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Propagation of SH-Waves through an irregular surface due to an infinite rigid plane boundary

Sanehaa
Assistant Professor, Department of Physics, Tika Ram PG college Sonipat, Haryana (India)
Email: sanehaadahiya@gmail.com

Abstract

The problem of Scattering of Love waves due to an infinite rigid plane boundary is studied. The infinite rigid plane boundary is present in the homogeneous, isotropic and slightly dissipative surface layer. The Scattered & reflected waves are obtained by Wiener – Hopf technique and Fourier transformations. Numerical computation has been done and conclusion has been drawn from the graphs of amplitudes versus wave number of the scattered Love waves. The amplitude of the scattered and the reflected waves has been plotted against the wave number. The amplitude of the scattered waves falls off very rapidly as the wave number increases slowly. The scattered waves behave as decaying cylindrical waves at distant points. The amplitude of the reflected Love wave decreases rapidly with the wave number and ultimately becomes saturated. The amplitude of the scattered waves decreases rapidly with the increase in wave number and then it decreases at slower rate and ultimately becomes saturated which shows that love waves take a long time to dissipate and go on moving around earth surface for a long time.

Keywords: Fourier transforms, free surface, reflection, Shear waves, rigid plane boundary, Wiener – Hopf technique.

Introduction

During earthquake, Seismic Surface waves appear on the earth surface. The horizontal shifting of earth is caused by surface seismic Love waves. Love waves are the fastest surface wave that moves the ground from side-to-side. Confined to the surface of the crust, Love waves produce entirely horizontal motion. The motion of particles of Love waves forms a horizontal line perpendicular to the direction of Propagation. During the earthquakes seismic waves such as Love waves are generated from the interior of earth. Love waves are almost entirely responsible for the damage and destruction associated with earthquakes. They are transmitted and reflected due to the presence of irregularities like mountains, rocks, surface impedance etc. in the earth crust. The scattering of Love waves due to discontinuities present in the earth's crust results in large amplification of these waves. Many authors have discussed the propagation of Love waves and different authors assumed different forms of irregularities at the interface.

This problem is based on a paper by Sato [12] who studied the problem of reflection and transmission of Love waves at a vertical discontinuity in a surface layer. Mal [9] used perturbation technique to study the propagation of Love waves in a layer of non uniform thickness lying over a half-space. Asghar and Zaman [1] have studied the diffraction of Love waves by a finite rigid barrier in layer overlying a half-space. The transmitted waves are calculated analytically and the caes of the infinite rigid barrier is obtained as a special case of this problem. Deswal and Mudgal [5] studied the problem of scattering of love waves due to the presence of a rigid barrier of finite depth in the crustal layer of the earth. Jing F. et al. [7] have studied propagation behavior of Love waves in a piezoelectric layered structure with inhomogeneous initial stress. Tomar & Kaur [14] studied a problem of reflection and transmission of a plane SH-wave at a corrugate interface between a dry sandy half space and an anisotropic elastic half space. They used the Rayleigh's method of approximation for studying the problem. Chattopadhay et al. [2] have studied SH waves propagation in a monoclinic layer over a semi-infinite elastic medium with irregularity. Chattopadhay et al. [3] also studied the problem of propagation of shear waves in a monoclinic layer with irregularity lying between two isotropic semi-infinite elastic medium. Singh [13] discussed the propagation of Love wave at a layer medium bounded by irregular boundary surface. Liang et al. [8] has been studied the propagation of Love waves in a piezoelectric material structure with a mass layer. The effect of the thickness of the mass layer on the properties of the Love waves has been discussed in details.

In this paper, Scattering of Love waves due to an irregular surface layer in the presence of infinite rigid boundary has been discussed. The irregularity is in form of a finite rigid plane boundary in the surface layer $-H \le z \le 0$ overlying a half space z ≥ 0. The reflected, transmitted and scattered waves have been obtained by Fourier transformations [11] and Wiener-Hopf technique [10]. The numerical computation has been done by taking the barriers of different sizes.

Problem Formulation

The problem is two dimensional and plane of propagation is the zx-plane. The x-axis lies along the interface and z-axis has been taken vertically downwards. A solid layer of thickness H ($-H \le z \le 0,\ -\infty < x < \infty$) lies over a solid half space ($z \ge 0,\ -\infty < x < \infty$). The semi-infinite rigid plane boundary is taken along the interface $z = -h,\ x \ge 0$ in a surface layer. The Velocities and rigidities of the shear waves in the crustal layer and solid half space are U_2, α_2 and U_1, α_1 respectively .The geometry of the problem is shown in figure 1.

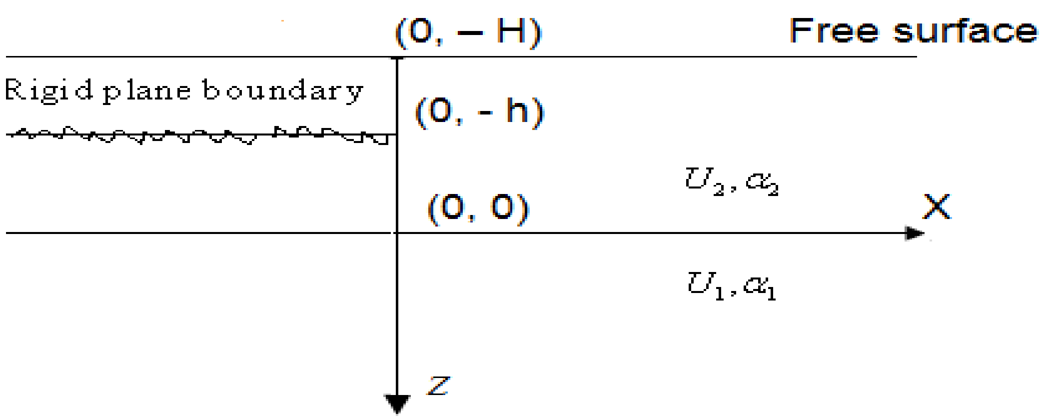

Figure1: Geometry of the problem

The incident Love wave is represented by

$$v_{0,1} = A\cos\varphi_{2N} He^{-(\varphi_{1N}z + ik_{1N}x)}, \quad z \ge 0, \tag{1}$$

$$v_{0,2} = A\cos\varphi_{2N}(z+H)e^{-ik_{1N}x}, \quad -H \le z \le 0, \tag{2}$$

where,

$$\varphi_{1N} = \sqrt{k_{1N}^2 - k_1^2}\ ,\ \varphi_{2N} = \sqrt{k_2^2 - k_{1N}^2},\quad ,\ |k_1| < |k_{1N}| < |k_2|$$

and k_{1N} is a root of the equation

$$\tan \varphi_{2N} H = \mu \frac{\varphi_{1N}}{\varphi_{2N}}, \quad \mu = \frac{\alpha_1}{\alpha_2}, \tag{3}$$

