4A^4}}}$$

(46)

## 3. Discussion of parameter

Problem (5) has a periodic solution with exact period $T$ and $T_0=\dfrac{2\pi}{\omega}$ that $\omega$ is constant and also

$$\frac{T}{T_0}=\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{1+4q^2A^2\cos^2\Phi}\,d\Phi \qquad (47)$$

In, first and second order perturbation method was used and $\dfrac{T}{T_0}$ was given as follows:

first order : $\dfrac{T_1}{T_0}=1+q^2A^2$ (48)

second order : : $\dfrac{T_2}{T_0}=1+q^2A^2-q^4A^4$, (49)

Relations of (12-13) are valid only for $((qA)^2\le 0.1(see[8]),)$, but (10) is valid for any values $(qA)^2$ . Also we have 2

$$Lim\frac{T_{exact}}{T_{(He)}}=Lim\frac{\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{1+4q^2A^2\cos^2\Phi}\,d\Phi}{\sqrt{1+2q^2A^2}}\approx 0.90,$$

$(qA)^2\to\infty$

And

$$Lim\frac{T_{exact}}{T_{(NHPM)}}=\frac{\frac{2}{\pi}\int_0^{\frac{\pi}{2}}\sqrt{1+4q^2A^2\cos^2\Phi}\,d\Phi}{\sqrt{5-9q^2A^2-\sqrt{16+56q^2A^2+53q^4A^4}}}\approx 0.97$$

$(qA)^2\to\infty$

Also, for any value of $(qA)^2$ , relative error is,

$$0\le\left|\frac{T_{exact}-T_{He}}{T_{He}}\right|\le\%10$$

but for proposed method according (10) is,

$$0\le\left|\frac{T_{exact}-T_{(NHPM)}}{T_{(NHPM)}}\right|\le\%3$$

**CONCLUSION:**

In this paper straight and nonlinear dissemination conditions are fathomed by utilizing Homotopy Perturbation Method. Diagnostic arrangement acquired by this technique is attractive same as the correct outcomes to these models.In this paper, we utilize another homotopy bother technique to get the methodology of a first-make inhomogeneous PDE. In this framework, each rot of the source work f(x,y) prompts to another homotopy. Regardless, we build up a system to get the best rot of f(x, y) which gifts us to get the methodology with scarcest estimation and enable the meeting of the approach. This examination shows that the demolish of the source work in a general sense impacts the measure of estimations and the connecting with of the meeting of the strategy.

Veering from the standard one, breaking separated the source work f(x, y) genuinely is a direct and to a mind boggling degree affecting instrument for figuring the colossal position or adversarial plots with less computational work. Extraordinary in relationship with all other correct systems, it gives us a sensible way to deal with oversee alter and control the meeting zone clearly of movement by picking fitting estimations of accomplice parameter h, aide work H(t) and right hand facilitate controller L.

Furthermore we showed that homotopy inconvenience procedure is the eminent occasion of homotopy examination system. here are some basic concentrations to make here. In any case, we have extraordinary adaptability to pick the aide parameter h, associate limit H(t) and right hand straight director L and the fundamental guesses.

Second the HAM was gave off an impression of being clear, yet able investigative numeric arrangement for handling distinctive nonlinear issues. A new homotopy method with Galerkin method for obtaining parameter such that it has a high accuracy in comparison with similar work such that HPM.

Homotopy Perturbation Method is effective technique to tackle direct nonlinear issues and gives rapidly focalized approximations that prompt correct arrangement. Homotopy bother technique tackled nonlinear issues straightforwardly without linearizing the issue.

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