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A Solution of Fuzzy Trilevel Quadratic Fractional Programming Problem through Interactive Fuzzy Goal Programming Approach

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ABSTRACT:-*The purpose of this paper is to study the fuzzy Trilevel Quadratic Fractional Programming Problem (TLQFPP) through Fuzzy Goal Programming procedure. A TLQFPP is a special case of multilevel quadratic fractional programming problem with three levels and fuzzy TLQFPP contains fuzzy parameters as coefficients in its cost of objective function, the resources and the technological coefficients. In this paper, we are considering those fuzzy parameters as the triangular fuzzy numbers. Firstly, we are converting this type of fuzzy TLQFPP into a deterministic tri-objective TLQFPP by using Zadeh extension principle and then interactive fuzzy goal programming procedure is used to solve this equivalent deterministic Trilevel Triobjective Quadratic Fractional Programming Problem (TLTOQFPP) by using respective membership functions. An illustrative numerical example for fuzzy trilevel quadratic fractional programming problem is provided to reveal the applicability of the proposed method.*

Keywords*: Quadratic Programming, Fractional programming, Fuzzy Goal Programming, Triangular Fuzzy Numbers, Multilevel hierarchical optimization*

1. INTRODUCTION

The Trilevel programming problem is a class of multi-level programming problem (MLPP) which contains three decision makers. First level objective function is known as the upper level decision maker and second level objective function is considered as middle level decision maker and the third one is called as the lower level decision maker. Trilevel decision making optimization problem is a mathematical model set by the planner in which each level of a hierarchy has its own objective function and decision space which is not fully determined by itself but evaluated with the interference of other levels. In these types of problems, control tools of each level may enable him to impact the policies of other levels and as a result of that participation, it improves the objective function of each level. For example, in an executive board, decentralized firm, and top management, or headquarters, to build a decision such as a budget of any firm; each division governs a production planes by knowing a budget completely. In 1988, Anandalingam [2] studied mathematical programming model for decentralized bi-level programming problem (DBLPP) as well as MLPP based on Stackelberg solution concept. The multi-level fractional quadratic programming problems are special types of MLPPs. In which, the objective function of each level of MLPP is taken as the ratio of two quadratic functions. This type of model is very useful in bank balance sheet management, health care, finance corporate planning etc. Due to these applications, it attracts the keen interest of researchers in its theory. In a few decades earlier, the various researchers introduced many such problems and their solution procedures. Some important existing solution approaches for solving multilevel programming problems are such as the decent method, the extreme point search, the solution-procedure based on Karush-Kuhn Tucker (KKT) conditions, and many more. But these methods are not much successful to solve the various MLPPs rather than in solving simple types of multilevel programming problems. The concept of maximizing decision was introduced by Bellman and Zadeh [5] in fuzzy decision making problems. But Zimmermann [17] introduced firstly the use of fuzzy set theory in decision making optimization problems and theory of fuzzy linear programming was introduced by Tanaka et al. [14]. After that the various approaches were introduced in the literature of Bilevel Programming Problems as well as in Multilevel Programming Problems. Mainly Linear Tri-level programming has been studied by some researchers. Bard and Falk [4] firstly introduced the necessary conditions for the linear tri-level programming problem which was focused on Stackelberg game theory. After that, White [16] proposed a penalty function approach in which objective function of each level decision maker is optimized by forcing a constraint set. The Upper-level decision-maker produces his goals by specifying a constraint set, then the middle-level decision-maker reflexes an action within a constraint set determined by the action of the upper-level decision-maker, and finally the lower-level decision-maker produces an action within a constraint set determined by the actions of decision-makers at the upper-level and middle-level. Obviously, this approach conflict that lower level decision power dominates upper level decisions. Also as said earlier, there is some technical inefficiency in solving the optimization problems by using existing methods like KKT conditions or penalty functions based Multilevel Programming approaches. To overcome these inefficiencies, Lai [8] applied the concepts of membership function on such problems in 1996 and this concept was extended further by Shih et al [13], but this approach is lengthy one for solution procedure. To overcome this type of problem, the fuzzy goal programming approach (FGP) was proposed by Mohamed [9] and this approach was extended by Pramanik and Roy [12] to solve the multilevel linear programming problems. Pop and Stancu Minasian [11] solved the fully fuzzified linear fractional problems by representing all the variables and parameters with triangular fuzzy numbers. Baky [3] solved the various decentralized multiobjective programming problems by using the fuzzy goal programming approach. Also, Chang [6] recommended the goal programming approach for fuzzy multiobjective fractional programming problems. Pal and Gupta [10] studied the multi-objective fractional decision-making problems by formulating fuzzy goal programming with the help of a genetic algorithm. Abousina and Baky [1] suggested fuzzy goal programming procedure to solve bilevel multi-objective linear fractional programming problems. Lachhwani [7] also used fuzzy goal programming approach for multi-level linear fractional programming problems. Recently, C.Veeramany [15] used a method to solve fuzzy

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linear fractional programming problem by using Zadeh extension principle. Here, in this paper we are extending this approach of using Zadeh's extension principle for solving Fuzzy trilevel Quadratic Fractional Programming Problems (FTLQFPP). This method works according with three characteristic features which are usually applied in the various solution procedures of decision making problems. Firstly, the Fuzzy trilevel Quadratic Fractional Programming Problems (FTLQFPP) is converted into the deterministic Trilevel Triobjective Quadratic Fractional Programming Problem (TLTOQFPP) by using the Zadeh's Principle. Secondly, fuzzy goals are designated by each level decision maker in the form of fractional membership functions which are linearised further by using the Taylor series approach and finally, an interactive fuzzy goal programming procedure is adopted to solve TLTOQFPP.