In two dimensions, the wave equation is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} + \frac{\varepsilon}{c^2}\frac{\partial \phi}{\partial t}, \tag{4}$$

where c is the velocity of propagation and $\varepsilon > 0$ is the damping constant . If the displacement be harmonic in time, then

$$\phi(x,z,t) = v(x,z)e^{-i\omega t} \tag{5}$$

and equation (4) reduce to

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + k^2 v = 0. \tag{6}$$

The wave equation (6) in the present study can be written as

$$(\nabla^2 + k_j^2)\, v_j = 0, \quad j = 1,2 \tag{7}$$

where $k_j = \dfrac{(\omega^2 + i\varepsilon\omega)^{1/2}}{U_j} = k_j^{'} + ik_j^{"}$

The total displacement is given by

$$v = v_{0,1} + v_1, \qquad z \ge 0 \quad , \quad -\infty < x < \infty, \tag{8}$$

$$= v_{0,2} + v_2, \quad -h \le z \le 0 \quad , \quad -\infty < x < \infty, \tag{9}$$

$$= v_{0,2} + v_3, \quad -H \le z \le -h \quad , \quad x \ge 0. \tag{10}$$

The boundary conditions are

(i) $$\frac{\partial v_3}{\partial z}=0, \qquad x\geq 0, \quad z=-H, \tag{11}$$

(ii) $$v_{0,2}+v_2=0, \qquad z=-h, \quad x\leq 0, \tag{12}$$

(iii) $$v_{0,2}+v_3=0, \quad z=-h, \quad x\leq 0, \tag{13}$$

(iv) $$v_2=v_3, \quad \frac{\partial v_2}{\partial z}=\frac{\partial v_3}{\partial z}, \quad z=-h, \quad x\geq 0, \tag{14}$$

(v) $$v_1=v_2, \ \alpha_1\frac{\partial v_1}{\partial z}=\alpha_2\frac{\partial v_2}{\partial z}, \ z=0, \quad -\infty<x<\infty. \tag{15}$$

The boundary conditions (12) and (13) implies that there is no displacement across the rigid plane boundary. Using equation (12), (13) in (2), we write

$$v_2=v_3=-A\cos\varphi_{2N}(z+H)e^{-ik_{1N}x}, \quad z=-h, \quad x\leq 0, \tag{16}$$

Solution of the Problem

Taking Fourier transform of equation (7), we obtain

$$\int_{-\infty}^{\infty}\left(\frac{\partial^2 v_j}{\partial x^2}+\frac{\partial^2 v_j}{\partial z^2}+k_j^2 v_j\right)e^{ipx}dx=0, \quad j=1,2$$

$$\frac{d^2\bar{v}_j(p,z)}{dz^2}-\varphi_j^2\bar{v}_j(p,z)=0, \tag{17}$$

where $\bar{v}_j(p,z)$ represents Fourier transform of $v_j(x,z)$ defined as

$$\bar{v}_j(p,z)=\int_{-\infty}^{\infty}v_j(x,z)e^{ipx}dx, \quad p=\alpha+i\beta$$

$$=\int_{-\infty}^{0}v_j(x,z)e^{ipx}dx+\int_{0}^{\infty}v_j(x,z)e^{ipx}dx$$

$$=\overline{v_{j-}}(x,z)+\overline{v_{j+}}(x,z)$$

and $\varphi_j=\pm\sqrt{p^2-k_j^2}$.

If for a given z, as $|x| \to \infty$ and $M, \tau > 0$, $|v_j(x,z)| \sim Me^{-\tau|x|}$, then $\bar{v}_{j+}(p,z)$ is analytic in $\beta > -\tau$ and $\bar{v}_{j-}(p,z)$ is analytic in $\beta < \tau\ (= lm(k_j))$. By analytic continuation, $\bar{v}_j\ (p,z)$ and its derivatives are analytic in the strip $-\tau < \beta < \tau$ in the complex p - plane[4]. Choosing the sign of φ_j such that its real part is always positive for all p, we find the solutions to equation (17)

$$\bar{v}_1\ (p,z) = E(p)e^{-\varphi_1 z}, \quad z \ge 0, \tag{18}$$

$$\bar{v}_2(p,z) = F(p)e^{-\varphi_2 z} + G(p)e^{\varphi_2 z}, \quad -H \le z \le 0. \tag{19}$$

Using condition (15) in equations (18) and (19), we find

$$F(p) = \frac{E(p)}{2\varphi_2}(\varphi_2 + \mu\varphi_1) \text{ and } G(p) = \frac{E(p)}{2\varphi_2}(\varphi_2 - \mu\varphi_1) \tag{20}$$

Now, eliminating $F(p)$ *and* $G(p)$ from (19), we get

$$\bar{v}_2(p,z) = E(p)\frac{[\varphi_2 \cosh\varphi_2 z - \mu\varphi_1 \sinh\varphi_2 z]}{\varphi_2} \tag{21}$$

Differentiating equation (21) with respect to z, we obtain

$$\bar{v}_2'(p,z) = E(p)[\varphi_2 \sinh\varphi_2 z - \mu\varphi_1 \cosh\varphi_2 z] \tag{22}$$

Putting $z = -h$ and denoting $\bar{v}_2'(p,-h)$ by $\bar{v}_2'(p)$ etc., we obtain

$$\bar{v}_2'(p) = -A(p)[\varphi_2 \sinh\varphi_2 z + \mu\varphi_1 \cosh\varphi_2 z] \tag{23}$$

Now, eliminating E(p) from (21) and (23), we find

$$\bar{v}_2\ (p,z) = -\frac{\varphi_2 \cosh\varphi_2 z - \mu\varphi_1 \sinh\varphi_2 z}{\varphi_2(\varphi_2 \sinh\varphi_2 h + \mu\varphi_1 \cosh\varphi_2 h)}\bar{v}_2'(p) \tag{24}$$

Putting $z = -h$ in (24) and denoting $\bar{v}_2\ (p,-h)$ by $\bar{v}_2\ (p)$, we obtain

$$\bar{v}_2\ (p) = [\bar{v}_{2+}(p) + \bar{v}_{2-}(p)] = -\frac{\varphi_2 \cosh\varphi_2 h + \mu\varphi_1 \sinh\varphi_2 h}{\varphi_2(\varphi_2 \sinh\varphi_2 h + \mu\varphi_1 \cosh\varphi_2 h)}$$

$$\times[\bar{v}_{2+}'(p)+\bar{v}_{2-}'(p)]. \qquad (25)$$

Now, multiplying equation (7) by e^{ipx} and integrating from 0 to ∞ $(j=3)$, we find

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(p,z)]-\varphi_2^2\,\bar{v}_{3+}(p,z)=\left(\frac{\partial v_3}{\partial x}\right)_{x=0}-ip(v_3)_{x=0}\ . \qquad (26)$$

It is obvious that $\varphi_2=\varphi_3$.

