Basics of Digital Filter and its Design Techniques

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ABSTRACT

A Brief view on the study, design and Evolution of digital filters is the focus of this paper. Every aspects related to the digital filters will be explained in this paper. Initiating from the basics of digital filters, we will go through the types of digital filters used in digital signal processing. Filter window also know as window function which is used in filter designing will also further discussed in the paper. The ambition of this paper is to provide dossier to the reader regarding all basic aspects of digital filters which are necessary part of signal processing.

Keywords: *Digital; processing; response filters*

INTRODUCTION

Digital Signal Processing (DSP) providesextreme flexibility, great performance (in terms of efficiency and selectivity) and strong environment stability.Digital signal processing technique has low equipment productioncosts than traditional analog techniques. Digital signal processors have been replacedmore complex and inefficient (in terms of production cost) microprocessor circuitry; an example of this isthe emergence of DSP in cellular base stations.

Digital Signal Processing consists of many levels and each level has variety of components, Digital filter is one of the important components used when digital signal has been processed.In signal processing, a filter is a device or process that removes some unwanted component or featurefrom a signal . Filtering is a class of signal processing; the feature of filters is the complete or partial suppression of some aspect of the signal. More often, this means removing some frequencies and not others in order to suppress interfering signals and reduce background noise.

According to the Fourier transform theory the linear convolution of two sequences in the time
Domain is the same as multiplication of two correspondingspectral sequences in the frequency domain. Thus Filtering is the multiplication of the signal spectrum by thefrequency domain impulse response of the filter.However, filters do not completely act in the frequency domain; especially in the field of image processing many other targets for filtering exist. Correlations can be removed for certain frequency components

and not for others without having to act in the frequency domain.

1. TRANSFER FUNCTION

A digital filter is characterized by its transfer function, or equivalently, its differential equation. Mathematical analysis of the transfer function can describe how it will respond to any input. As such, designing a filter consists of developing specifications appropriate to the problem (for example, a second-order low pass filter with a specific cut-off frequency), and then producing a transfer function which meets the specifications.

The transfer function for a linear, time-invariant, digital filter can be expressed as a transfer function in the:

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_N z^{-N}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$

.

The order of the filter is of *N* or *M*.

This is the form for a recursive filter with both the inputs (Numerator) and outputs (Denominator), which typically leads to an IIR infinite impulse response behavior but if the denominator made equal to unity i.e. no feedback, then this becomes an FIR or finite impulse response filter.

2. TYPES OF DIGITAL FILTERS

Digital filters used in Digital signal Processing are of two types one is infinite impulse response and second is finite impulse response. IIR and FIR filters are discussed below: -

2.1 Infinite Impulse Response

Infinite impulse response (IIR) is a property applying to many linear time-invariant systems. Common examples of linear time-invariant systems are most electronic and digital filters. Systems with this property are known as IIR systems or IIR filters, and are distinguished by having an impulse response which does not become exactly zero past a certain point, but continues indefinitely. This is in contrast to a finite impulse response in which the impulse response h(t) does become exactly zero at times t > T for some finite T, thus being of finite duration.

The transfer functions pertaining to IIR analog electronic filters have been extensively studied and optimized for their amplitude and phase characteristics. These continuous-time filter functions are described in the Laplace domain. Desired solutions can be transferred to the case of discrete-time filters whose transfer functions are expressed in the z domain, through the use of certain mathematical techniques such as the bilinear transform, impulse invariance, or pole–zero matching method.

Mathematically Infinite Response Filter is expressed as:-

IIR filters are recursive, with the output depending on both current and previous inputs as well as previous outputs. The general form of an IIR filter is thus:

$$\sum_{m=0}^{M-1} a_m y_{n-m} = \sum_{k=0}^{n-1} b_k x_{n-k}$$

2.2 Finite Impulse Response

In signal processing, a finite impulse response (FIR) filter is a filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time. This is in contrast to infinite impulse response (IIR) filters, which may have internal feedback and may continue to respond indefinitely (usually decaying).

Mathematically Finite Response Filter is expressed as:-

In the case of linear time-invariant FIR filters, the impulse response is exactly equal to the sequence of filter coefficients:

$$y_n = \sum_{k=0}^{n-1} h_k x_{n-k}$$

2.3 FIR v/s IIR

A FIR filter has a number of useful properties which sometimes make it preferable to an infinite impulse response (IIR) filter. FIR filters:

- Require no feedback. This means that any rounding errors are not compounded by summed iterations. The same relative error occurs in each calculation. This also makes implementation simpler.
- Are inherently stable, since the output is a sum of a finite number of finite multiples of the input values, so can be no greater than $\sum |b_i|$ times the largest value appearing in the input.