SOME BASIC NOTATIONS

In this section, we are explaining the basic definitions of fuzzy sets, fuzzy numbers and membership functions which are given below:-

Definition 2.1.:-

A Fuzzy set $\tilde{F}_i$ on a real space R is a set of ordered pairs $\{(x, \mu_{\tilde{F}_i}(x)/x \in R)\}$, where $\mu_{\tilde{F}_i}(x): \rightarrow [0,1]$ is called as the membership function of fuzzy set.

Definition2.2.:-

A convex fuzzy set,$\tilde{F}_i$, on a real space R is a fuzzy set in which:

$\forall \quad x, y \in R, \forall \lambda \in [0,1]\ \mu\tilde{F}_i\,(\lambda x + (1-\lambda)y \geq min\,[\mu\tilde{F}_i\,(x), \mu\tilde{F}_i(y)]$.

Definition 2.3.:-

A fuzzy set $\tilde{F}_i$, on real space R, is called positive if its membership function is such that $\mu\tilde{F}_i(x) = 0, \forall x \leq 0$

Definition 2.4:-

A convex fuzzy set $\tilde{F}$ is called as triangular fuzzy number (TFN) if it can be defined as

$\tilde{F} = (x, \mu\tilde{F}_i\ (x))$ where: $\mu\tilde{F}_i = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \ , \\ 0, & otherwise \end{cases}$

For simplicity, we can represent TFN by three real parameters (a, b, c) which are $(a \leq b \leq c)$ will be denoted by the triangle a, b, c (Fig.1).

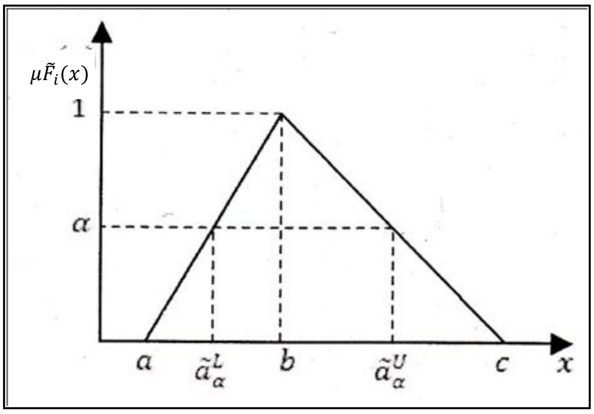

Fig.1: Triangular Fuzzy Number

Definition 2.5.:-

In any multilevel optimization problem, f_{ij} is objective function for decision maker of any level. Let f_{ij}^*, f_{ij}^{min} , f_{ij}^{max} are ideal, minimum and maximum values for f_{ij}. Then, the decision of any considered level can be formulated as follows:

Find x

So as to satisfy

$$f_{ij} \begin{pmatrix} \leq \\ \cong \\ \geq \end{pmatrix} f_{ij}^*$$

Subject to $x \epsilon S$

Where, f_{ij}^{*} is the perspective goal value for the objective function f_{ij}, $\begin{pmatrix} \leq \\ \cong \\ \geq \end{pmatrix}$ represents different fuzzy relations.

Let $(f_{ij}{}^{*}, f_{ij}^{max})$ be the tolerance interval selected to ij^{th} objective function f_{ij}. Thus membership function is defined as

$$\mu_{ij}\left(f_{ij}(x)\right) = \begin{cases} 1, & \text{if } f_{ij}(x) \leq f_{ij}{}^{*} \\ \frac{f_{ij}^{max} - f_{ij}}{f_{ij}^{max} - f_{ij}{}^{*}}, & f_{ij}{}^{*} \leq f_{ij}(x) \leq f_{ij}{}^{max} \\ 0, & f_{ij} \geq f_{ij}{}^{max} \end{cases}$$

Where $f_{ij}{}^{*}$ is called an ideal value and $f_{ij}{}^{max}$ is tolerance limit for f_{ij}

Similarly, Let $(f_{ij}{}^{min}, f_{ij}^{*})$ be the tolerance interval selected to ijth objective function f_{ij}. Thus membership function is defined as

$$\mu_{ij}\left(f_{ij}(x)\right) = \begin{cases} 1, & \text{if } f_{ij}(x) \geq f_{ij}{}^{*} \\ \frac{f_{ij} - f_{ij}{}^{min}}{f_{ij}{}^{*} - f_{ij}{}^{min}}, & f_{ij}{}^{min} \leq f_{ij}(x) \leq f_{ij}{}^{*} \\ 0, & f_{ij} \leq f_{ij}{}^{min} \end{cases}$$

Definition 2.6.:-

Membership functions are linearized by using Taylor series approach. The suggested procedure for fractional objectives can be continued as follow:

Obtain $\tilde{x}_i{}^{*} = (\tilde{x}_{i1}{}^{*}, \tilde{x}_{i2}{}^{*}, \dots \dots \dots, \tilde{x}_{ip_i}{}^{*})$ which is the value that is used to maximize the ij-th membership function $\mu_{ij}\left(f_{ij}(x)\right)$ associated with ij-th objective $f_{ij}(x)$.

$$\tilde{\mu}_{ij}\left(f_{ij}(x)\right) \cong \left[\left.\frac{\mu_{ij}\left(f_{ij}(\tilde{x}_i{}^{*})\right)}{\partial x_1}\right|_{\tilde{x}_i{}^{*}} (x_1 - \tilde{x}_{i1}{}^{*}) + \left.\frac{\mu_{ij}\left(f_{ij}(\tilde{x}_i{}^{*})\right)}{\partial x_2}\right|_{\tilde{x}_i{}^{*}} (x_2 - \tilde{x}_{i2}{}^{*}) + \dots \dots \dots \dots \dots \dots + \left.\frac{\mu_{ij}\left(f_{ij}(\tilde{x}_i{}^{*})\right)}{\partial x_m}\right|_{\tilde{x}_i{}^{*}} (x_m - \tilde{x}_{im}{}^{*}) \right]$$

Definition 2.7.:-

In trilevel programming problems there are three independent decision makers. Let $x_i \in R^n$ (n=1,2,3) be a vector variable which indicates the i^{th} decision variable and $f_i: R^n \to R^n$ be the i^{th} level objective function under the linear constraint vector $Ax(\leq, =, \geq)b$ which is a set of m equations and its right hand side has real or fuzzy variables. This type of programming problem is read as Tri-level Quadratic Programming Problem (TLQPP), and it can be formulated as following:-

$[1^{st}\ Level]$

$$\max_{x_1} f_1 = C_1 x + \frac{1}{2} x^T D_1 x$$

where x_1 solves and x_1 is vector of decision variable

$[2^{nd}\ Level]$

$$\max_{x_2} f_2 = C_2 x + \frac{1}{2} x^T D_2 x$$

where x_2 solves and x_2 is vector of decision variable

$[3^{rd}\ Level]$

$$\max_{x_3} f_3 = C_3x + \frac{1}{2}x^T D_3 x$$

subject to

$$Ax \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} b$$

$x \geq 0$ ……………………………….(1)

where $f_1 and\ f_2$ are objective functions of the first level decision maker (FLDM), and second level decision maker (SLDM) and f_3 is third level decision maker;

C_i are (1×3) matrices and D_i are 3×3 real matrices for $i = 1,2,3$. $A = (a_{pq})_{m \times n}$,matrix of coefficients and $b = (b_1, b_2, \dots \dots \dots, b_m)^T$. The first-level decision maker has control over $x_1 \in R^{n_1}$, and second-level decision maker has control over the $x_2 \in R^{n_2}$ and so on.

2. PROBLEM FORMULATION

We are considering a problem of trilevel quadratic programming whose each objective function has fractional form i.e. $f_i = \frac{f^N{}_i}{f^D{}_i}$ under the linear constraint vector $Ax(\leq, =, \geq)b$ which is a set of m equations. This type of programming problem can be stated as Trilevel Quadratic Fractional Programming Problem (TLQFPP), and it can be formulated as following:-

$[1^{st}$ Level$]$

$$\max_{x_1} f_1 = \frac{f^N{}_1}{f^D{}_1} = \frac{\breve{C}_{11}x + \frac{1}{2}x^T\breve{D}_{11}x}{\breve{C}_{12}x + \frac{1}{2}x^T\breve{D}_{12}x}$$

where x_1 solves and x_1 is vector of decision variable

$[2^{nd}$ Level$]$

$$\max_{x_2} f_2 = \frac{f^N{}_2}{f^D{}_2} = \frac{\breve{C}_{21}x + \frac{1}{2}x^T\breve{D}_{21}x}{\breve{C}_{22}x + \frac{1}{2}x^T\breve{D}_{22}x}$$

where x_2 solves and x_2 is vector of decision variable

$[3^{rd}$ Level$]$

$$\max_{x_3} f_3 = \frac{f^N{}_3}{f^D{}_3} = \frac{\breve{C}_{31}x + \frac{1}{2}x^T\breve{D}_{31}x}{\breve{C}_{32}x + \frac{1}{2}x^T\breve{D}_{32}x}$$

where x_3 solves and x_3 is vector of decision variable

subject to

$$\breve{A}x \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \breve{b}$$

$x \geq 0$

………………………………………………. (2)

where f_1, f_2, f_3 are objective functions of the first level decision maker (FLDM), second level decision maker (SLDM) and third level decision maker (TLDM) respectively; $\breve{C}_{ij}$ are matrices of order (1×3) and $\breve{D}_{ij}$ are 3×3 real matrices for $i = 1,2,3$ and $j = 1,2$. $\breve{A} = (\breve{a}_{pq})_{m \times n}$ is a matrix of coefficients and $\breve{b} = (\breve{b}_1, \breve{b}_2, \dots \dots \dots, \breve{b}_m)^T$. The matrices $\breve{C}_{ij}, \breve{D}_{ij}, \breve{A}$ and $\breve{b}$ contains triangular fuzzy numbers as their elements. The first-level decision maker has control over $x_1 \in R^{n_1}$, and second-level decision maker has control over $x_2 \in R^{n_2}$ and $x_3 \in R^{n_3}$.