Changing p to - p in equation (26), we get

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(-p,z)]-\varphi_2^2\,\bar{v}_{3+}(-p,z)=\left(\frac{\partial v_3}{\partial x}\right)_{x=0}+ip(v_3)_{x=0}\,. \qquad (27)$$

Subtracting equation (27) from (26), we obtain

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(p,z)-\bar{v}_{3+}(-p,z)]-\varphi_2^2[\bar{v}_{3+}(p,z)-\bar{v}_{3+}(-p,z)]$$

$$=-2ip(v_3)_{x=0}. \qquad (28)$$

Using equation (16) in equation (28), we find

$$\frac{d^2}{dz^2}[\bar{v}_{3+}(p,z)-\bar{v}_{3+}(-p,z)]-\varphi_2^2[\bar{v}_{3+}(p,z)-\bar{v}_{3+}(-p,z)]$$

$$=2ipA\cos\varphi_{2N}(z+H). \qquad (29)$$

The solution of equation (29) is given as

$$\bar{v}_{3+}(p,z)-\bar{v}_{3+}(-p,z)=J_1(p)e^{-\varphi_2 z}+J_2(p)e^{\varphi_2 z}$$

$$-\frac{2ipA\cos\varphi_{2N}(z+H)}{p^2-k_{1N}^2}. \qquad (30)$$

Using boundary condition (11) in equation (30), we find

$$J_1(p) = \frac{e^{-\varphi_2 H}}{e^{\varphi_2 H}} J_2(p)$$

Then Equation (30) becomes

$$\bar{v}_{3+}(p,z) - \bar{v}_{3+}(-p,z) = J(p)\cosh\varphi_2(z+H) - \frac{2ipA\cos\varphi_{2N}(z+H)}{p^2 - k_{1N}^2}. \qquad (31)$$

where $J(p) = 2J_2(p)e^{-\varphi_2 H}$.

Differentiating equation (31) with respect to z, we have

$$\bar{v}'_{3+}(p,z) - \bar{v}'_{3+}(-p,z) = J(p)\varphi_2 \sinh\varphi_2(z+H) + \frac{2ipA\varphi_{2N}\sin\varphi_{2N}(z+H)}{p^2 - k_{1N}^2} \qquad (32)$$

Putting $z = -h$ in equations (31) and (32) and denoting $\bar{v}'_{3+}(p,-h)$ by $\bar{v}'_{3+}(p)$ etc., we write

$$\bar{v}_{3+}(p) - \bar{v}_{3+}(-p) = J(p)\cosh\varphi_2\delta - \frac{2ipA\cos\varphi_{2N}\delta}{p^2 - k_{1N}^2}. \qquad (33)$$

and

$$\bar{v}'_{3+}(p) - \bar{v}'_{3+}(-p) = J(p)\,\varphi_2\sinh\varphi_2\delta + \frac{2ipA\varphi_{2N}\sin\varphi_{2N}\delta}{p^2 - k_{1N}^2}. \qquad (34)$$

where $\delta = H - h$.

Now, after eliminating J(p) between (33) and (34), equation (34) is written as

$$\bar{v}_{3+}(p) - \bar{v}_{3+}(-p) = \frac{\coth\varphi_2\delta}{\varphi_2}\left[\bar{v}'_{3+}(p) - \bar{v}'_{3+}(-p) - \frac{2ipA\varphi_{2N}\sin\varphi_{2N}\delta}{p^2 - k_{1N}^2}\right]$$

$$-\frac{2ipA\cos\varphi_{2N}\delta}{p^2-k_{1N}^2} \qquad (35)$$

Taking Fourier transformation of (14), we obtain

$$\bar{v}_{2+}(p)=\bar{v}_{3+}(p) \quad and \quad \bar{v}'_{2+}(p)=\bar{v}'_{3+}(p). \qquad (36)$$

From equation (32) and (33), we write

$$\bar{v}_{2+}(p)-\bar{v}_{2+}(-p)=\frac{\coth\varphi_2\delta}{\varphi_2}\left[\bar{v}'_{2+}(p)-\bar{v}'_{2+}(-p)-\frac{2ipA\varphi_{2N}\sin\varphi_{2N}\delta}{p^2-k_{1N}^2}\right]$$

$$-\frac{2ipA\cos\varphi_{2N}\delta}{p^2-k_{1N}^2} \qquad (37)$$

Now, eliminating $\bar{v}'_{2+}(p)$ from (25) and (37), we obtain

$$\frac{f_1(p)\bar{v}_{2+}(p)}{f_2(p)\sinh\varphi_2\delta}+\frac{iA\cos\varphi_{2N}\delta}{p+k_{1N}}$$

$$=\bar{v}_{2+}(-p)-\frac{f_1(p)\bar{v}_{2-}(p)}{f_2(p)\sinh\varphi_2\delta}-\frac{\coth\varphi_2\delta}{\varphi_2}\bar{v}_{2-}(p)$$

$$-\frac{\coth\varphi_2\delta}{\varphi_2}\left[\bar{v}'_{2+}(-p)+\frac{2ipA\varphi_{2N}\sin\varphi_{2N}\delta}{p^2-k_{1N}^2}\right], \qquad (38)$$

where,

$$g_1(p)=\varphi_2\sinh\varphi_2 H+\mu\varphi_1\cosh\varphi_2 H, \qquad (39)$$

$$g_2(p)=\varphi_2\sinh\varphi_2 h+\mu\varphi_1\cosh\varphi_2 h. \qquad (40)$$

The equation (38) is the Wiener-Hopf type differential equation (Noble, 1958) whose solution will give $\bar{v}_{2+}(p)$.

Solution of Wiener-Hopf equation

For solution of equation (38), we factorize $\left(\frac{\varphi_2\delta}{\sinh\varphi_2\delta}\right)\frac{g_1(p)}{g_2(p)}$ as

$$\frac{\varphi_2\delta}{\sinh\varphi_2\delta}\frac{g_1(p)}{g_2(p)}=M_+(p)M_-(p). \tag{41}$$

Where

$$M_+(p)=M_-(-p)=\frac{L_+(p)}{J_+(p)}\prod_{n=1}^{\infty}\frac{(p+p_{1n})}{(p+p_{2n})}$$

According to infinite product theorem, we can write

$$\frac{\sinh\theta_2\delta}{\theta_2\delta}=\prod_{n=1}^{\infty}(p_n^2\delta_n^2+p^2\delta_n^2)=J(p)=J_+(p)J_-(p), \tag{42}$$

where $p_n^2\delta_n^2=1-k_n^2\delta_n^2,\ \ \delta_n=\delta/n\pi$.