3. DESIGNING OF DIGITAL FILTERS

As we know digital filters are of two types one is Finite impulse response (FIR) and another is Infinite impulse response (IIR). Both FIR and IIR filters have different designing techniques and there are varieties of techniques which are available for design of digital filters. So let us discuss these techniques: -

3.1 Design Techniques of FIR Filters

There are various window functions which are used for FIR filter design. As we know Finite-duration impulse response filters (FIR) is based on frequency response specifications and the filter implementation is done by taking Fourier transform of the desired frequency response specifications.

Basically following are the types of window functions which are used in FIR filter designing.

- Rectangular window
- Bartlett (Triangular window)
- Blackmann window
- Hamming window
- Hanning window
- Kaiser window

Let us discuss some of the most commonly used window function for FIR filter designing:-

3.1.1 Rectangular Window

As we know there are different types of windows used to design FIR filter and Rectangular window is most widely used window for designing FIR filter. So let us discuss the design of FIR using Rectangular window. The rectangular window is denoted by W_R (n) and it is expressed as: -

$$w_R(n) \triangleq \begin{cases} 1, & |n| \leq \frac{M-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

Now, let us consider its Fourier transform: -

$$W_R(\omega) = M \cdot \text{asinc}_M(\omega) \triangleq \frac{\sin\left(M\frac{\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$$

The DTFT of a rectangular window is shown in Fig.3.1.

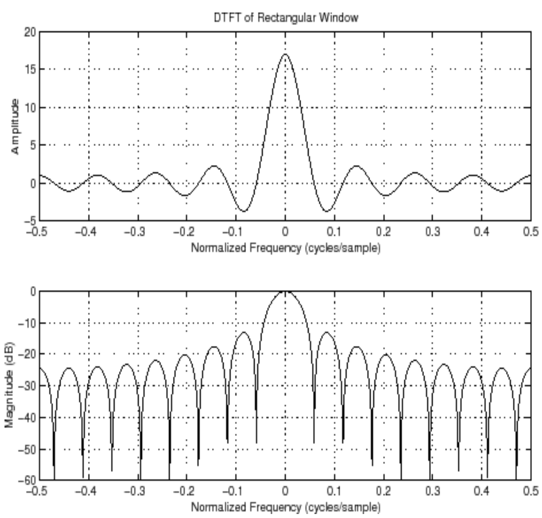


Figure 3.1: Rectangular window discrete-time Fourier transform.

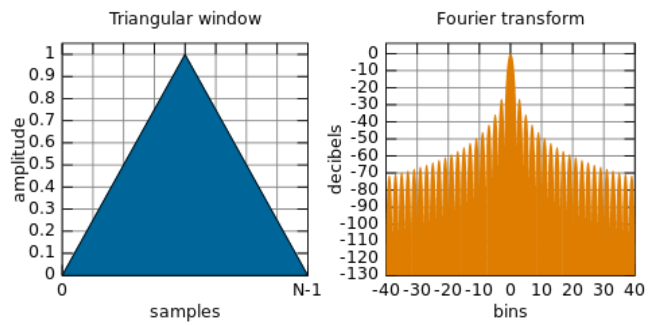
3.1.2 Triangular Window

The triangular window is denoted by W_T (n) and it is expressed as: -

$$w(n) = 1 - \left|\frac{n - \frac{N-1}{2}}{\frac{L}{2}}\right|,$$

Where L can be N N+1, or N-1 The last one is also known as Bartlett window.

Fourier transform of this window can be obtained in similar manner as discussed for rectangular window.

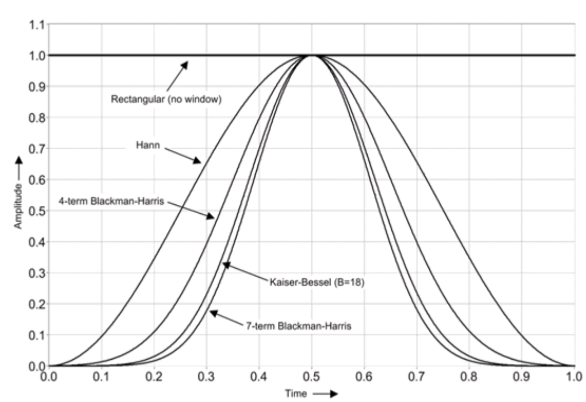


3.1.3 Blackmann Window

The blackmann window has a bell like shape of its time domain samples. It is expressed mathematically as: -

W_B (n)= $\{0.42-0.5\cos(2\pi n/(N-1)) + 0.08\cos(4\pi n/(N-1))\}$ For $0 \leq n \leq M-1$

whereM is N/2 for N even
and (N+1)/2 for N odd.