3. TRANSFORMATION OF FUZZY TLQFPP INTO DETERMINISTIC FORM

By using Zadeh extension principle, we can transform the above mentioned fuzzy TLQFPP into a deterministic TLTOQPP in the following way:-

[1^{st} Level]

$$\max_{x_1}(f_{11}, f_{12}, f_{13}) = \left(\frac{C^1{}_{11}x+\frac{1}{2}x^T D^1{}_{11}x}{C^3{}_{12}x+\frac{1}{2}x^T D^3{}_{12}x}, \frac{C^2{}_{11}x+\frac{1}{2}x^T D^2{}_{11}x}{C^2{}_{12}x+\frac{1}{2}x^T D^2{}_{12}x}, \frac{C^3{}_{11}x+\frac{1}{2}x^T D^3{}_{11}x}{C^1{}_{12}x+\frac{1}{2}x^T D^1{}_{12}x}\right)$$

where x_1 solves and x_1 is vector of decision variable

[2^{nd} Level]

$$\max_{x_2}(f_{21}, f_{22}, f_{23}) = \left(\frac{C^1{}_{21}x+\frac{1}{2}x^T D^1{}_{21}x}{C^3{}_{22}x+\frac{1}{2}x^T D^3{}_{22}x}, \frac{C^2{}_{21}x+\frac{1}{2}x^T D^2{}_{21}x}{C^2{}_{22}x+\frac{1}{2}x^T D^2{}_{22}x}, \frac{C^3{}_{21}x+\frac{1}{2}x^T D^3{}_{21}x}{C^1{}_{22}x+\frac{1}{2}x^T D^1{}_{22}x}\right)$$

where x_2 solves and x_2 is vector of decision variable

[3^{rd} Level]

$$\max_{x_3}(f_{31}, f_{32}, f_{33}) = \left(\frac{C^1{}_{31}x+\frac{1}{2}x^T D^1{}_{31}x}{C^3{}_{32}x+\frac{1}{2}x^T D^3{}_{32}x}, \frac{C^2{}_{31}x+\frac{1}{2}x^T D^2{}_{31}x}{C^2{}_{32}x+\frac{1}{2}x^T D^2{}_{32}x}, \frac{C^3{}_{31}x+\frac{1}{2}x^T D^3{}_{31}x}{C^1{}_{32}x+\frac{1}{2}x^T D^1{}_{32}x}\right)$$

subject to

$$A^1x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^1, A^2x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^2, A^3x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^3$$

.... (3)

where f_{11}, f_{12}, f_{13} are objective functions of the first level decision maker (FLDM), f_{21}, f_{22}, f_{23} for second level decision maker (SLDM) and f_{31}, f_{32}, f_{33} for third level decision maker (TLDM); $C^1{}_{ij}, C^2{}_{ij}, C^3{}_{ij}$ are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{C}_{ij}$ respectively and $C^1{}_{ij}, C^2{}_{ij}, C^3{}_{ij}$ are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{D}_{ij}$ respectively for $i = 1,2,3$ and $j = 1,2$. A^1, A^2, A^3 are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{A}$ and b^1, b^2, b^3 are matrices formed by taking of first, second and third number as a deterministic from triangular fuzzy numbers of $\breve{b}$.

4. FUZZY GOAL PROGRAMMING PROCEDURE

In Trilevel multiobjective problems, if an imprecise aspiration level is assigned to each of the objectives in each level of Trilevel tri-objective quadratic fractional programming problem (TLMOQPP) then these fuzzy objectives are taken as the fuzzy goals and those goals are evaluated as the fractional membership functions by defining the tolerance limits for achievements of their aspiration levels.

5.1. Construction of fractional membership functions

Let $(x_1{}^{U_{ij}}, x_2{}^{U_{ij}}, x_3{}^{U_{ij}}, f_{ij}{}^{max})$ and $(x_1{}^{L_{ij}}, x_2{}^{L_{ij}}, x_3{}^{L_{ij}}, f_{ij}{}^{min})$ be the best and worst optimal solutions of each objective function f_{ij} of every decision maker over the region S, when solved individually. Then, the fuzzy goals appear as $f_{ij} \le f_{ij}{}^{max}$ and their respective membership functions can be defined as the following:-

$$\mu_{ij}\left(f_{ij}(x)\right) = \begin{cases} 1, & \text{if } f_{ij}(x) \ge f_{ij}{}^{max} \\ \dfrac{f_{ij} - f_{ij}^{min}}{f_{ij}^{max} - f_{ij}{}^{min}}, & f_{ij}{}^{min} \le f_{ij}(x) \le f_{ij}{}^{max} \\ 0, & f_{ij} \le f_{ij}{}^{min} \end{cases}$$