If $p=\pm p_{1n}$ and $p=\pm p_{2n}$ are zeros of $g_1(p)$ and $g_2(p)$ respectively, we write

$$\frac{g_1(p)}{g_2(p)}=\prod_{n=1}^{\infty}\frac{(p^2-p_{1n}^2)}{(p^2-p_{2n}^2)}\frac{F_1(p)}{F_2(p)}, \tag{43}$$

where,

$$F_1(p)=\frac{g_1(p)}{\prod_{n=1}^{\infty}(p^2-p_{1n}^2)}$$

$$F_2(p)=\frac{g_2(p)}{\prod_{n=1}^{\infty}(p^2-p_{2n}^2)} \tag{44}$$

and $F_1(p)$ and $F_2(p)$ have no zeros. Also, we write

$$L(p)=\frac{F_1(p)}{F_2(p)}=L_+(p)L_-(p), \tag{45}$$

where,

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$$\log L_+(p) = \frac{1}{\pi}\int_0^\infty \frac{\theta_1 - \theta_2}{s - ip} ds - \frac{1}{\pi}\int_0^{k_1} \frac{\phi_1 - \phi_2}{s + p} ds - \frac{1}{\pi}\int_{k_1}^{k_2} \frac{ds}{s + p} \tag{46}$$

and

$$\tan\theta_1 = \frac{\mu\sqrt{(s^2 + k_1^2)}\cos H\sqrt{(s^2 + k_2^2)}}{\sqrt{(s^2 + k_2^2)}\sin H\sqrt{(s^2 + k_2^2)}} \tag{47}$$

$$\tan\theta_2 = \frac{\mu\sqrt{(s^2 + k_1^2)}\cosh\sqrt{(s^2 + k_2^2)}}{\sqrt{(s^2 + k_2^2)}\sinh\sqrt{(s^2 + k_2^2)}} \tag{48}$$

$$\tan\phi_1 = \frac{\mu\sqrt{(k_1^2 - s^2)}\cos H\sqrt{(k_2^2 - s^2)}}{\sqrt{(k_2^2 - s^2)}\sin H\sqrt{(k_2^2 - s^2)}} \tag{49}$$

$$\tan\phi_2 = \frac{\mu\sqrt{(k_1^2 - s^2)}\cosh\sqrt{(k_2^2 - s^2)}}{\sqrt{(k_2^2 - s^2)}\sinh\sqrt{(k_2^2 - s^2)}}. \tag{50}$$

Now, from (41), (43) and (45), we find

$$\frac{\varphi_2\delta}{\sinh\varphi_2\delta}\frac{g_1(p)}{g_2(p)} = \frac{L_+(p)L_-(p)}{J_+(p)J_-(p)}\prod_{n=1}^{\infty}\frac{(p^2 - p_{1n}^2)}{(p^2 - p_{2n}^2)} = M_+(p)M_-(p), \tag{51}$$

and

$$M_+(p) = M_-(-p) = \frac{L_+(p)}{J_+(p)}\prod_{n=1}^{\infty}\frac{(p + p_{1n})}{(p + p_{2n})}, \tag{52}$$

where $|M_+(p)| \to \sqrt{|p|}$, as $|p| \to \infty$.

We now decompose $\dfrac{\coth\varphi_2\delta}{\varphi_2\delta}$ as

$$\frac{\coth\varphi_2\delta}{\varphi_2\delta} = f_+(p) + f_-(p), \tag{53}$$

where,

$$f_{+}(p)=f_{-}(-p)=-\frac{1}{2k_2\delta(p+k_2)}+\sum_{n=1}^{\infty}\frac{1}{p_n\delta(p+ip_n)}. \qquad (54)$$

Using equation (38) and (51) in equation (35), we obtain

$$\frac{M_{+}(p)\bar{v}_{2+}(p)}{\delta\sqrt{p+k_2}}+\frac{iA\cos\varphi_{2N}\delta\sqrt{p-k_2}}{(p+k_{1N})M_{-}(p)}$$

$$+\frac{iA\theta_{2N}\sin\varphi_{2N}\delta\, f_{+}(p)\sqrt{p-k_2}}{(p+k_{1N})M_{-}(p)}$$

$$=\frac{\bar{v}_{2+}(-p)\sqrt{p-k_2}}{M_{-}(p)}-\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta\sqrt{p-k_2}\, f_{-}(p)}{(p+k_{1N})M_{-}(p)}$$

$$-\frac{f_{-}(p)}{M_{-}(p)}\left[\bar{v}_{2-}^{'}(p)+\bar{v}_{2+}^{'}(-p)+\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta}{p-k_{1N}}\right]\sqrt{p-k_2}$$

$$-\frac{f_{+}(p)}{M_{-}(p)}\left[\bar{v}_{2-}^{'}(p)+\bar{v}_{2+}^{'}(-p)+\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta}{p-k_{1N}}\right]\sqrt{p-k_2}. \qquad (55)$$

which can be expressed as

$$\frac{M_+(p)\bar{v}_{2+}(p)}{\delta\sqrt{p+k_2}}+\frac{iA\cos\varphi_{2N}\delta}{\delta(p-k_{1N})}\left[\frac{M_+(p)}{\sqrt{p+k_2}}-\frac{M_+(k_{1N})}{\sqrt{k_2+k_{1N}}}\right]$$

$$+\frac{iA\cos\varphi_{2N}\delta\sqrt{-k_2-k_{1N}}}{(p+k_{1N})M_-(-k_{1N})}+\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta\, f_-(-k_{1N})\sqrt{-k_2-k_{1N}}}{(p+k_{1N})M_-(-k_{1N})}$$

$$+A\varphi_{2N}\sin\varphi_{2N}\delta\sum_{n=1}^{\infty}\frac{\sqrt{-k_2-ip_n}}{p_n\delta(p+ip_n)K_-(-ip_n)(ip_n-k_{1N})}$$
$$+\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta\sqrt{-2k_2}}{2k_2\delta(p+k_2)M_-(-k_2)(k_2-k_{1N})}$$
$$-\frac{\sqrt{-p^2k_2}}{2k_2\delta(p+k_2)}\left[\bar{v}_{2-}'(-k_2)+\bar{v}_{2+}'(k_2)-\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta}{k_2+k_{1N}}\right]\times\frac{1}{M_-(-k_2)}$$
$$-\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta}{p+k_{1N}}\left[\frac{1}{2k_2\delta(k_2-k_{1N})}-\sum_{n=1}^{\infty}\frac{i}{p_n\delta(-k_{1N}+ip_n)}\right]\frac{\sqrt{-k_2-k_{1N}}}{M_-(-k_{1N})}$$
$$+\sum_{n=1}^{\infty}\frac{i\sqrt{-k_2-ip_n}}{p_n\delta(p+ip_n)}\left[\bar{v}_{2-}'(-ip_n)+\bar{v}_{2+}'(ip_n)-\frac{iA\varphi_{2N}\sin\varphi_{2N}\delta}{ip_n+k_{1N}}\right]\frac{1}{M_-(-ip_n)}$$

$$=O_-(p). \qquad (56)$$

In equation (56), $O_-(p)$ include the terms which are analytic in $\beta<\tau$ and left hand member of above equation is analytic in the region $\beta>-\tau$. Therefore by analytic continuation each member tends to zero in its region of analyticity as $|p|\to\infty$. Hence by Liouville's theorem, the entire function is identically zero. So equating to zero the left hand side of equation (56), we obtain

$$\bar{v}_{2^+}(p)=\left[\frac{iA\cos\varphi_{2N}\delta M_+(k_{1N})\sqrt{p+k_2}}{(p-k_{1N})\sqrt{k_2+k_{1N}}}\right.$$
$$+\frac{A\varphi_{2N}\sin\varphi_{2N}\delta\sqrt{k_2+k_{1N}}}{(p+k_{1N})M_+(k_{1N})}.\delta\sqrt{p+k_2U}$$