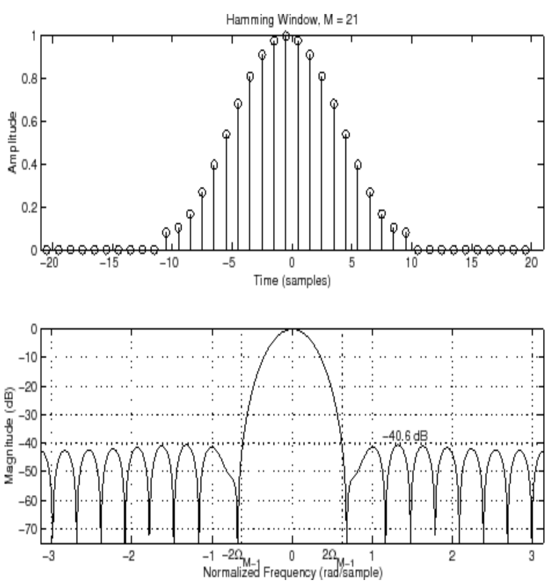
3.1.4 Hamming Window

Hamming is most commonly used window in speech processing and it is given as: -

$$W_H(n) = \left\{0.54-0.46\cos(2\pi n/N-1)\right\} \quad 0\leq n\leq M-1$$

where M is N/2 for N even and (N+1)/2 for N odd.

This window also has bell like shape. Its first and last samples are not zero.



3.2 Design Techniques of IIR Filters

In order to design the Infinite impulse response filter (IIR), first analog filter is designed. Then analog filter is converted into digital filter. Because of the following reasons we design IIR filter from analog filter: -

i. The procedure of designing analog filter is highly advanced.

ii. Implementation becomes simple.

Basically following methods are used in IIR filter designing.

- Impulse invariant method
- Approximation of derivatives
- Bilinear transformation

Let us discuss some of the most commonly used methods for IIR filter designing:-

3.2.1 Impulse Invariant Method

Impulse invariant is a technique for designing discrete-time IIR filter from continuous-time filters in which the impulse response of the continuous-time system is sampled to produce the impulse response of the discrete-time system.

The impulse-invariant method converts analog filter transfer functions to digital filter transfer functions in such a way that the impulse response is the same (invariant) at the sampling instants. Thus, if $\gamma(t)$ denotes the impulse-response of an analog (continuous-time) filter, then the digital (discrete-time) filter given by the impulse-invariant method will have impulse Response $\gamma(nT)$, where T denotes the sampling in seconds. Moreover, the order of the filter is preserved, and IIR analog filters map to IIR digital filters. However, the digital filter's frequency response is an aliased version of the analog filter's frequency response.

To derive the impulse-invariant method, we begin with the analog transfer function

$$\Gamma_a(s) \stackrel{\Delta}{=} \frac{B_a(s)}{A_a(s)} \stackrel{\Delta}{=} \frac{b_a(0)s^{N-1}+b_a(1)s^{N-2}+\cdots+b_a(N-2)s+b_a(N-1)}{s^N+a_a(1)s^{N-1}+\cdots+a_a(N-1)s+a_a(N)}$$

And perform a partial fraction expansion (PFE) down to first-order terms

$$\Gamma_a(s) \;=\; \sum_{i=1}^{N} \frac{K_i}{s-s_i},$$

Where s_i is the i the pole of the analog system, and K_i is its residue. Assume that the system is at least marginally stable so that there are no poles in the right-half plane ($\mathrm{re}\{s_i\} \leq 0$). Such a PFE is always possible when $\Gamma(s)$ is a strictly proper transfer function (more poles than zeros). Performing the inverse Laplace transform on the partial fraction expansion we obtain the impulse response in terms of the system poles and residues:

$$\gamma_a(t) \;=\; \sum_{i=1}^{N} K_i e^{s_i t}, \quad t \geq 0.$$

We now sample at intervals of T seconds to obtain the digital impulse response

$$\gamma_d(n) \stackrel{\Delta}{=} \gamma_a(nT) \;=\; \sum_{i=1}^{N} K_i e^{s_i nT}, \quad n = 0, 1$$

Taking the z transform gives the digital filter transfer function designed by the impulse-invariant method:

$$\Gamma_d(z) \;=\; \sum_{i=1}^{N} \frac{K_i}{1-e^{s_iT}z^{-1}} \;\stackrel{\Delta}{=}\; \frac{B_d(z)}{A_d(z)}$$

We see that the s -plane poles s_i have mapped to the z -plane poles

$$\boxed{z_i \;\stackrel{\Delta}{=}\; e^{s_iT}}$$

And the residues have remained unchanged. Clearly we must have $-\pi < \mathrm{im}\{s_i\}T < \pi$, i.e., the analog poles must lie within the bandwidth spanned by the digital sampling rate $f_s = 1/T$.

CONCLUSION

Digital filters are one of the most important parts of Digital Signal Processing (DSP) as it is used for removing some unwanted component from signal. With the instant advancement in digital technologies, digital filters began to offer valuable economical service to many of the filtering problem. However digital filters can perform such tasks which are not possible with analog filters.

Digital filters are playing crucial role in today's signal processing as noise can be easily introduced in our signal and we want to remove this noise, so digital filters are employed.

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