4.2.Linearization of fractional membership functions

Here, fractional membership functions associated with each objective function are linearized by using Taylor series approach. According to which, the fractional membership functions can be linearized at the neighborhood of the point of optimal solution $\left(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}}\right)$

$$\mu^*_{ij} = \mu_{f_{ij}}\left(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}}\right) + +\left(x_1 - x_1^{U_{ij}}\right)\frac{\partial \mu_{f_{ij}}}{\partial x_1}\left[\left(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}}\right)\right] + +\left(x_2 - x_2^{U_{ij}}\right)\frac{\partial \mu_{f_{ij}}}{\partial x_2}\left[\left(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}}\right)\right] + +\left(x_3 - x_3^{U_{ij}}\right)\frac{\partial \mu_{f_{ij}}}{\partial x_n}\left[\left(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}}\right)\right]$$

4.3.Construction of membership functions for decision variables

The tolerance of decision variable which is controlled by the upper level decision maker is used to find the satisfactory solution. Thus, it is required to construct membership function for those decision variables which are controlled by the upper level decision maker after getting the optimal solution of respective level. i.e. the optimal solution at ith level of TLMOPP is $\left(x_1{}^i, x_2{}^i, x_3{}^i\right)$ and it controls the decision variable x_j whose positive and negative tolerance limits are $t_j{}^R$ and $t_j{}^L$ respectively, then the linear membership function for this controlled variable can be defined as the following:-

$$\mu_{x_j}(x_j) = \begin{cases} \dfrac{x_j - \left(x_j^i - t_j{}^L\right)}{t_j{}^L}, & x_j^i - t_j{}^L \le x_j \le x_j^i \\ \dfrac{\left(x_j^i + t_j{}^R\right) - x_j}{t_j{}^R}, & x_j^i \le x_j \le x_j^i + t_j{}^L \\ 0, & \text{otherwise} \end{cases}$$

It may be noted that the decision maker may desire to shift the range of x_j.

4.4. Interactive Fuzzy Goal Programming Approach

Here, the fuzzy goal programming model given by Baky [1] is used to solve TLTOPP by constructing linear membership functions as explained in section 4.1-4.3. Thus by using the fuzzy goal programming procedure given by Baky, we can construct the fuzzy goal programming model for first level as the following:-

Find X so as to

$$\text{Min } Z = W^-{}_{11}\ d^-{}_{11} + W^-{}_{12}\ d^-{}_{12} + W^-{}_{13}\ d^-{}_{13}$$

and satisfy

$$\mu_{f_{11}} + d^-{}_{11} - d^+{}_{11} = 1,$$
$$\mu_{f_{12}} + d^-{}_{12} - d^+{}_{12} = 1,$$
$$\mu_{f_{13}} + d^-{}_{13} - d^+{}_{13} = 1$$

$$A^1x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^1, A^2x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^2, A^3x\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b^3$$

$d^-{}_{ij}\ , d^+{}_{ij} \ge 0$ with

$d^-{}_{ij}\ , d^+{}_{ij} = 0$, ………………………….. (4)

where $d^-{}_{ij}$ and $d^+{}_{ij}$ represents the upper and over deviational variables and

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$W^-{}_{11} = \frac{1}{f_{11}^{max} - f_{11}{}^{min}}$, $W^-{}_{12} = \frac{1}{f_{12}^{max} - f_{12}{}^{min}}$,

$$W^-{}_{13} = \frac{1}{f_{13}^{max} - f_{13}{}^{min}}$$

let the solution of FGP model (4) is $(x_1{}^1, x_2{}^1, x_3{}^1)$ and the value of each objective function f_{ij} at this point is $f_{ij}{}^1$, now we will find membership functions as $\mu_{f_{ij}} = \frac{f_{ij} - f_{ij}{}^1}{f_{ij1}^{max} - f_{ij}{}^1}$ and positive and negative tolerance limits are $t_1{}^R$ and $t_1{}^L$ respectively. Thus, the FGP model for second level can be described as the following:-

Find X so as to

$$Min\ Z = \sum_{k=1}^{3} W^-{}_{1k}\ d^-{}_{1k} + \sum_{k=1}^{3} W^-{}_{2k}\ d^-{}_{2k} + W^L{}_1\ (d^{L-}{}_1 + d^{L+}{}_1) + W^R{}_1\ (d^{R-}{}_1 + d^{R+}{}_1)$$

and satisfy

$$\mu_{f_{11}} + d^-{}_{11} - d^+{}_{11} = 1,$$

$$\mu_{f_{12}} + d^-{}_{12} - d^+{}_{12} = 1,$$

$$\mu_{f_{13}} + d^-{}_{13} - d^+{}_{13} = 1$$

$$\mu_{f_{21}} + d^-{}_{21} - d^+{}_{21} = 1,$$

$$\mu_{f_{22}} + d^-{}_{22} - d^+{}_{22} = 1,$$

$$\mu_{f_{23}} + d^-{}_{23} - d^+{}_{23} = 1$$

$$\frac{x_1 - (x_1^1 - t_1{}^L)}{t_1{}^L} + d^{L-}{}_1 - d^{L+}{}_1 = 1,$$

$$\frac{(x_1^1 + t_1{}^R) - x_1}{t_1{}^R} + d^{R-}{}_1 - d^{R+}{}_1 = 1$$

$$A^1 x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b^1, A^2 x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b^2, A^3 x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b^3$$