$$\left.+\frac{iW}{\sqrt{p+k_2}\sqrt{2k_2}}.\frac{1}{M_+(k_2)}+\sqrt{p+k_2}\sum_{n=1}^{\infty}\frac{\sqrt{k_2+ip_n}}{p_n(p+ip_n)M_+(ip_n)}V\right]$$

$$\times \frac{1}{M_{+}(p)}, \qquad (57)$$

where,

$$U = \frac{\cot \varphi_{2N}\delta}{\varphi_{2N}} - \frac{1}{\delta(k_2^2 - k_{1N}^2)} + \sum_{n=1}^{\infty} \frac{2}{\delta(p_n^2 + k_{1N}^2)}, \qquad (58)$$

$$V = \bar{v}_{2^-}'(-ip_n) + \bar{v}_{2^+}'(ip_n) - \frac{2Ap_n\varphi_{2N}\sin\varphi_{2N}\delta}{p_n^2 - k_{1N}^2}, \qquad (59)$$

$$W = \bar{v}_{2^-}'(-k_2) + \bar{v}_{2^+}'(k_2) - \frac{2iAk_2\varphi_{2N}\sin\varphi_{2N}\delta}{k_2^2 - k_{1N}^2}. \qquad (60)$$

The displacement $v_2(x,z)$ is obtained by inversion of Fourier transform as given below

$$v_2(x,z) = \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \bar{v}_2(p,z)e^{-ipx}dp$$

$$= \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \frac{-1}{\varphi_2}\left[\frac{\varphi_2\cosh\varphi_2 z - \mu\varphi_1\sinh\varphi_2 z}{\varphi_2\sinh\varphi_2 h + \mu\varphi_1\cosh\varphi_2 h}\right] \times\left[\bar{v}_{2^+}'(p) + \bar{v}_{2^-}'(p)\right]e^{-ipx}dp \qquad (61)$$

where $\bar{v}_{2^+}(p)$ is given in equation (61) .

The Scattered Waves

The incident Love waves are scattered when these waves encounter with surface irregularities like rigid plane boundary in the crustal layer of earth. For finding the scattered component of the incident Love waves, we evaluate the integral in equation (61). There is a branch point $p = -k_1$ in the lower half-plane. We put $p = -k_1 - it$; t being small. The branch cut is obtained by taking $\mathrm{Re}(\varphi_1) = 0$. Now, $\varphi_1^2 = p^2 - k_1^2$ gives $\varphi_1 = \pm i\bar{\varphi}_1$ and $\varphi_2 = \bar{\varphi}_2$ .The contour of integration is shown in figure 2. The imaginary part of φ_1 has different signs on two sides of the branch cut.

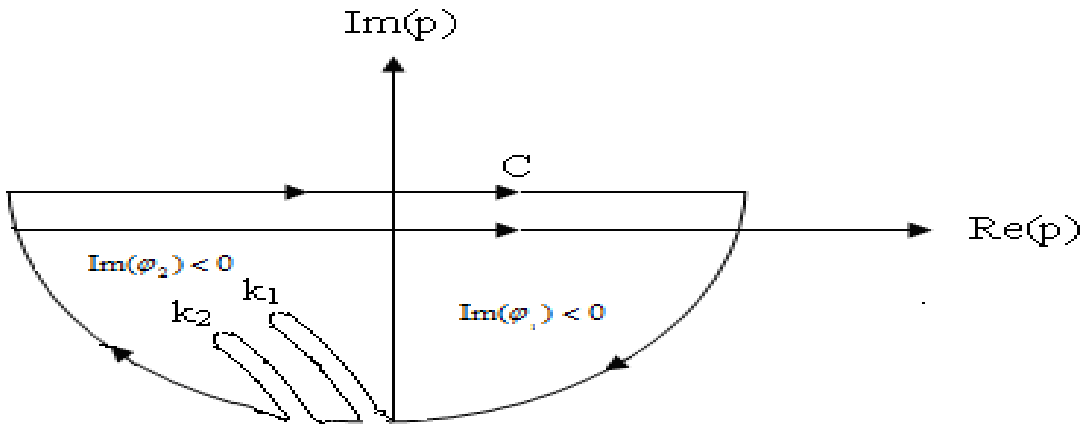

Figure 2 : The contour of integration in complex p-plane

Now integrating equation (61) along two sides of branch cut, we get

$$v_{2,1}(x,z) = \frac{i}{2\pi}\int_0^{\infty}[\{\bar{v}_2\,(p,z)\}_{\varphi_1 = i\bar{\varphi}_1} - \{\bar{v}_2\,(p,z)\}_{\varphi_1 = -i\bar{\varphi}_1}]e^{-xk_1''}e^{-sx}ds$$

$$= \frac{i}{\pi}\left[\int^{\infty}\frac{\zeta(t)\mu(2k_1''t)^{1/2}\sinh\bar{\varphi}_2(z+H)}{\bar{\varphi}_2^{2}\sinh^2\bar{\varphi}_2 H + \mu^2\bar{\theta}_1^2\cosh^2\bar{\varphi}_2 H}\right]\times e^{-k_1''x}e^{-sx}ds$$

$$= e^{-k_1''x}\int_0^{\infty} t^{1/2}Q(t)\,e^{-sx}ds, \qquad (62)$$

where,

$$Q(t) = \frac{i\mu(2k_1'')^{1/2}}{\pi}\left[\frac{\zeta(t)\cosh\bar{\varphi}_2(z+H)}{\bar{\varphi}_2^2\sinh^2\bar{\varphi}_2\,H + \mu^2\overline{\varphi_1^2}\cosh^2\bar{\varphi}_2\,H}\right] \qquad (63)$$

and

$$\zeta(t)=\left[\frac{U}{(2k_2)^{1/2}(k_1+k_2+is)M_+(k_2)}-\sum_{n=1}^{\infty}\frac{i(k_2+ip_n)^{1/2}V}{p_n(k_1+ip_n+is)M_+(ip_n)}-\frac{iAW\delta\sin\varphi_{2N}\delta}{(k_1+k_{1N}+is)M_+(k_{1N})}(k_2+k_{1N})^{1/2}\right]$$

$$\times\frac{(k_1+iu+k_2)^{1/2}}{M_-(-k_1-iu)} \qquad (64)$$

Now, the integral $\int_0^{\infty}s^{1/2}Q(s)e^{-sx}ds$ can be evaluated using the result of Ewing et al. (1957), we write

$$\int_0^{\infty}s^{1/2}Q(s)e^{-sx}ds=\frac{Q(0)\Gamma(3/2)}{x^{3/2}}+\frac{Q'(0)\Gamma(5/2)}{x^{5/2}}+\frac{Q''(0)\Gamma(7/2)}{x^{7/2}}+......... \qquad (65)$$

where $\Gamma(x)$ is Gamma function. Using equation (65), equation (62)

$$e^{-k_1''x}\int_0^{\infty}s^{1/2}Q(s)e^{-sx}ds=\left[\frac{Q(0)\Gamma(3/2)}{x^{3/2}}+\frac{Q'(0)\Gamma(5/2)}{x^{5/2}}+\frac{Q''(0)\Gamma(7/2)}{x^{7/2}}+..........\right]e^{-k_1''x} \qquad (66)$$

Neglecting first and higher order derivatives of $Q(s)$ at $s=0$. Equation (63) becomes written as

$$v_{2,1}(x,z)=\frac{Q(0)\Gamma(3/2)}{x^{3/2}}e^{-k_1''x} \qquad (67)$$

where,

$$Q(0)=\frac{\mu(2k_1'')^{1/2}}{\pi}\left[\frac{\xi(0)\cos\varphi_2''(z+H)}{\varphi_2''^2\sin^2\varphi_2''H}\right] \qquad (68)$$

and $\varphi_2''=\sqrt{k_2^2-k_1^2}$. Hence, we have

$$v_{2,1}(x,z)=\frac{-\mu(2k_1'')^{1/2}}{2\sqrt{\pi}x^{3/2}}\left[\frac{\zeta(0)\cos\varphi_2''(z+H)}{\varphi_2''^2\sin^2\varphi_2''H}\right]e^{-k_1''x} \qquad (69)$$

The equation (69) represents the scattered waves in the lower half of the plane due to the presence of plane boundary $-H \le z \le -h, \ \ x = 0$ in the crustal layer $-H \le z \le 0$.

The Reflected and Transmitted Waves

The incident Love waves are not only scattered but they are reflected also by the surface irregularity. For finding the reflected component, we evaluate the integral in equation (58) in upper half plane when $x < 0$. There is a pole at $p = k_{1N}$ and the corresponding wave is given as

$$v_{2,2}(x,z) = A_n \cos\varphi_{2N}(z+H)e^{-ik_{1N}x}, \qquad x<0, \ -H \le z \le -h, \qquad (70)$$

where,

$$A_n = \left[\frac{U}{\delta(2k_2)^{1/2}(k_{1N}-k_2)^{1/2}M_+(k_2)} + \sum_{n=1}^{\infty} \frac{i(k_2+ip_n)^{1/2}(k_{1N}-k_2)^{1/2}}{p_n\delta(k_{1N}-ip_n)} V \right]$$

$$\times \frac{\sin\varphi_{2N}\delta}{\cos\varphi_{2N}H\left[\frac{d}{dp}g_1(p)\right]_{p=k_{1N}}} \qquad (71)$$

and

$$\left[\frac{d}{dp}g_1(p)\right]_{p=k_{1N}} = k_{1N}\left[\frac{\varphi_{1N}H+\mu}{\varphi_{1N}}\cos\varphi_{2N}H + \frac{1+\mu\varphi_{1N}H}{\varphi_{2N}}\sin\varphi_{2N}H\right] \qquad (72)$$

The amplitude of reflected Love wave is given by taking modulus of A_n. Now we evaluate the integral (61) in lower half-plane when $x > 0$. We see that $p = -ip_n$ are the zeros of $\sinh\theta_2\delta$ and therefore, $\frac{1}{M_+(-ip_n)} = 0$. Also, $\frac{1}{M_-(ip_n)} = 0$ in upper half-plane where $x < 0$. Therefore, $p = \pm ip_n$ are not poles of the integrand and the residue due to the pole at $p = -k_{1N}$ contributes to

$$v_{2,3}(x,z) = B_n \cos\varphi_{2N}(z+H)e^{ik_{1N}x}, \quad x>0, \ -H \le z \le 0 \qquad (73)$$

where,

$$B_n = \left[-\frac{U}{\delta(2k_2)^{1/2}(k_2-k_{1N})^{1/2}M_+(k_2)} \quad -\sum_{n=1}^{\infty}\frac{i(k_2+ip_n)^{1/2}(k_2-k_{1N})^{1/2}}{p_n\delta(ip_n-k_{1N})}V \right]$$

$$\times\frac{\sin\varphi_{2N}\delta}{\cos\varphi_{2N}H\left[\frac{d}{dp}g_1(p)\right]_{p=-k_{1N}}} \qquad (71)$$

and

$$\left[\frac{d}{dp}g_1(p)\right]_{p=-k_{1N}} = -\left[\frac{d}{dp}g_1(p)\right]_{p=k_{1N}}. \qquad (74)$$

Equation (73) gives the transmitted waves and their amplitudes are given by taking the modulus of $B_{m\cdot}$

We again evaluate the integral in equation (61) in upper half plane when $x<0$. Let p_{2n} be the roots of the equation

$$g_2(p) = \varphi_2 \sinh\varphi_2 h + \mu\varphi_1 \cosh\varphi_2 h = 0 \qquad (75)$$

$$\text{i.e.} \qquad \tan\varphi'_{2n}h = \mu\frac{\varphi'_{1n}}{\varphi'_{2n}}, \qquad (76)$$

where,

$$\varphi'_{2n} = \sqrt{k_2^2 - p_{2n}^2}, \qquad \varphi'_{1n} = \sqrt{p_{2n}^2 - k_1^2} \qquad (77)$$

There is a pole at $p=-p_{2n}$, the residue due to which contributes to

$$v_{2,2}(x,z) = C_n \sin\varphi'_{2n}(z+h)e^{-ip_{2n}x}, \qquad x<0,\ -h\le z\le 0, \qquad (78)$$

where,

$$C_n = \left[\sum_{n=1}^{\infty} \frac{iU}{(2k_2)^{1/2}(p_{2n}+k_2)^{1/2} M_+(k_2)} + \sum_{n=1}^{\infty} \frac{(k_2+ip_n)^{1/2}(k_2+p_{2n})^{1/2} V}{p_n(p_{2n}+ip_n) M_+(ip_n)} - \sum_{n=1}^{\infty} \frac{JA\delta\varphi_{2N} \sin\varphi_{2N} \delta(k_2-k_{1N})^{1/2}(k_2+p_{2n})^{1/2}}{p_n(p_{2n}+K_{1N}) M_+(k_{1N})} \right] \times \frac{\mu\varphi_1^{'}}{M_+(p_{2n})\cos\varphi_{2n}^{'} h \left[\frac{d}{dp} g_2(p) \right]_{p=p_{2n}}} \qquad (79)$$

and

$$\left[\frac{d}{dp} f_2(p) \right]_{p=p_{2n}} = k_{1N} \left[\frac{\varphi_{1n}^{'} h + \mu}{\varphi_{1n}^{'}} \cos\varphi_{2n}^{'} h + \frac{1+\mu\varphi_{1n}^{'}}{\varphi_{2n}^{'}} \sin\varphi_{2n}^{'} h \right] p_{2n}. \qquad (80)$$

The equation (78) represents the reflected Love waves of n^{th} mode in the surface layer with thickness h. These are absent on the boundary z=-h. If h=0 i.e. there is no surface barrier, then the reflected Love waves propagate with a velocity that is equal to that of shear waves in half space.