$d^-{}_{ij}$, $d^+{}_{ij} \ge 0$ with

$d^-{}_{ij}$, $d^+{}_{ij} = 0$, ...(5)

where $d^-{}_{ij}$ *and* $d^+{}_{ij}$ represents the upper and over deviational variables

and $W^-{}_{ij} = \frac{1}{f_{ij}^{max} - f_{ij}{}^1}$, $W^L{}_1 = \frac{1}{t_1{}^L}$, $W^R{}_1 = \frac{1}{t_1{}^R}$

let the solution of FGP model (5) is $(x_1{}^2, x_2{}^2, x_3{}^2)$ and the value of each objective function f_{ij} at this point is $f_{ij}{}^2$, now we will find membership functions as $\mu_{f_{ij}} = \frac{f_{ij} - f_{ij}{}^2}{f_{ij1}^{max} - f_{ij}{}^2}$ and new positive and negative tolerance limits for x_1 are $t_1{}^R$ and $t_1{}^L$ respectively; also consider the positive and negative tolerance limits for x_2 are $t_2{}^R$ and $t_2{}^L$ respectively. Thus, the FGP model for third level can be described as the following:-

Find X so as to

$$Min\ Z = \sum_{k=1}^{3} W^-{}_{1k}\ d^-{}_{1k} + \sum_{k=1}^{3} W^-{}_{2k}\ d^-{}_{2k} + \sum_{k=1}^{3} W^-{}_{3k}\ d^-{}_{3k} + \sum_{k=1}^{2} (W^L{}_k\ (d^{L-}{}_k + d^{L+}{}_k) + +W^R{}_k\ (d^{R-}{}_k + d^{R+}{}_k))$$

and satisfy

$$\mu_{f_{11}} + d^-{}_{11} - d^+{}_{11} = 1,$$

$$\mu_{f_{12}} + d^-{}_{12} - d^+{}_{12} = 1,$$

$$\mu_{f_{13}} + d^-{}_{13} - d^+{}_{13} = 1$$

$$\mu_{f_{21}} + d^-{}_{21} - d^+{}_{21} = 1,$$

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$\mu_{f_{22}} + d^{-}{}_{22} - d^{+}{}_{22} = 1,$

$\mu_{f_{23}} + d^{-}{}_{23} - d^{+}{}_{23} = 1$

$\mu_{f_{31}} + d^{-}{}_{31} - d^{+}{}_{31} = 1,$

$\mu_{f_{32}} + d^{-}{}_{32} - d^{+}{}_{32} = 1,$

$\mu_{f_{33}} + d^{-}{}_{33} - d^{+}{}_{33} = 1$

$$\frac{x_1 - (x_1^2 - t_1{}^L)}{t_1{}^L} + d^{L-}{}_1 - d^{L+}{}_1 = 1,$$

$$\frac{(x_1^2 + t_1{}^R) - x_1}{t_1{}^R} + d^{R-}{}_1 - d^{R+}{}_1 = 1$$

$$\frac{x_2 - (x_2^2 - t_2{}^L)}{t_2{}^L} + d^{L-}{}_2 - d^{L+}{}_2 = 1,$$

$$\frac{(x_2^2 + t_2{}^R) - x_2}{t_2{}^R} + d^{R-}{}_2 - d^{R+}{}_2 = 1$$

$$A^1 x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b^1, A^2 x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b^2, A^3 x \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b^3$$

$d^{-}{}_{ij}\ , d^{+}{}_{ij} \ge 0$ with

$d^{-}{}_{ij}\ , d^{+}{}_{ij} = 0,$ .... (6)

where $d^{-}{}_{ij}$ and $d^{+}{}_{ij}$ represents the upper and over deviational variables

and $W^{-}{}_{ij} = \frac{1}{f_{ij}^{max} - f_{ij}{}^2}$,

$$W^L{}_1 = \frac{1}{t_1{}^L}, W^R{}_1 = \frac{1}{t_1{}^R},$$

$$W^L{}_2 = \frac{1}{t_2{}^L}, W^R{}_2 = \frac{1}{t_2{}^R}$$

let the solution of FGP model (6) is $(x_1{}^3, x_2{}^3, x_3{}^3)$ and the value of each objective function f_{ij} at this point is $f_{ij}{}^3$, which is the required satisfactory solution.

5. NUMERICAL EXAMPLE

Consider the following Fuzzy Trilevel Quadratic Fractional Programming problem whose objective function of each level contains fractional function with numerator and denominator as quadratic one:-

<u>TLQFPP</u>

$[1^{st}$ Level]

$$\text{Max}_{x_1} f_1 = \frac{[6]x_2{}^2 + [4]x_2x_3 + [8]}{[5]x_3{}^2 + [8]x_1{}^2 + [12]}$$

$[2^{nd}$ Level]

$$\text{Max}_{x_2} f_2 = \frac{[2]x_1{}^2 + [4]x_1x_2 + [5]}{[2]x_3{}^2 + [10]x_1x_2 + [6]}$$

$[3^{rd}$ Level]

$$\text{Max}_{x_3} f_3 = \frac{[4]x_3{}^2 + [6]x_2x_1 + [2]x_2x_3 + [5]}{[2]x_2{}^2 + [10]}$$

$[3]x_1 + [5]x_2 - [2]x_3 \le [20],$

$[8]x_1 - [12]x_2 + [6]x_3 \le [10],$

$x_1, x_2, x_3 \ge 0$ ....(7)

Let us assume that the various fuzzy numbers used in above problem are

$[2] = (1,2,3), [3] = (1,3,5), [4] = (4,4,5)$

$[5] = (3,5,7), \quad [6] = (5,6,7), [8] = (7,8,9),$

$[10] = (8,10,12), [12] = (11,12,13),$

$[20] = (15,20,25)$

Thus, its equivalent deterministic multiobjective Trilevel quadratic fractional programming problem can be formed as the following:-

<u>TLTOQFPP</u>

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[1st Level]

$$\mathrm{Max}_{x_1} f_{11} = \frac{5x_2^2 + 4x_2x_3 + 7}{7x_3^2 + 9x_1^2 + 13},$$

$$f_{12} = \frac{6x_2^2 + 4x_2x_3 + 8}{5x_3^2 + 8x_1^2 + 12},$$

$$f_{13} = \frac{7x_2^2 + 5x_2x_3 + 9}{3x_3^2 + 7x_1^2 + 11}$$

[2nd Level]

$$\mathrm{Max}_{x_2} f_{21} = \frac{1x_1^2 + 4x_1x_2 + 3}{3x_3^2 + 12x_1x_2 + 7},$$

$$f_{22} = \frac{2x_1^2 + 4x_1x_2 + 5}{2x_3^2 + 10x_1x_2 + 6},$$

$$f_{23} = \frac{3x_1^2 + 5x_1x_2 + 7}{1x_3^2 + 8x_1x_2 + 5}$$

[3rd Level]

$$\mathrm{Max}_{x_3} f_{31} = \frac{4x_3^2 + 5x_2x_1 + 1x_2x_3 + 3}{3x_1^2 + 12},$$

$$f_{32} = \frac{4x_3^2 + 6x_2x_1 + 2x_2x_3 + 5}{2x_1^2 + 10},$$

$$f_{33} = \frac{5x_3^2 + 7x_2x_1 + 3x_2x_3 + 7}{1x_1^2 + 8}$$

$$1x_1 + 3x_2 - 1x_3 \le 15,$$

$$3x_1 + 5x_2 - 2x_3 \le 20,$$

$$5x_1 + 7x_2 - 3x_3 \le 25,$$

$$7x_1 - 11x_2 + 5x_3 \le 8,$$

$$8x_1 - 12x_2 + 6x_3 \le 10,$$

$$9x_1 - 13x_2 + 7x_3 \le 12,$$

$$x_1, x_2, x_3 \ge 0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; (8)$$

Optimize solution for each level decision maker of model (8), when taken individually, is Listed in the following table:-

Decision variables	f_{ij}^{max}	$(x_1^{U_{ij}}, x_2^{U_{ij}}, x_3^{U_{ij}})$	f_{ij}^{min}	$(x_1^{L_{ij}}, x_2^{L_{ij}}, x_3^{L_{ij}})$
f_{11}	5.88	(0,3.752,0.422)	0	(1.143,0,0)
f_{12}	7.06	(0,3.787,0.503)	0	(1.143,0,0)
f_{13}	9.51	(0,3.907,0.782)	0	(1.143,0,0)
f_{21}	0.62	(1.143,0,0)	0.0012	(0,14.625,28.875)
f_{22}	1.27	(1.143,0,0)	0.0030	(0,14.625,28.875)
f_{23}	2.18	(1.143,0,0)	0.0083	(0,14.625,28.875)
f_{31}	6.24	(0,4.42,9.93)	0.06	(0,3.57,0)
f_{32}	10.01	(0,5.77,12.42)	0.14	(0,3.57,0)
f_{33}	24.75	(0,9.54,19.43)	0.88	(0,0,0)

Table1:- optimized values of all objective functions at optimized points by taking them individually in the optimization procedure

Software LINGO 15 is used to find the optimize solution of each type of optimizing problem in this numerical example.

Firstly, we take the point $(0,3.752,0.422)$ at which the membership functions of first level are formed and thus we get the following FGP model for the first level as the following:-

$$\min = 0.17d^-_{11} + 0.13d^-_{12} + 0.10d^-_{13};$$
$$-0.006x_1 + 0.48x_2 - 0.21x_3 + d^-_{11} - d^+_{11} = 1.72;$$
$$-0.006x_1 + 0.53x_2 - 0.17x_3 + d^-_{12} - d^+_{12} = 1.82;$$

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$$-0.007x_1 + 0.51x_2 - 0.04x_3 + d^-_{13} - d^+_{13} = 1.85;$$
$$1x_1 + 3x_2 - 1x_3 \leq 15,$$
$$3x_1 + 5x_2 - 2x_3 \leq 20,$$
$$5x_1 + 7x_2 - 3x_3 \leq 25,$$
$$7x_1 - 11x_2 + 5x_3 \leq 8,$$
$$8x_1 - 12x_2 + 6x_3 \leq 10,$$
$$9x_1 - 13x_2 + 7x_3 \leq 12, \quad x_1, x_2, x_3 \geq 0$$