Numerical Computations and Discussion of Results

The incident Love waves are scattered as well as reflected due to the presence of rigid plane boundary in the surface layer of earth. The scattered Love waves given in the equation (69) are of the form $\frac{e^{-k_1^{''}x}}{x^{3/2}}$, which shows that scattered waves decreases rapidly for large values of x and they behave as decaying cylindrical waves at the distant points. The scattered waves propagate with the speed of waves in the half space and not with that of waves in the layer. The scattered waves are also absent on the boundary $z=-h$. The numerical calculations are carried out by taking $h = 0.49 km.$, $H = 0.50 km.$, $\mu = 2$, $U_2/U_1 = 3/4, z = -H$

and taking $k_2\delta$ to be small. For scattered Waves, the graph showing the variation of amplitude versus the wave number of the scattered waves has been shown in figure 3.

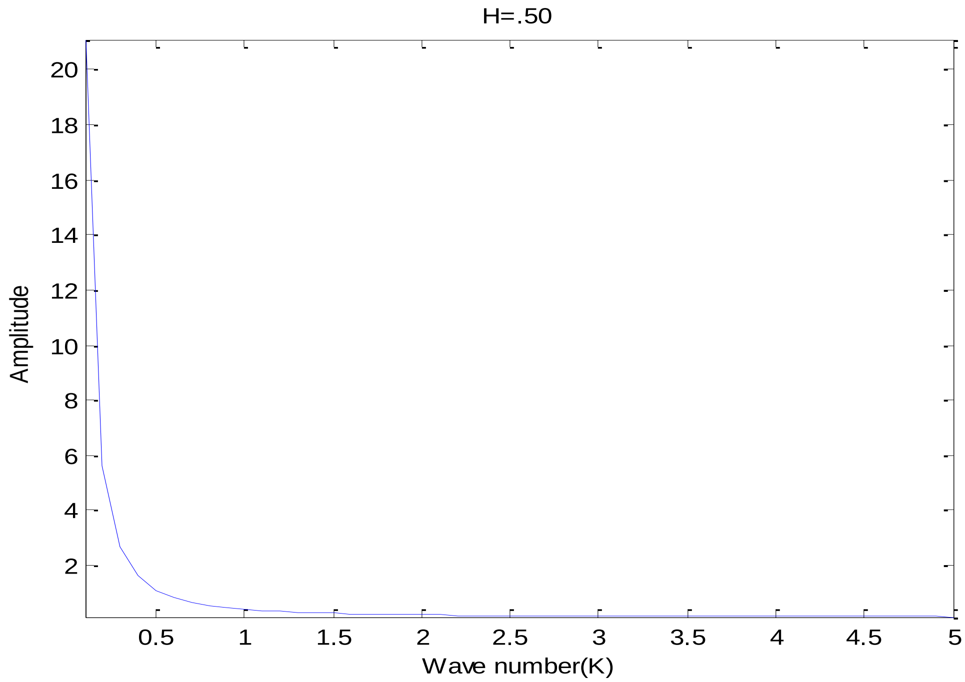

Figure 3:Variation of amplitude versus wave number k of scattered waves

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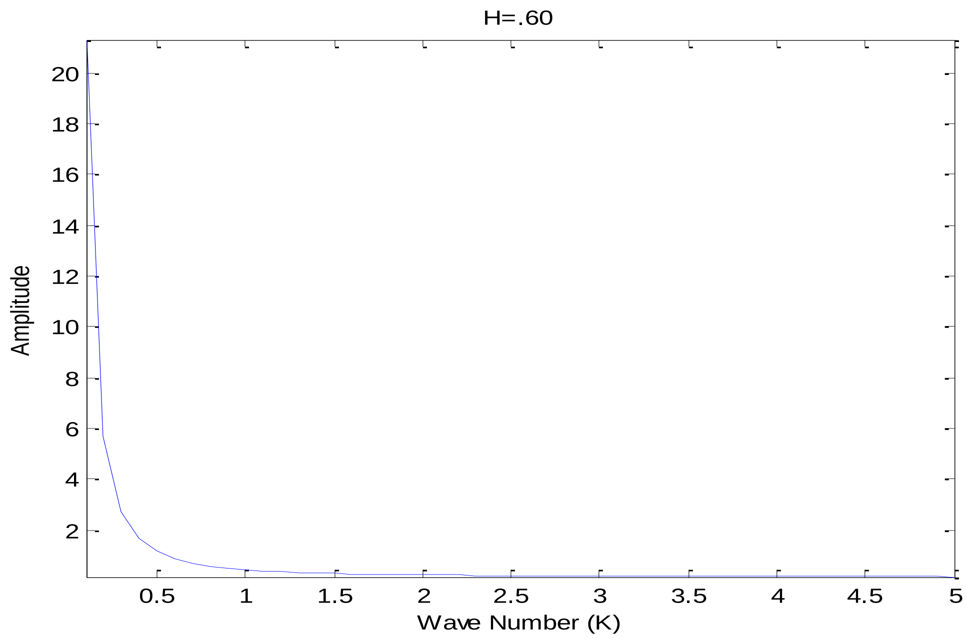

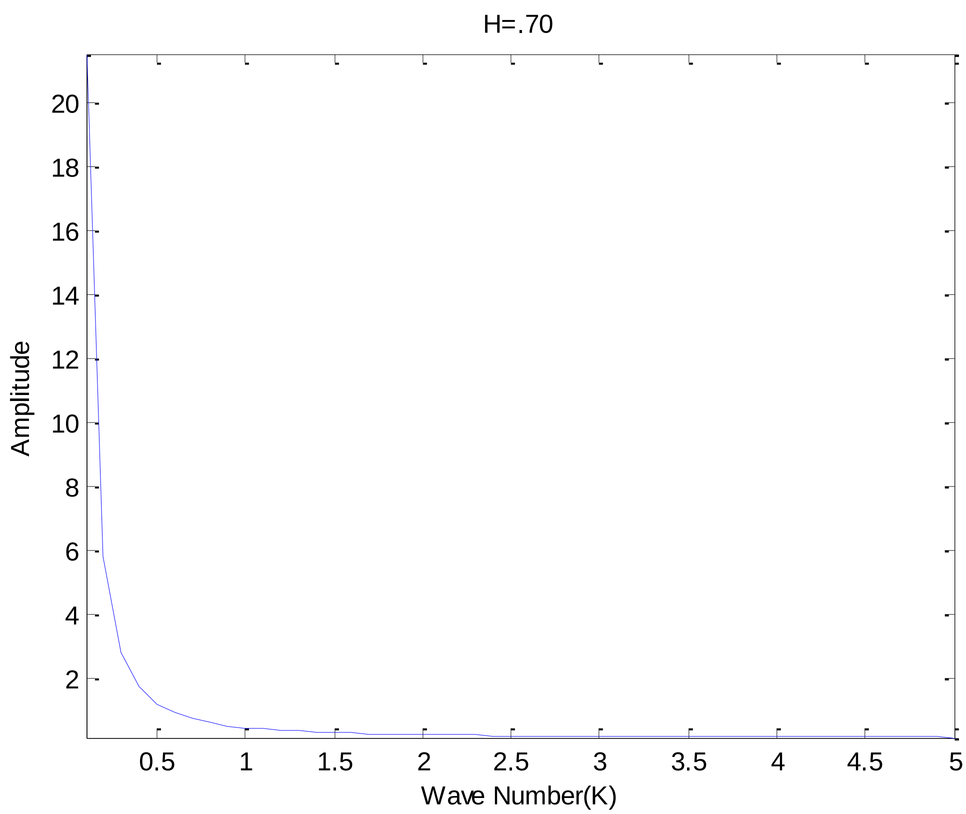

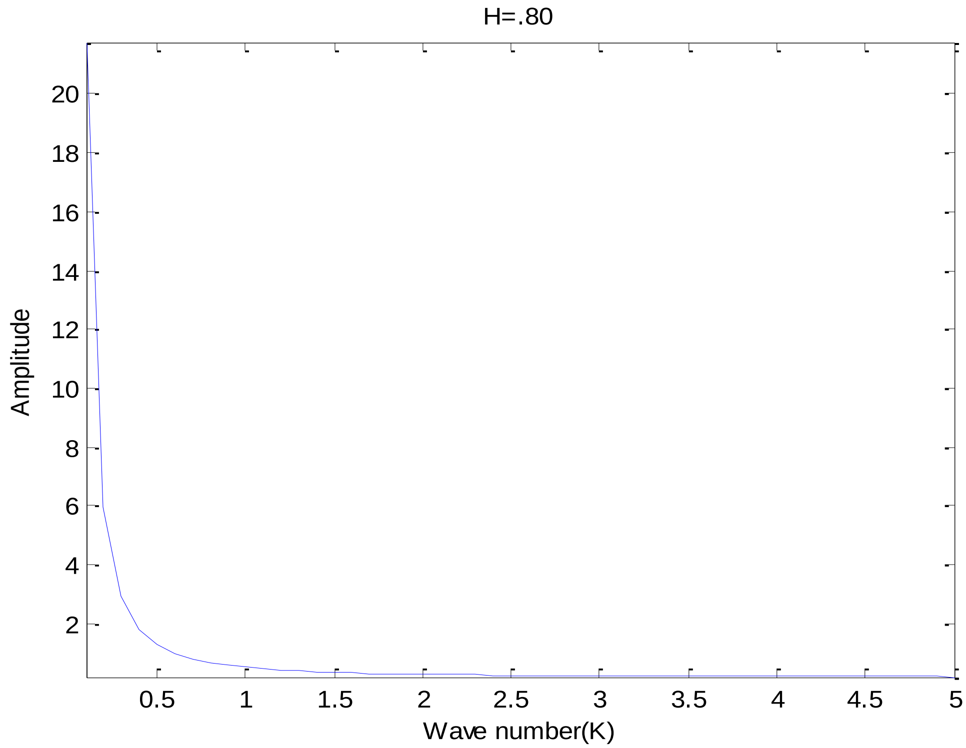
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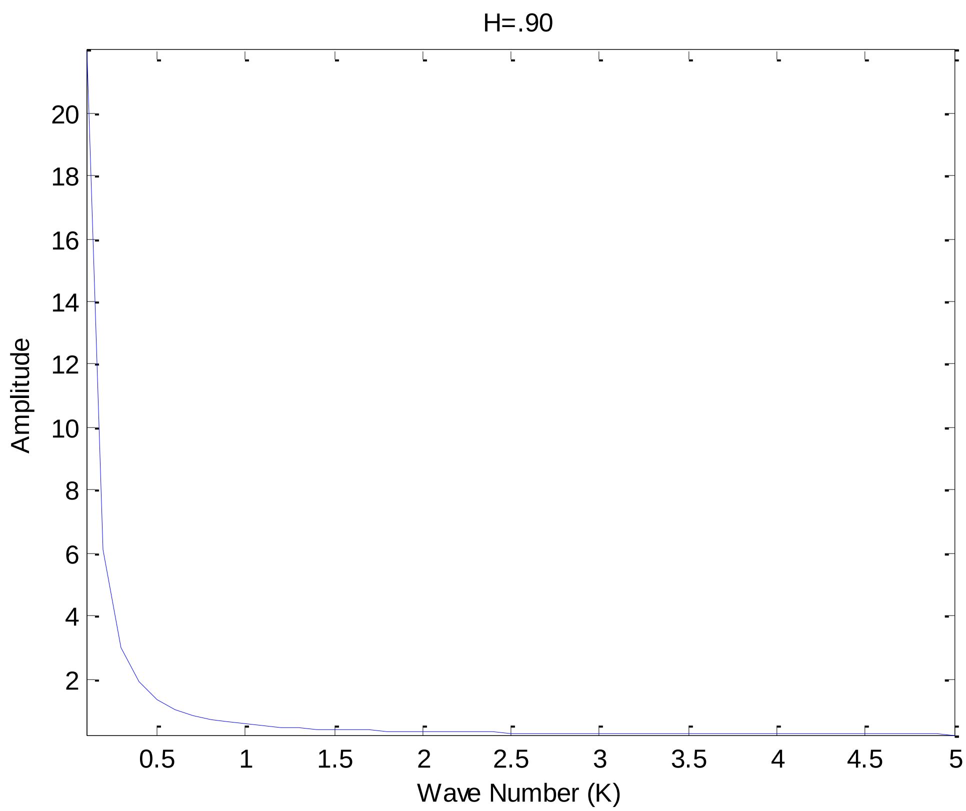
Amplitude
20
18
16
14
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Wave number(K)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Amplitude
20
18
16
14
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Wave Number (K)

The graph shows that the amplitude of the scattered waves depends on the wave number and hence on the wavelength of the scattered wave. The reflected and transmitted waves are given in equations (70) and (73) and their amplitudes are given in equations (71) and (74) respectively. For mathematical computation and graphical purpose, we have fixed h = 0.49 km. and graphs has been plotted by taking H = 0.50, 0.60, 0.70, 0.80, 0.90 km. The comparison of graphs shows that the amplitude of the scattered Love waves depends upon barrier-size to some extent.

Conclusions

Scattering of Love waves due to an irregular surface layer in the presence of infinite rigid boundary has been discussed. The Weiner-Hopf technique is applied to find the displacement and the result obtained is used to evaluate the scattered, reflected and transmitted waves in layer, when Love wave is incident on an irregular boundary in the form of infinite rigid plane boundary. The effect of the wave number of scattered waves is shown graphically. Variation of amplitude against wave number for different barrier is shown graphically. From the above discussion we conclude that:

1. The amplitude of the scattered waves decreases sharply with the slightly increment in the value of wave number which highlights that as the wave number increases, the amplitude decreases rapidly but approaches to zero after long interval of time. Due to this reason the scattered Love waves are considered one of the most destructive seismic waves during earthquake.
2. Amplitude of the reflected waves is significantly affected by the shape of irregularity.

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