Its solution comes as $x_1 = 0$, $x_2 = 3.64$, $x_3 = 0.16$,

Thus, we find new membership functions for first and second level decision makers by using the values of decision variables as $(x_1, x_2, x_3) = (0,3.64,0.16)$ and we take positive limits for first decision variable as $0.5(t_1{}^R = 0.5)$ and forms the FGP model for second level decision maker as the following:-

$$\min = 0.17d^-_{11} + 0.13d^-_{12} + 0.10d^-_{13} + 1.62d^-_{21} + 0.79d^-_{22} + 0.46d^-_{23} + 2d^{R^-}{}_1 + 2d^{R^+}{}_1;$$
$$-0.007x_1 + 0.49x_2 + 0.038x_3 + d^-_{11} - d^+_{11} = 1.82;$$
$$-0.008x_1 + 0.53x_2 + 0.045x_3 + d^-_{12} - d^+_{12} = 1.89;$$
$$-0.008x_1 + 0.50x_2 + 0.097x_3 + d^-_{13} - d^+_{13} = 1.86;$$
$$-0.80x_1 + 0x_2 - 0.093x_3 + d^-_{21} - d^+_{21} = 0.30;$$
$$-1.80x_1 + 0x_2 - 0.069x_3 + d^-_{22} - d^+_{22} = 0.34;$$
$$-1.82x_1 + 0x_2 - 0.041x_3 + d^-_{23} - d^+_{23} = 0.36;$$

$$-2x_1 + d^{R^-}{}_1 - d^{R^+}{}_1 = 0;$$
$$1x_1 + 3x_2 - 1x_3 \leq 15,$$
$$3x_1 + 5x_2 - 2x_3 \leq 20,$$
$$5x_1 + 7x_2 - 3x_3 \leq 25,$$
$$7x_1 - 11x_2 + 5x_3 \leq 8,$$
$$8x_1 - 12x_2 + 6x_3 \leq 10,$$
$$9x_1 - 13x_2 + 7x_3 \leq 12, \quad x_1, x_2, x_3 \geq 0$$

Its solution comes as $x_1 = 0$, $x_2 = 3.57$, $x_3 = 0$,

Thus, we find new membership functions for first, second and third level decision makers by using the values of decision variables as $(x_1, x_2, x_3) = (0,3.57,0)$ and we take positive limits for first and second decision variable as 0.5 $(t_1{}^R = 0.5, t_2{}^R = 0.5)$ and forms the FGP model for third level decision maker as the following:-

$$\min = 0.17d^-_{11} + 0.13d^-_{12} + 0.10d^-_{13} + 1.62d^-_{21} + 0.79d^-_{22} + 0.46d^-_{23} + 0.16d^-_{31} + 0.10d^-_{32} + 0.04d^-_{33} + 2d^{R^-}{}_1 + 2d^{R^+}{}_1 + 2d^{R^-}{}_2 + 2d^{R^+}{}_2;$$
$$-0.007x_1 + 0.48x_2 + 0.19x_3 + d^-_{11} - d^+_{11} = 1.79;$$
$$-0.008x_1 + 0.52x_2 + 0.17x_3 + d^-_{12} - d^+_{12} = 1.86;$$
$$-0.008x_1 + 0.49x_2 + 0.18x_3 + d^-_{13} - d^+_{13} = 1.81;$$
$$-0.94x_1 + 0x_2 + 0x_3 + d^-_{21} - d^+_{21} = 0.31;$$
$$-2.03x_1 + 0x_2 + 0x_3 + d^-_{22} - d^+_{22} = 0.34;$$
$$-2.04x_1 + 0x_2 + 0x_3 + d^-_{23} - d^+_{23} = 0.36;$$
$$0.058x_1 - 0.0040x_2 + 0.011x_3 + d^-_{31} - d^+_{31} = 0.98;$$
$$0.061x_1 - 0.0060x_2 + 0.020x_3 + d^-_{32} - d^+_{32} = 0.97;$$
$$0.050x_1 - 0.0050x_2 + 0.022x_3 + d^-_{33} - d^+_{33} = 0.98;$$

$$-2x_1 + d^{R^-}{}_1 - d^{R^+}{}_1 = 0;$$

$-2x_2 + {d^{R}}^{-}{}_2 - {d^{R}}^{+}{}_2 = 0;$

$1x_1 + 3x_2 - 1x_3 \leq 15,$

$3x_1 + 5x_2 - 2x_3 \leq 20,$

$5x_1 + 7x_2 - 3x_3 \leq 25,$

$7x_1 - 11x_2 + 5x_3 \leq 8,$

$8x_1 - 12x_2 + 6x_3 \leq 10,$

$9x_1 - 13x_2 + 7x_3 \leq 12, \quad x_1, x_2, x_3 \geq 0$

Its solution comes as $x_1 = 0$, $x_2 = 0$, $x_3 = 1.60$,

Thus, we get the satisfactory solution for model (7) as the following:-

$x_1 = 0, \quad x_2 = 0, \quad x_3 = 1.60,$

6. CONCLUSION

In this paper, we solve the fuzzy Trilevel Quadratic Fractional Programming Problem by using interactive fuzzy goal programming procedure and this study can be extended to solve nonlinear multilevel and nonlinear multiobjective programming problems and also this study can be further applied by taking fuzzy parameters other than fuzzy triangular numbers. It is wished that the approach presented in this paper can contribute to future study of hierarchical optimization problems.

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